Def Sci J, Vol 41, No 2, April 1991, pp 149-156

Application of Raleigh-Ritz Method for Determining Optimum Blunt-Nosed Missile Geometries

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ABSTRACT

Raleigh-Ritz method has been applied to solve the problem of determining optimum blunt-nosed missile shapes of minimum ballistic factor. It has been found that in both the cases, i.e., (i) length & diameter, and (ii) diameter and wetted area given in advance the optimum shapes are purely regular upto critical values of the friction parameters, defined in the two cases and beyond these values the shapes are regular followed by cylinders.

1. INTRODUCTION

The study of minimum ballistic factor missile shapes are of practical importance in connection with the design of re-entry missiles. A keen interest has thus been shown by various authors like Berman[1], Miele and Huang[2], Tawakley and Jain[3-6] for determining shapes of sharp-nosed missiles of minimum ballistic factor. All these studies take zero radius of curvature at the nose. However, such shapes experience severe local aerodynamic heating during re-entry (Miele[7]). For this reason, the problem of minimising the ballistic factor for blunt-nosed bodies in hypersonic flow is of paramount importance. Heideman[8] investigated blunt-nosed missile shapes of minimum ballistic factor for given length, diameter and nose radius via the calculus of variations. Tawakley and Jain[9] studied the problem of finding a blunt-nosed missile of minimum ballistic factor again via calculus of variations for the case when the

Received 20 November 1989

wetted area, the diameter and the nose radius are known and the length is free. In this paper, the Raleigh-Ritz method is applied to determine shapes of blunt-nosed missiles of minimum ballistic factor under the assumption that the surface-averaged skin-friction is constant for the cases when (i) the length, diameter and nose radius, and (ii) the wetted area, diameter and nose radius, are known.

2. FORMULATION OF THE PROBLEM

For an axisymmetric blunt-nosed slender body at zero angle of attack, the drag (D), the wetted area (S), and the volume (V) are given by

$$\frac{D}{\quad} = \int_0 y\,(y'^3 + C_{f/2})\,dx + \frac{y_0^2}{2} \tag{1}$$

$$S = 2\pi \int_0 y\,dx + \pi y_0^2 \tag{2}$$

$$V = \pi \int_0 y^2 dx \tag{3}$$

where l denotes the length, y_0 the nose radius, C_f the surface- averaged skin-friction coefficient, q the free stream dynamic pressure, x and y the axial and radial coordinates, and y' the derivative dy/dx. If we take $X = x/l$ and $Y = 2y/d$ as the dimensionless coordinates in the x and y directions respectively, where d is the diameter of the body, then it can be shown that in the cases where (i) l, d, and (ii) s, d are known *a priori*, and S and l are free, then

$$\text{(i)}\quad \frac{l^3}{d^2}\ \frac{D}{qV} = \frac{I_1 + K_1 I_2 + K_2}{I_3} \tag{4}$$

$$\text{(ii)}\quad \frac{S'^3}{\pi^3 d^5} \cdot \frac{D}{qV} = \frac{I_1\ I_2^3}{\quad} + \frac{K_3\ I_2}{\quad} \tag{5}$$

where $I_1 = \int^1 YY'^3\,dx \qquad I_2 = \int_0^1 Y\,dx, \qquad I_3 = \int^1 Y^2\,dx$

$$S' = S - \pi(d/2)^2\, Y_0^{\,2}$$

$$Y' = \frac{dY}{dX} \tag{6}$$

$$K_1 = 4C_f\, \frac{l^3}{d^3}$$

$$K_2 = 2 \cdot \frac{l^2}{d^2} \cdot Y_0^{\,2}$$

$$K_3 = 4C_f\, \frac{S'^3}{\pi^3 d^6} + \frac{2S'^2}{\pi^2 d^4}\; Y_0^{\,2} \tag{7}$$

3. SOLUTION OF THE PROBLEM

Raleigh-Ritz method requires that the optimum curve to be obtained within the class of continuous curves satisfying

$$Y = Y_0 + X(\ \ + a - Y_0) - aX^2 \tag{8}$$

where 'a' is a parameter to be determined from the mathematical analysis of the problem. This expression satisfies the boundary conditions that initially $X = 0$, $Y = Y_0$ and finally $X = 1$, $Y = 1$.

In order to find the value of the parameter a for which the values of Eqns.(4) and (5) are minimum, we evaluate the integrals I_1, I_2 and I_3 with the help of Eqn.(8) and get

$$I_1 = \frac{1}{2}(1 + Y_0)(1 - Y_0)^3 - \frac{1}{3}(1 - Y_0)^3 a + \frac{1}{2}(1 + Y_0)(1 - Y_0)a^2 \tag{9}$$

$$I_2 = \frac{1}{2}(1 + Y_0) + \frac{1}{6}a \tag{10}$$

$$I_3 = \frac{1}{3}(1 + Y_0 + Y_0^2) + \frac{1}{6}(1 + Y_0)a + \frac{1}{30}a^2 \tag{11}$$

For minimum value of $\dfrac{S'^3}{\pi^3 d^5}\dfrac{D}{qV}$

$$\frac{d}{} \left(\frac{S'^3}{\pi^3 d^5}\frac{D}{qV} \right) = 0$$

i.e.

$$\begin{aligned}
&9(1 - Y_0^2)a^6 + (110 + 126Y_0 - 126Y_0^2 - 110Y_0^3)a^5 + (591 + 1212Y_0 \\
&- 1212\, Y_0^3 - 591Y_0^4)a^4 + (1660 + 4900Y_0 + 2920\, Y_0^2 - 2920\, Y_0^3 \\
&- 4900\, Y_0^4 - 1660\, Y_0^5)a^3 + \{(2223 - 216K_3) + 9360Y_0 + 10251Y_0^2 \\
&- 10251Y_0^4 - 9360Y_0^5 - 2223Y_0^6\}a^2 + \{(918 - 1296K_3) + (6318 \\
&- 1296\, K_3)Y_0 + 11286Y_0^2 + 5886Y_0^3 - 5886Y_0^4 - 11286\, Y_0^5 - 6318\, Y_0^6 \\
&- 918Y_0^7\}a + 135\{-(1 + 8K_3) - 4(1 + 8K_3)Y_0 - (8K_3 - 2)Y_0^2 \\
&+ 12Y_0^3 - 12Y_0^5 - 2Y_0^6 + 4Y_0^7 + Y_0^8\} = 0
\end{aligned} \tag{15}$$

Here the Newton-Raphson method may be employed to get the value of a for known values of Y_0 and K_3. Knowing the value of a, the value of $\dfrac{S'^3}{\pi^3 d^5}\dfrac{D}{qV}$ is calculated from Eqn.(14) and the optimal geometry from Eqn.(8).

4. NUMERICAL SOLUTION

In order that optimum curve should have a maximum at $X = 1,\ Y = 1$, we find from Eqn.(8) that $a = 1 - Y_0$

This condition in conjunction with Eqn. (13) will enable us to know the value of the friction parameter K_1 in terms of Y_0 and K_2 for which the optimum shape will be regular. Similarily, in the second case when S, d are given, the above condition together with Eqn.(15) will give the value of friction parameter K_3 in terms of y_0 for which the optimum shape will be regular. These values of K_1 and K_3 are called the critical values and represented as K_{c1} and K_{c3} respectively. Having calculated the values of the parameter a from Eqns.(13) and (15) for various sets of values of Y_0, K_1 and K_2; and Y_0 and K_3 for both the cases. The optimum curves of minimum ballistic factor (Figs.(1) to (4)) have been drawn with the help of Eqn.(8). Graphs have also been drawn between $(l^3/d^2).(D/q\ v)$ and friction parameter K_1 for known values of Y_0 and K_2 in case (i) (Fig. 5) and between $(S'^3/\pi^3 d^5).(D/q\ v)$ and friction parameter K_3 for given values of Y_0 for case (ii) (Fig. 6).

Case (i) : When Length and Diameter of the Body are Given

By substituting the values of I_1, I_2, I_3 from Eqns.(9)–(11) in Eqn.(4), we get

$$\frac{l^3}{d^2}\frac{D}{qV} = \left[15\{(1 + Y_0)(1 - Y_0)^3 + (1 + Y_0)K_1 + 2K_2\}\right.$$
$$\left. + 5\{K_1 - 2(1 - Y_0)^3\}a + 15(1 - Y_0^2)a^2\right] / \left[10(\quad + Y_0\right.$$
$$+ Y_0^2$$

$$\frac{l^3}{d^2}\frac{D}{qV}$$

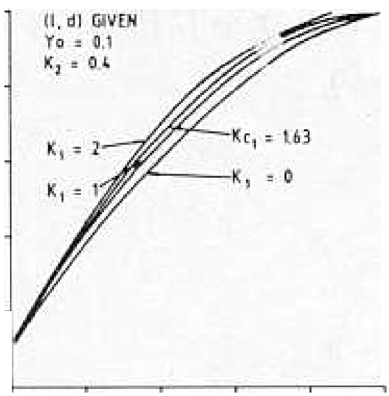
$$\frac{d}{da}\left(\frac{l^3}{d^2}\frac{D}{qV}\right)$$

$$[\{17 + 26 Y_0 + 17 Y_0^2)(1 - Y_0) - K_1\}$$
$$(9 + 12 Y_0 + Y_0^2) - K_1(1 + Y_0) - 2K_2\} a - 5\{(7 + 10 Y_0 + 7 Y_0^2$$
$$+ 6 K_2 (1 + Y_0)\}] = 0$$

Case (ii) : When Wetted Area and Diameter of the Body are Given

Again by substituting the values of I_1, I_2, I_3 from Eqns. (9)–(11) in Eqn.(5), we get

$$\frac{S^3}{\pi^3 d^5}\frac{D}{qV} = [(3 - 3Y_0)a^5 + (25 + 33Y_0 \quad 33Y_0^2 - 25Y_0^3)a^4 + (66 + 192Y_0$$
$$192Y_0^2 - 66Y_0^4)a^3 + 54(\quad + 5Y_0 + 4Y_0^2 - 4Y_0^3 - 5Y_0^4 -$$
$$a +$$



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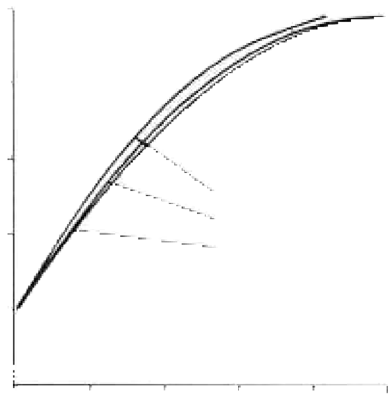

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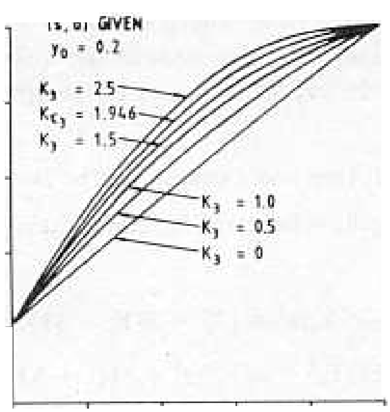

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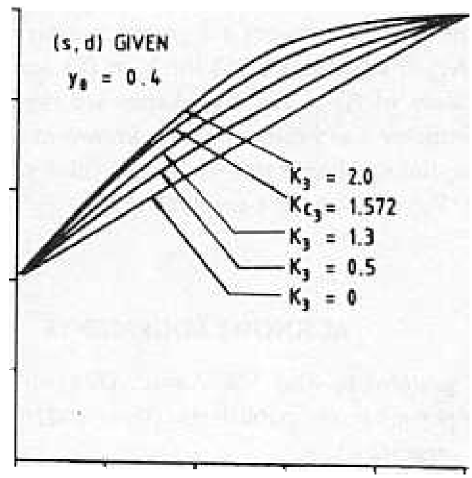

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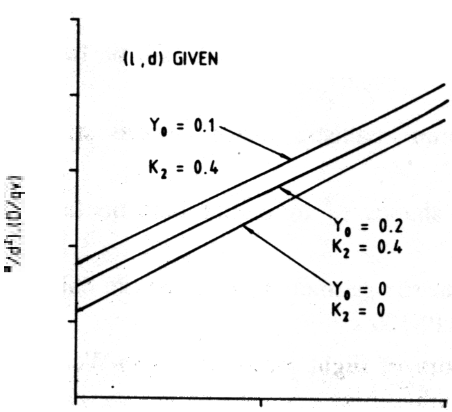

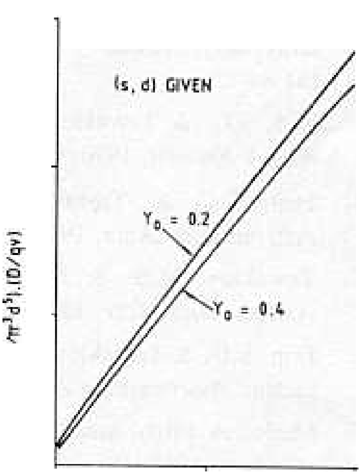

d^2).

Figure Relationship

5. CONCLUSIONS

Raleigh-Ritz method has been applied to solve the problem of finding minimum ballistic factor missile shapes. It has been found that in case length and diameter are given in advance, the optimum shapes are purely regular upto a critical value of the friction parameter $Kc_1 = 1.630$ and 0.363 for $Y_0 = 0.1$ and 0.2, and $K_2 = 0.4$ and 0.8 respectively. For values of $K_1 > Kc_1$ the shapes are regular followed by cylinders. Similarly, in case diameter and wetted area are known in advance, a similar tendency is exhibited in the optimum shapes upto a critical value of friction parameter $Kc_3 = 1.946$ and 1.572 for $Y_0 = 0.2$ and 0.4 respectively.

ACKNOWLEDGEMENTS

The author is grateful to Shri VK Arora, Director, Defence Science Centre, Delhi for according permission to publish this paper and to Dr VB Tawakley, Scientist E for his help and suggestions.

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