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Unsteady Axisymmetric Rotational Flow of Dusty Elastico-Viscous Liquid

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ABSTRACT

This paper reports the flow of elastico-viscous liquid embedded with particles in an oscillating cylinder. Explicit expressions are obtained for the velocities of liquid and dust particles by the technique of Laplace transforms. Numerical computations of the velocity fields are carried out for different values of mass concentration and relaxation time of the dust particles and varying elastic elements in the liquid.

1. INTRODUCTION

The study of rotational transient flow of non-Newtonian fluids in both semi-infinite field and bounded field is of practical need for certain industrial processes to have the description of fluid mechanical phenomena exhibited by non-Newtonian materials. Srivastava[1] and Tandon[2] analysed the propagation of small disturbances in an Oldroyd fluid contained in a semi-infinite circular cylinder due to the slow rotation of a disc at the base. Srivastava considered the radius of the disc to be same as the radius of the cylinder while Tandon has considered it to be smaller. Rao and Rao[3] have investigated the rectilinear oscillations of a circular cylinder about a mean position along a diameter in an infinitely extended micropolar fluid. Tandon and Chandra[4]

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have discussed the unsteady motion inside and outside of an infinite cylinder which suddenly starts rotating impulsively about an axis in an incompressible Oldroyd's two-parametric fluid, not three-parametric one as claimed by the authors. Recently Mukherjee and Mukherjee[5] have considered the unsteady axisymmetric rotational flow of elastico-viscous liquid due to the time-dependent rotation of a circular cylinder.

However, studies on dusty non-Newtonian fluid flows and rheological aspects of such flows have not received much attention though the studies of dusty non-Newtonian fluid flows are likely to have some industrial and chemical engineering applications on the problems of polluted oil extraction, polymer extrusion and paint spraying. Based upon the theoretical model proposed by Saffman[6], Srivastava[7] has analysed the unsteady flow of dusty Rivlin-Ericksen fluid through a channel. Bagchi and Maiti[8] have studied the unsteady flow of dusty elastico-viscous liquid through a channel with arbitrary time-varying pressure gradient.

This paper deals with the rotational flow of dusty elastico-viscous liquid. The expressions for the velocity fields of the liquid and the dust particles are obtained explicitly. The effect of elastic element in the liquid, the mass concentration and the relaxation time of dust particles on the velocity profiles of liquid and dust particles are studied graphically. This paper is likely to have some bearing on the problems of transport of solid particles suspended in non-Newtonian fluids through pipes.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider the flow of an incompressible elastico-viscous liquid (Kuvshinski[9] type) for which the stress-strain relation is

$$\left(1 + \lambda_0 \frac{D}{Dt}\right) p'_{ik} = 2\mu e_{ik} \tag{1}$$

where

$$\frac{D}{Dt} p'_{ik} = \frac{\partial p'_{ik}}{\partial t} + v_m \frac{\partial p'_{ik}}{\partial x_m} \tag{2}$$

and

$$p_{ik} = -p\delta_{ik} + p'_{ik} \tag{3}$$

p is the static pressure, δ_{ik} is the associated metric tensor and p'_{ik} is a tensor usually related to the rate of strain e_{ik} by the equation of state (1), λ_0 is the relaxation time and μ is the coefficient of viscosity.

In the present problem, it is assumed that the particles are spherical in shape and uniform in size and the bulk-concentration (concentration by volume) of dust is very small. Following Saffman, it is assumed that steady Stokes law of resistance between the particles and fluid is applicable. However the mass concentration of dust can be of the order of unity by allowing the ratio of the density of the dust and fluid to be large. For sufficiently small particles, the velocity of sedimentation will be small compared with a characteristic velocity of the flow and can be neglected.

The equations of motion of a dusty elastico-viscous liquid obeying Eqn. (1) are

$$\rho\left(\frac{\partial u_i}{\partial t} + u_k u_{i,k}\right) = p_{ik,k} + KN(v_i - u_i) \tag{4}$$

$$m\left(\frac{\partial v_i}{\partial t} + v_k v_{i,k}\right) = K(u_i - v_i) \tag{5}$$

$$u_{i,i} = 0 \tag{6}$$

$$\frac{\partial N}{\partial t} + (Nv_i)_{,i} = 0 \tag{7}$$

where u_i, v_i are the local velocity vectors of liquids and dust particles respectively, ρ the density, K the Stokes resistance coefficient (for spherical particles of radius d, it is $6\pi\mu d$), N the number density of dust particles and m the mass of a dust particle.

Initially the liquid and dust particles are at rest. We consider the flow of a dusty elastico-viscous liquid in an infinitely long circular cylinder of radius a which oscillates with constant frequency about the axis of the cylinder. In the cylindrical polar system of co-ordinates (r, θ, z), the z-axis is chosen along the axis of the cylinder. The physics of the problem suggests

$$u_i \equiv (O, u(r, t), O)$$

and

$$v_i \equiv (0, v(r, t), O)$$

Using Eqns. (I) - (7); we get the equations of motion of dusty elastico-viscous liquid as

$$\left(1 + \lambda_0 \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) + \frac{KN_0}{\rho}\left(1 + \lambda_0 \frac{\partial}{\partial t}\right)(v - u) \tag{8}$$

$$\frac{\partial v}{\partial t} = \frac{K}{m}(u - v) \tag{9}$$

where ν is the kinematic viscosity of the liquid and the number density of dust particles is $N = N_0$, a constant throughout the motion.

Initial and boundary conditions for the problem are

$$\left.\begin{aligned} u(r, t) &= \frac{\partial u(r, t)}{\partial t} = 0 \\ v(r, t) &= \frac{\partial v(r, t)}{\partial t} = 0 \end{aligned}\right\} \text{ at } t = 0 \text{ and for all } r \tag{10}$$

$$u = u_0 e^{-i\Omega t} \quad \text{on} \quad r = a \Big\} \; t > 0 \quad (11)$$
$$u \text{ is finite on } r = 0$$

where u_0 is the characteristic velocity and Ω is thd imposed oscillation.

Using the non-dimensional variables

$$\bar{u} = \frac{u}{u_0}, \; \bar{v} = \frac{v}{u_0}, \; \bar{r} = \frac{r}{a}, \; \bar{t} = \frac{t\nu}{a^2}, \; \bar{\Omega} = \frac{\Omega a^2}{\nu}$$

Eqns. (8) - (11) in non-dimensional form are written as (dropping bars)

$$\left(1 + \alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) + \beta\left(1 + \alpha\frac{\partial}{\partial t}\right)(v - u) \quad (12)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}(u - v) \quad (13)$$

$$u = \frac{\partial u}{\partial t} = 0; \; v = \frac{\partial v}{\partial t} = 0 \text{ at } t = 0 \text{ and for all } r \quad (14)$$

$$u = e^{-i\Omega t} \text{ on } r = 1; \text{ u is finite on } r = 0; \text{ and for } t > 0 \quad (15)$$

where a (= $\lambda_0 \nu/a^2$) nondimensional elastic parameter, f (=mN_0/ρ) the mass concentration of dust partides, and τ (= $m\nu/Ka^2$) the dimensionless relaxation time of dust particles and $\beta = (f/\tau)$.

3. SOLUTION OF THE PROBLEM

Using the Laplace transform technique in Eqns. (12) and (13) subject to initial and boundary conditions in Eqns. (14) and (15), it turns out that the expressions for the velocity profile of liquid and dust particles can be represented by the Laplace inversion integral in the form

$$u = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{1}{(p + i\Omega)} \frac{I_1\left[r\left\{\frac{p(1 + \alpha p)(1 + f + p\tau)}{(p\tau + 1)}\right\}^{1/2}\right]}{I_1\left[\left\{\frac{p(1 + \alpha p)(1 + f + p\tau)}{(p\tau + 1)}\right\}^{1/2}\right]} \cdot e^{pt} dp \quad (16)$$

$$v = \frac{1}{\tau} e^{-t/\tau} \int_0^t u(r, \lambda)\, e^{\lambda/\tau}\, d\lambda \quad (17)$$

where γ is greater than the real parts of the singularities of the integrand and $Re\,(p) > 0$.

On evaluating Eqns. (16) and (17), we have the expressions for the velocity profile of liquid as

$$u = \frac{I_1[rA]}{I_1[A]} e^{-i\Omega t} - 2\sum_n \sum_j \frac{\beta_n}{Q(p_{nj})} \frac{1}{(p_{nj} + i\Omega)} \times \frac{J_1(r\beta_n)}{J_0(\beta_n)} e^{p_{nj}t} \tag{18}$$

and of dust particles as

$$v = \frac{I_1[rA]}{I_1[A]} \frac{e^{-i\Omega t} - e^{-t/\tau}}{(1 - i\Omega\tau)} - 2\sum_n \sum_j \frac{\beta_n}{Q(p_{nj})} \frac{1}{(p_{nj} + i\Omega)} \frac{J_1(r\beta_n)}{J_0(\beta_n)} \times \frac{1}{(1 + p_{nj}\tau)} (\exp(p_{nj}t) - e^{-t/\tau}) \tag{19}$$

where

$$A = \left\{ \frac{-i\Omega(1 - i\alpha\Omega)(1 + f - i\Omega\tau)}{(1 - i\Omega\tau)} \right\}^{1/2}$$

β_n's are roots of

$$J_1(\beta) = 0 \tag{20}$$

and p_{nj}'s are the roots of the cubic equation

$$\frac{p_n(1 + \alpha p_n)(1 + f + p_n\tau)}{(p_n\tau + 1)} = -\beta_n^2, \ n = 0, 1, 2, \ldots \tag{21}$$

$$Q(p_{nj}) = [\{(1 + 2\alpha p_{nj})(1 + f + p_{nj}\tau) + p_{nj}\tau(1 + \alpha p_{nj})\} \times (p_{nj}\tau + 1) - \tau p_{nj}(1 + \alpha p_{nj})(1 + f + \tau p_{nj})]/(1 + p_{nj}\tau)^2 \tag{22}$$

It is clear from Eqn. (21) that all roots of p_n (for any n = 0, 1, 2,...) are either negative or one negative and other two complex. From the physics of the problem we consider those values of p_{nj} in Eqns. (18) and (19) for which exp $(p_{nj}t) \to 0$ as $t \to \infty$.

The non-dimensional skin-friction on the wall of the cylinder is given by

$$\tau_{r\theta}|_{r=1} = (1 + i\alpha\Omega)\left[\frac{AI_1'[A]}{I_1[A]} - 1\right] e^{-i\Omega t} - 2\sum_n \sum_j \frac{\beta_n^2(1 - \alpha p_{nj})}{Q(p_{nj})(p_{nj} + i\Omega)} \exp(p_{nj}t) \tag{23}$$

It is evident from Eqns. (18) and (19) that velocity of liquid and dust particles become same as the relaxation time tends to zero, i.e., when the dust particles become very fine. In the absence of elastic parameter and dust particles, the expression for the velocity profile of liquid particles is same as that obtained by Mukherjee and Bhattacharya[10] (if it is made dimensionless).

4. DISCUSSION

The analysis of the present study reveals that the solution contains three pertinent flow parameters, viz., a (the dimensionless elastic parameter), f (the mass concentration of dust particles) and τ (the relaxation time of dust particles). The behaviour of these parameters, therefore, yields a physical insight into the problem. Keeping this in view, the numerical computations of real part of Eqns. (18), (19) and (20) have been carried out to represent graphically the velocity fields, Skin-friction at the plate walls for different values of a, f, τ.

The velocity of liquid and dust particles are depicted in Figs. 1-4 against r for different values of a, f and τ. Figures 1 and 2 show the effect of f on u and v (with τ fixed) while Figs. 3 and 4 depict the variation of u and v due to the change of relaxation time of dust particles (with f fixed) for different values of elastic parameter. From Fiis. 1 and 3 it is seen that u increases with increasing a for fixed τ and f, i.e., the effect of elastic element in the liquid is to increase the velocity of liquid particles. Also it is observed that both mass concentration (f) and relaxation time (T) increase the velocity of liquid for any a. Figure 2 shows that flow occurs in reverse direction, (i.e., in the direction of decreasing θ) for a = 0, 1, 2 and τ = 0.5. As f increases both forward flow and back flow exist and the region of forward flow increases with the increase in f for any value of a. It is evident from Fig. 4 that as τ increases, the magnitude of the velocity of dust particles increases with fixed f.

Table 1 shows that the magnitude of skin-friction increases with the increase in elastic' parameter for f = 0.2, τ = 0.5 at t = 5. The negative values of skin-friction indicate that the shearing stress acts in the decreasing θ direction at t= 5.

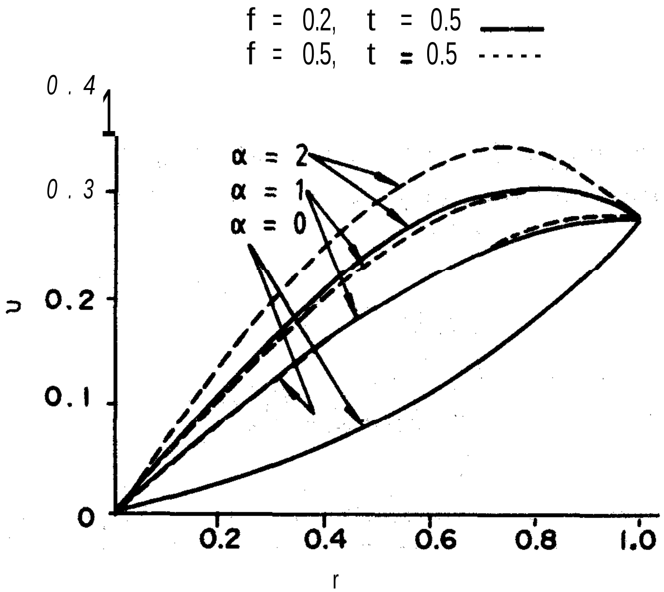

Figure 1. Velocity profile of liquid particles at t = 5 when τ = 0.5

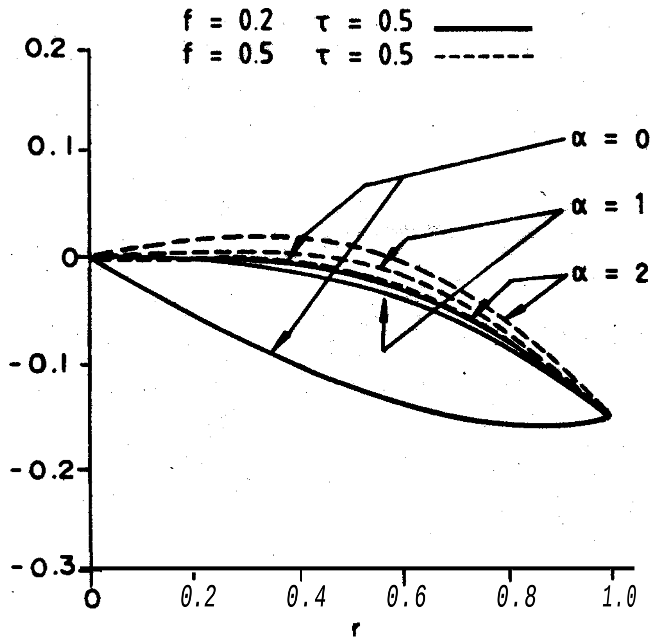

Figure 2. Velocity profile of dust particles at $t = 5$ when $\tau = 0.5$.

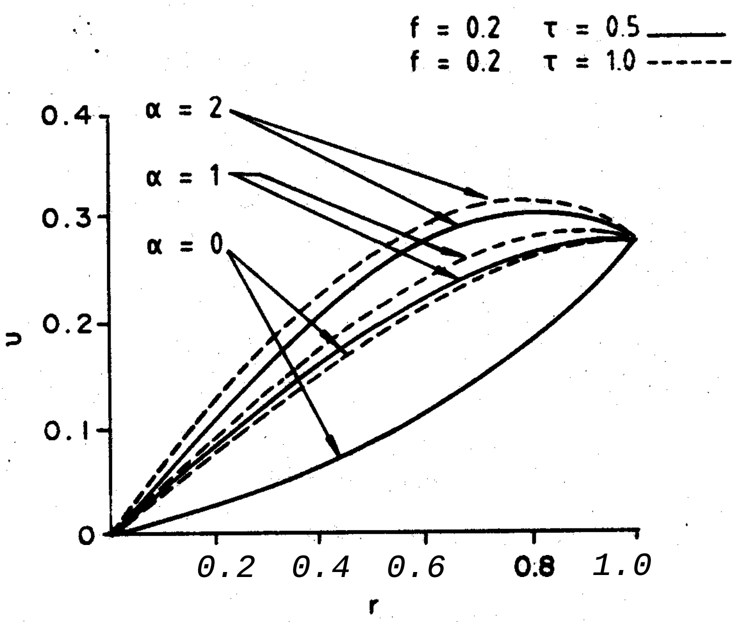

Figure 3. Velocity profile of liquid particles at $t = 5$ when $f = 0.2$.

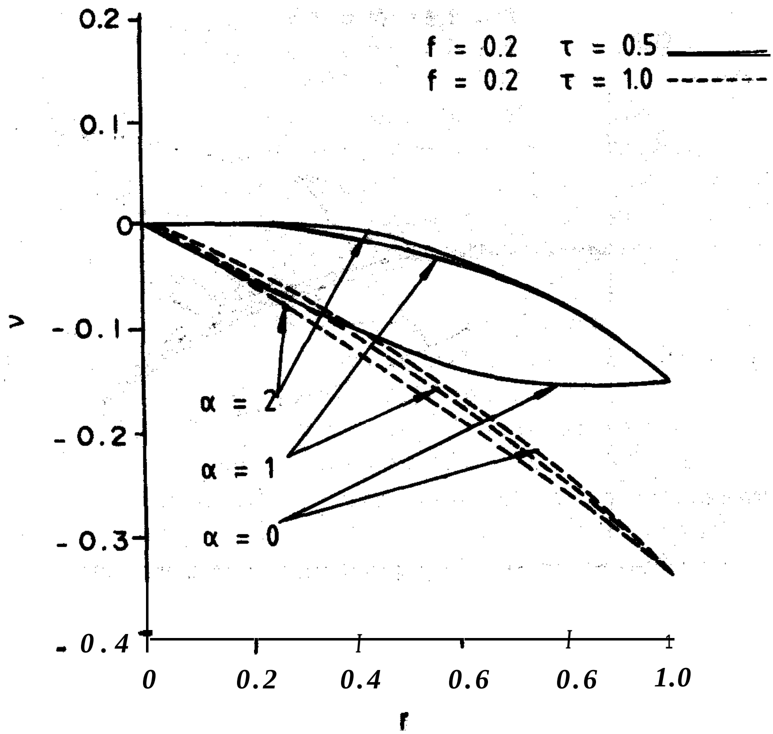

Figure 4. Velocity profile of dust particles at $t = 5$ when $f = 0.2$.

Table 1. Skin-friction on the wall at $t = 5$, $f = 0.2$ and $\tau = 0.5$

α	$\tau_{r\theta}\ _{r=1}$
1.0	-0.0273578
1.5	-0.053213
2.0	-0.912364
2.5	-1.563151
3.0	-2.251595

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