Def Sci J, Vol 41, No 3, July 1991, pp 241-249

Signal Processing in Vibration Analysis—Advanced Techniques

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ABSTRACT

Utilisation of vibration signature for trend analysis, health monitoring and defect location in machinery has been accepted as a handy tool and standard practice throughout industries for early prediction of emerging problems and safeguarding costly machinery. In the present scenario, frequency spectrum analysis of vibration signature is not the only or ideal solution. New signal processing techniques are emerging to meet more exacting and specific demands.

1. INTRODUCTION

The complexity of mechanical systems has, in recent years, increased markedly and the maintenance of such systems has become an important and inevitable task in industry. Visual inspection and physical assessment alone no longer provide adequate early warning to any emerging problem in a complex system to restrict down the time, maintenance cost and propagation of damage. Vibration signal analysis has emerged as an extremely useful and essential early warning technique for predicting onset of defect in its nascent form; thus giving adequate indication and time to plan preventive measures. The vibratory character relations between vibration amplitude and frequency or time, etc obtained from the recorded vibration signal of a running machine system is known as machine signature.

Several methods to examine the performance or identify the fault sources of machine systems, such as gears, engines, bearings etc by analysing the machine vibration signature have been reported. For example, in a frequency spectrum plot

Received 27 July 1988, re-revised 11 December 1990

of a coupled rotating machine if the second harmonic of the rotational fundamental frequency component is higher than the fundamental in amplitude, it shows a state of shaft misalignment. Similarly appearance of increasing number of side bands to mesh frequency component of a gearing system indicates wear in gear tooth. There is no single best technique; the best technique depends on the type and degree of diagnosis needed. In most cases machine vibration signature analysis alone is sensitive enough to indicate the condition of a system or to provide complete protection.

A conventional vibration signature analysis method based on frequency spectrum analysis of received signal is prone to error because of the surrounding noise contamination of the recorded signal. Frequency spectrum-based analysis also fails to bring out finer or minute deviations in a given signal, which may be early indicators to an emerging defect/fault. Research therefore continues in the field of signal processing to find accurate techniques that can bring out even the minutest of signature deviations to serve as early indicators to the maintainers and operators for health monitoring of modern complex and sensitive machine systems. Some of these advanced signal processing techniques finding increasing applications in the field of vibration analysis are briefly described below under two heads, namely, time domain analysis and frequency domain analysis.

2. TIME DOMAIN ANALYSIS

2.1 Kurtosis Value

Frequency spectrum of machine bearing (ball/rolling element) vibration changes as a bearing gets worn out or damaged. Normal frequency-based vibration spectrum signature analysis technique cannot predict onset of state deterioration and defect propagation till such time deterioration is severe enough to cause visual variation in vibration frequency spectrum. Kurtosis value analysis on the other hand clearly brings out even minutest of variations in the vibration characteristics of machine bearing through estimation of change in shape of important frequency components of the vibration spectrum. For Kurtosis value analysis, it is essential to bandpass the vibration signal around the monitoring frequency and obtain a series of vibration amplitudes against time plots to obtain central moments.

Normally, vibration frequency curves of a sound bearing are peaked near the event frequencies, while those of a worn bearing may be flat near the centre. The coefficient of Kurtosis (β_2), for determining the extent of peakness or flatness of the curve near the centre is given by :

$$\beta_2 = \frac{m_4}{m_2^2} \tag{1}$$

where m_2 and m_4 are the second and fourth central moments. The β_2 value of a normal Gaussian curve is 3. Lepto Kurtic curve which is peaked near the centre has a value of $\beta_2 > 3$ and Platic Kurtic curve which is flat near the centre has a value of $\beta_2 < 3$.

The fourth statistical moment of a machine bearing vibration signal results in a finite Kurtosis value which indicates a change in the bearing condition. A Gaussian distribution indicates acceptable limit of bearing. Measurement of Kurtosis value in selected frequency bands (i.e., at desired frequencies related to machine operation) and at different times makes it possible to determine the severity of damage in bearings. The main advantage of Kurtosis value estimation is its early prediction of the onset of bearing failure.

2.2 Bispectral Analysis

Changes in vibration frequency spectrum pattern of a machine gear box become noticeable only when a certain degree of deterioration sets in progressively. In a sensitive mechanical system, such as a gas turbine drive, the requirement of earliest possible detection of defects often arises. This is only possible through a study of phase variations of a critical frequency component, in a narrow band vibration signal of the system, with respect to a reference frequency component. The effect of error in the processed signal due to stray noise pick up is also eliminated in bispectral analysis owing to random phase characteristics of the noise.

Bispectral analysis of gear signatures are carried out to distinguish abnormal states from the normal one. Let the received vibration signal, including stray noise, be

$$x(t) = \sum_{n=0}^{\infty} a_n \cos [2\pi n f_0(t) + \phi_n(t)] \tag{2}$$

where f_0 is fundamental frequency of vibration and $\phi_n(t)$ is the noise corruption. Then bispectrum of the signal at two equispaced frequency components f_1 and f_2 about f_0 is a Fourier transform of third order correlation function expressed as

$$B(f_1 f_2) = \sum_{\substack{m,n=-\infty \\ m,n\neq 0}}^{\infty} \tfrac{1}{2} < a_m a_n a_{m+n} > \text{x} < \exp [j(\phi_m + \phi_n - \phi_{m+n})] >$$

$$\delta(f_1 - mf_0)\,(f_2 - nf_0) \tag{3}$$

where δ (.) is Kronecker delta and < > denotes ensemble averaging. For a clearer spectral feature of the modulus characteristics, it is more appropriate to derive bicoherence, i.e., normalised bispectrum with respect to power spectrum which is given by

$$Bic(f_1 f_2) = B(f_1 f_2)\ |\ P(f_1)\ P(f_2)\ P(f_1 f_2)\ |^{1/2} \tag{4}$$

Bispectral analyser consists of bandpass filters, multipliers and integrators. Since bispectrum is a statistical quantity which indicates the dependency among three frequency components whose frequencies f_1, f_2 and f_0 statisfy condition $f_1+f_2+f_0=0$, the bispectrum of any Gaussian noise vanishes completely. The technique is, therefore, effective to detect even such a delicate abnormality of a machine system that appears only as a change of the relative phase among frequency components while the amplitude of each frequency component remains unchanged. When scoring has grown in a gear face the moduli of bispectra of the vibration signal are reduced markedly comparing to those of normal state, while conventional spectral analysis fails to distinguish the different gear surface conditions.

2.3 Changing Variance

This technique is extremely effective for extraction of weak (least prominent) periodic signals submerged in noise or other non-periodic disturbances. A typical case occurs for ball bearings with localised defects, where by each rotating ball produces a slightly different impact. Not only the existance of such a phenomenon is completely ignored by an averaging process commonly adopted in vibration amplitude vs frequency or time plot analysis, but also the averaged component is often reduced in magnitude, and in extreme cases rendered undetectable. Computation of variances is extremely useful in such cases of vibration of rotating parts. It exhibits high sensitivity to the developing malfunctions; the presence of unstable repetitive pattern simultaneously increases the variance and decreases the average.

The method is suitable for processing repetitive vibration signatures of unstable character and involves time domain averaging of signal and estimation of variances. It is done in real-time, hence the running variance is computed instead of the true variance. If sufficient samples are taken, the running variance approaches the true variance. If i is the period number then running average

$$\bar{x}_i = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{5}$$

and running variance

$$Vr = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x}_i)^2 \tag{6}$$

where x_i is the amplitude of vibration at the frequency of interest. For greater accuracy in estimation of variances at least 25 to 70 samples of a signal is needed; samples one period apart are analysed.

Bladed machinery tend to generate high frequency vibration signatures with many spectral lines making the analysis difficult through frequency spectrum approach. Many a time changes in individual spectral component can indicate changes in condition even though these changes alone do not affect the overall rms level of the signal. In such cases estimation of changes in variances is useful.

2.4 Summation Technique

Summation is another method for determining the degree of wear in machine components. It enhances the periodic characteristics of an event signature and suppresses noise, in a continuous signal. If $f(t)$ is the raw vibration waveform starting at $t = 0$ and repeating every T seconds, then sum function

$$F(\tau) = \sum_{n=1}^{n} |f[\tau + (n-1)n]|\ 0 \le \tau \le T \qquad (7)$$

where T is the event occurrence period, n is sample number and $\tau = T/n$

Digital summation technique takes signal only at time intervals of selected event occurrence and displays only that portion of the signal which is generated by the events under study.

Most machines have cyclic operation of different components which produce vibration pattern of repetitive nature, which repeats at a certain time interval. The advantage of this method is that a particular peak, through its location in the summed vibration signature and previous knowledge of machine kinematic, can be related to certain events in the machine. Unlike in a common frequency spectrum, where the effect of noise contamination cannot be effectively eliminated, the spectrum obtained through summation technique not only reduces noise factor of non-periodic nature; but also enhances the prominence of the periodic component in a signal by improving signal to noise ratio. This technique is very handy for detection of ball bearing faults and valve stickyness, malfunction of loose pin, damage to piston rings, etc in reciprocating engines.

3. FREQUENCY DOMAIN ANALYSIS

3.1 Coherence Function

In the field of environmental dynamics, it is desired frequently to identify and rank in order of severity, the dynamic sources contributing to the measured vibratory output of a machine system and also to understand the paths through which such sources contribute vibratory disturbances to the system, i.e., establishment of relations between inputs and outputs. Vibratory inputs from external sources can adversely affect performance of a machine system including onset of defects.

A number of techniques are available for vibratory source identification in a machine.. If the sources have narrow band frequency characteristics, frequency spectrum (frequency vs amplitude) analysis is often used for disturbance source detection. However, the resulting spectra can be ambiguous, making source detection extremely difficult; when the vibratory inputs and resultant response outputs in a machine exhibit peaks at coincident frequencies. In this situation a time domain approach can be applied using auto- and cross-correlation functions. Unfortunately, though correlation technique enables identification of paths of propagation of vibration frequency components exciting a machine system, it fails to detect the sources of excitation. Further, narrow band signal processing in correlation results in broadening of peaks and hence tends to overlap one peak over the other.

Attention has recently been diverted to using various forms of cohence function to solve the problem. This is a frequency domain function that provides easy identification of excitation sources contributing to the vibration of a machine system. The technique can also identify the extent of individual inputs contributing to an output, even where a number of non-independent excitation sources result in vibration of a machine system. There are three forms of coherence function—ordinary coherence function (OCF), multiple coherence function (MCF), and partial coherence function (PCF).

The OCF between any two signal inputs $x_i(t)$ and $x_j(t)$ is given by

$$\mathrm{OCF} = \gamma_{ij}^2 = \frac{|G_{ij}|^2}{G_{ii} \cdot G_{jj}}$$

where G_{ii} is the auto-spectrum of $x_i(t)$, G_{jj} is auto-spectrum of $x_j(t)$, and G_{ij} is cross-spectrum of $x_i(t)$ with $x_j(t)$. If the inputs are completely independent, the OCF will vanish for all combinations of $x_i(t)$ and $x_j(t)$ since G_{ij} will be zero having nil correlation. Thus any variation from zero indicates some degree of mutual coherence between the inputs If $\gamma_{ij}^2 = 1$, the signals $x_i(t)$ and $x_j(t)$ are linearly related and one of the inputs should be eliminated from analysis. The OCF between signal input $x_i(t)$ and total signal output $y(t)$ is

$$\gamma_{iy}^2 = \frac{|G_{iy}|^2}{G_{ii} G_{yy}} \tag{9}$$

In this case, $\gamma_{iy}^2 = 1$ implies that the other inputs are not contributing to the output. In such cases, the OCF shows the proportion of the total output resulting from an input.

MCF shows the coherence function relationship between the complete set of inputs $x_1(t), x_2(t), x_3(t) \ldots x_p(t)$ and the total output $y(t)$. It can be used as a diagnostic tool to examine how closely an assumed model matches the actual system. When all inputs are independent for all frequencies MCF is given by

$$\gamma_{xy}^2 = \gamma_{1y}^2 + \gamma_{2y}^2 + \ldots \gamma_{py}^2 = 1 \tag{10}$$

When this result is not obtained, it means that either noise is present in the input or output signal or both, or other inputs that exist have not been included, or the system is non-linear.

This equation does not apply if the inputs are not completely independent. However, a similar relation can be developed for such cases between the MCFs and various PCFs. Before PCFs can be calculated, the inputs must be modified or conditioned to remove the coupling effects (i.e., mutually interacting contributions of more than one signal input) caused by the lack of independence between the inputs.

Each PCF will then show the proportion of the output caused by a specific input, with the effects of all other inputs eliminated. The generalised PCF is given by

$$\gamma^2_{py \cdot 1 \cdot 2 \cdot 3 \ldots (p-1)} = \frac{|G_{py \cdot 1 \cdot 2 \cdot 3 \ldots (p-1)}|^2}{G_{pp \cdot 1 \cdot 2 \cdot 3 \ldots (p-1)}\, G_{yy \cdot 1 \cdot 2 \cdot 3 \ldots (p-1)}} \tag{11}$$

where $G_{pp.1.2.3....(p-1)}$ is auto-spectrum of one of the input signals with the contributions of all other input signals removed; $G_{yy.1.2.3....(p-1)}$ is auto-spectrum of total output signals with the output contributions of all other input signals removed; and $G_{py.1.2.3....(p-1)}$ is cross-spectrum one conditioned input signal to total output signal. Here $\gamma^2_{py.1.2.3....(p-1)}$ is a special subscript notation generally used for PCF, meaning that the input $x_p(t)$ has been conditioned on other inputs $x_1(t)$, $x_2(t)....x_{(p-1)}(t)$ to eliminate their effects. $x_y(t)$ is the total output and *'G'* stands for auto- and cross-spectrum.

It is to be remembered that coherence function identifies source only, and does not provide any information about the path since phase information is not taken into account. On the other hand, cross-correlation gives path information on account of retention of phase information but fails to identify the source.

3.2 Cepstrum Analysis

Vibration signature of a gear box generally presents a complex picture comprising of a large number of frequency components, harmonics and side bands. A conventional amplitude vs frequency plot of such a signal appears more complex for analysis due to amplitude and frequency modulation resulting from different transmission paths in a gear box. Cepstrum analysis in such a case simplifies data interpretation by suppressing effects of path modulation.

This technique is used to separate out the periodic effects in a complex vibration spectrum. It is a logarithm power spectrum. Cepstrum performs the same function as the auto-correlation, however, effects multiplied in the power spectral analysis are additive in the cepstrum analysis. In cepstrum the logarithmic amplitude but linear frequency scale emphasises the harmonic structure of the spectrum and reduces the influence of the somewhat random transmission path by which the signal travels from the source to the measured point.

If f_x is a time domain signal then its power spectrum is

$$F_{xx}(t) = |\mathcal{F}|f_x(t)||^2 \tag{12}$$

Where $\mathcal{F}$ represents forward Fourier transform. Hence, cepstrum is

$$C(\tau) = |\mathcal{F}|\log F_{xx}(t)|| \tag{13}$$

$$= \mathcal{F}^{-1}|\log F_{xx}(t)| \tag{14}$$

(taking inverse transform of logarithm of the power spectrum). The method serves as an effective tool for fault diagnosis in gear box.

4. CREST FACTOR, COMPOSITE EXCEEDENCE AND ORDER ANALYSIS

In addition to different methods of vibration signature analysis discussed so far, there are a few other additional methods which are also used in processing vibration signals. The ratio of the peak amplitude to the root mean square amplitude of a machine vibration waveform is called crest factor. The crest factor depends on the slope of the waveform. In a complex waveform, the crest factor may vary considerably, and may change with time; but its value is never less than unity. In normal spectrum analysis, averaged value of a frequency component peak varies very gradually, and hence only significant changes in the machine state can be detected. Whereas, in crest factor analysis, even minor changes in machine condition can easily be predicted since transients to rms values of a frequency component peak are compared. The value of transients to rms ratio increases with deterioration of machine components. Peak to rms ratio indicates the size of a fatigue spall in a rolling element. The crest factor value increases with spall size. In a pump, leaky valve and defective valve spring give rise to severity of impact and can be detected through crest factor analysis (i.e., an increase in crest factor value).

Composite exceedence is a method based on a count of the total number of positive and negative peaks in an acceleration time history of a machine vibration signal, in specific amplitude bands, corresponding to excellent and acceptable levels of machine vibration at frequency components of interest. Since acceleration amplitude variation within a chosen frequency band is observed against time base (the base can be expanded), even minor variations can be noted in a more effective way than possible in a narrow band frequency spectrum analysis. Detection of spalled tooth surface in gear pinion is promptly noticed by observing increase in composite exceedence value.

Order related vibratory forcing functions in machinery are of particular importance in monitoring operative health of machines in repeated cyclic operations. The orders are related to operating speed of a machine and the forcing function amplitudes are proportional to machine loading and operating conditions. Signal via a tracking filter is fed to a sample lock analyser, the frequency range controls are set to a band width based on the vibration signal to be analysed. Unlike frequency spectrum analysis, order analysis provides a 3-D picture of machine vibration signal relating amplitude, frequency and time. In other words, it presents a series of specific frequency band spectrums in selected (operation cycle-related) time sequences, and any variation in speed or performance of machine due to the onset of defect appears as distinct changes in the 3-D pattern. The set of order-related frequency spectrums, thus obtained at different rotational speeds of a machine indicates performance soundness or emerging defects based on the extent of scattering of frequency peaks and also appearance of additional frequency peaks. In reciprocating machinery order analysis provides greater information than common frequency spectrum analysis.

5. CONCLUSION

Besides being expensive, modern day machinery are critically designed to give optimum performance under extreme operating conditions. Under such demanding situations, it is essential for plant engineers to be able to asses machinery health fairly accurately at any point of time and also to have access to such techniques as can indicate emerging machinery defects at fairly early stages of development of such faults to avoid or restrict the extent of machinery damage.

Conventional frequency spectrum analysis of vibration signal is no longer adequate to meet the current demand for accurate defect diagnosis. New techniques are emerging, though there is no single signal processing technique to suit all situations. Each of these techniques has specific field of application in isolation or combination.

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