# Application of Computer Graphics to Performance Studies of Missile Warheads 

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#### Abstract

Intercept geometry of target aircraft and missiles play an important role in determining the effectiveness of the warhead. Factors such as fragment spatial distribution profile, damage capabilities, target and missile characteristics have been considered and visualised through computer graphics and optimum intercept angles have been arrived. Computer graphics has proved to be an important tool to enhance perception and conceptual design capabilities in the design environment.


## 1. INTRODUCTION

In modern warfare, the air threat from low level as well as high level flying aircraft is successfully met through guided missiles. A typical missile mission entails tracking of the target during its pre-launch phase and launching of the missile from launch control equipment. Subsequently it passes through gathering and guidance phases to reach crucial terminal phase when the warhead detonates at close proximity to the target to anhilate or to cause maximum damage. For successful mission, intercept geometry of target and missile warhead needs close attention. A computer model has been developed in respect of short range quick reaction surface-to-air missile with pre-fragment type warhead to determine optimum fragment-target intercept angles for maximum effectiveness of the warhead. Computer graphics has been employed to produce fragment front profile and target interceptions and has been found to be a prime tool in enhancing perception in the given design environment.

## 2. FRAGMENT FRONT-TARGET INTERCEPTION

In the design of pre-fragmented missile warheads, the designer will always endeavour to obtain a narrow spray zone and align this spray zone in a particular direction where the target-fragment interception will be maximum. An hypothetical dynamic fragment spray for a low level quick reaction missile is given in Table 1. The objective is to maximise the effectiveness of the warhead, through a comparison of measure of effectiveness of several warhead designs. Measure of effectiveness can be defined as a ratio of available kinetic energy of the fragment to the required energy for desired lêvel of damage multiplied by the number of such fragments.

Table 1. Fragment dynamic spatial distribution for hypothetical missile warhead

| Velocity of missile | $700 \mathrm{~m} / \mathrm{s}$ |
| :--- | :--- |
| Fragment shape | cubical |
| Fragment material | steel |
| Fragment mass | 4 g |
| Total number of fragments | 1756 |


| $A_{z}$ <br> $(\mathrm{deg})$ | $\bar{N}$ | $V_{d}$ <br> $(\mathrm{~m} / \mathrm{s})$ |
| :---: | ---: | :---: |
| $25-30$ | 5 | 1053 |
| $30-35$ | 44 | 1053 |
| $35-40$ | 44 | 1053 |
| $40-45$ | 5 | 1069 |
| $45-50$ | 10 | 1780 |
| $50-55$ | 110 | 1784 |
| $55-60$ | 232 | 1850 |
| $60-65$ | 248 | 1953 |
| $65-70$ | 357 | 1949 |
| $70-75$ | 491 | 1895 |
| $75-80$ | 197 | 1834 |
| $80-85$ | 13 | 1772 |

Note : $A$, is the angular zone of fragment beam considered
The lethality of a fragment depends on its relative striking velocity, $V_{s}$, at the target which in turn depends on the dynamic ejection velocity of the fragment, $V_{d}$, target velocity $V_{t}$, and the distance between the detonation point and the position of the target, $X_{s}$, at the fragment intercept time. The intercept geometry is shown in Fig. 1.

Fragment velocity at any point in their flight path can be determined by using the following equation.

$$
\begin{equation*}
V=V, . e^{-\left(C_{d} \cdot \rho_{a} \cdot A / m\right) \cdot X} \tag{1}
\end{equation*}
$$



Figure 1. Fragment-target intercept geometry.
where, $V_{x}$ is the velocity at range $X, V_{d}$ is the initial fragment dynamic velocity, $C_{d}$ is the coefficient of drag, $\rho$ is the density of air, $A$ is the projected area of fragment, $m$ is the fragment mass, and $X$ is the range.

The relative fragment striking velocity $V_{s}$ is determined by using the intercept geometry as follows :

$$
\begin{equation*}
V_{s}=\sqrt{\left(V_{x s} \cdot \cos \theta_{d}-V_{t}\right)^{2}+\left(V_{x s} \cdot \sin \theta_{d}\right)^{2}} \tag{2}
\end{equation*}
$$

where $\theta_{d}$ is the fragment target intercept angle.

### 2.1 Intercept Geometry

For intercept condition, the flight time of the target and fragment must be equal Thus from Fig. 1, it is found that

$$
\begin{equation*}
=(u+z) / V_{t} \tag{3}
\end{equation*}
$$

where $\bar{V}$ is the average velocity of the fragment in free air which is obtained by taking the time integral of the Eqn. (1) over the range $X_{s}$ and divided by $X_{s}$.

Ti

$$
\begin{align*}
& \nabla=V_{d} \cdot a \cdot X_{s} /\left(\exp \left(-a \cdot X_{s}\right)-1\right)  \tag{4}\\
& \alpha=C_{d} \cdot \rho_{a} \cdot A / m
\end{align*}
$$

and $(u+z)$ is the distance travelled by the target from the time of warhead burst to the interception.

From the geometry of attack, we have

$$
\begin{equation*}
\omega \cdot \sin \sigma_{t}=X_{s} \cdot \sin \theta_{t} \tag{5}
\end{equation*}
$$

where $\sigma_{t}$ is the angle of sight to the target.
Equation (3) can be rewritten using the geometrical relations, as

$$
\begin{equation*}
X_{s} / V=\left(\omega^{2} \cdot \cos \sigma_{t}+\sqrt{\left(X_{s}-\omega^{2} \cdot \sin ^{2} \sigma_{t}\right)}\right) / V_{t} \tag{6}
\end{equation*}
$$

The combination of Eqns. (6) and (4) gives $X_{s}$. The value of $X_{s}$ so obtained is used in Eqn. (1) to solve for $V_{x s}$ and in Eqn. (5) to get $\theta_{d}$. The value of $X_{s}$ and $\theta_{d}$ are then substituted into Eqn. (2) to obtain relative striking velocity $V_{s}$.

### 2.2 Warhead Performance

Consider a fragment spray in a small angular zone from $\theta_{1}$ to $\theta_{2}$ having $N$ number of fragments. Then the number of hits $N_{h}$ on a given target of presented area $A_{t}$ at a distance $R$ can be given by

$$
N_{h}=N \cdot A_{t} /\left[R^{2} \cdot 2 \pi \cdot\left(\cos \theta_{1}-\cos \theta_{2}\right)\right]
$$

After determining the fragment striking energy $E$ from the kinetic energy principles, the measure of effectiveness (MOE) can be determined from the following equation

$$
\begin{equation*}
\mathrm{MOE}=\frac{\text { fragment strike energy available }}{\text { minimum fragment energy required }} N_{h} \tag{8}
\end{equation*}
$$

## 3. COMPUTER CODE FOR THE ANALYSIS OF INTERCEPT GEOMETRY AND PERFORMANCE

A computer code has been evolved to solve the intercept geometry of fragment front, target aircraft and performance. In this analysis, orientation of missile with respect to line of sight for each angular zone is determined assuming that fragments from the angular zone under consideration only are hitting the target. To maximise the measure of effectiveness, the optimum orientation of missile axis with respect to line of sight has been worked out. Orientation of missile axis with respect to line of sight was determined for each angular zone assuming that fragments from the angular zone under consideration only are hitting the target in two modes, i.e., when the horizontal components of missile and target velocities are (a) in the same direction ( $\gamma_{1}$ ), and (b) in the opposite direction ( $\gamma_{2}$ ).

The parameters of intercept geometry have been computed through the code which forms the input to warhead performance evaluation and are presented in Table 2.

Performance parameters like the number of hits on the target, strike energy of the fragments and MOE have been determined. The results of the analysis are shown in Table 3. Various levels of strike energy are required to accomplish the desired damage to the target aircraft. However, for design purposes an average strike energy has been assumed as 6000 J for evaluating the MOE of the fragments hitting the target.

### 3.1 Graphics

Computer graphics plays an important role in the perception of physical phenomena, like fragment spatial distribution, missile-target interception, etc. A
computer code has been developed to present the spatial distribution of any pre-fragment type warhead in the graphic form as shown in Fig. 2. This gives a 2-dimensional view of the fragment distribution in space.

Table 2. Fragment-target intercept geometry

| Velocity of target |  |  | $300 \mathrm{~m} / \mathrm{s}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Velocity if missile |  |  | $700 \mathrm{~m} / \mathrm{s}$ |  |  |  |
| Angle of sight |  |  | 40 degrees |  |  |  |
| Miss distance |  |  | 10 m |  |  |  |
| $\begin{gathered} A_{\mathbf{z}} \\ (\mathrm{deg}) \end{gathered}$ | $\begin{gathered} X_{s} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} V_{\mathrm{s}} \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | $\begin{gathered} \psi \\ (\mathrm{deg}) \end{gathered}$ | $\begin{gathered} \gamma_{1} \\ \text { (deg) } \end{gathered}$ | $\begin{gathered} \gamma_{2} \\ \text { (deg) } \end{gathered}$ | $\begin{gathered} \theta_{d} \\ (\mathrm{deg}) \end{gathered}$ |
| 25-30 | 8.32 | 1139 | 10.6 | 38.1 | -16.9 | 129.4 |
| 30-35 | 8.32 | 1139 | 10.6 | 43.1 | -21.9 | 129.4 |
| 35-40 | 8.32 | 1139 | 10.6 | 48.1 | -26.9 | 129.4 |
| 40-45 | 8.34 | 1153 | 10.5 | 53.0 | -32.0 | 129.5 |
| 45-50 | 8.92 | 1770 | 6.1 | 53.6 | -41.4 | 133.9 |
| 50-55 | 8.93 | 1774 | 6.1 | 58.6 | -46.4 | 133.9 |
| 55-60 | 8.96 | 1831 | 5.9 | 63.4 | -51.6 | 134.1 |
| 60-65 | 9.01 | 1919 | 5.5 | 68.0 | -57.0 | 134.5 |
| 65-70 | 9.01 | 1916 | 5.5 | 73.0 | -62.0 | 134.5 |
| 70-75 | 8.98 | 1869 | 5.7 | 78.2 | -66.8 | 134.3 |
| 75-80 | 8.95 | 1817 | 5.9 | 83.4 | -71.6 | 134.1 |
| 80-85 | 8.92 | 1763 | 6.1 | 88.6 | -76.4 | 133.9 |

Note : $A_{z}$ is the fragment beam angular zone; and $\psi$ is fragment flight angle with respect to line of sight.


Figure 2. Dynamic fragment spatial distribution of a hypothetical missile warhead.
The fragment fronts generated by the warhead detonation will start moving in the space in various directions. Simultaneously the target aircraft is also moving in a
particular direction. In such a dynamic environment, the performance of the warhead not only depends on the design of the warhead, and velocities of missile and target but also on the orientation of the missile with respect to the line of sight. To visualise such dynamic environment clearly, computer animation can be considered as one of the powerful techniques. Hence a code has been developed using the computer graphics software. The graphic outputs at various stages of animation have been taken and presented in Figs. 3 and 4.

Table 3. Missile warhead performance parameters

| Missile velocity | $700 \mathrm{n} / \mathrm{s}$ |
| :--- | :--- |
| Target velocity | $300 \mathrm{~m} / \mathrm{s}$ |
| Angle of sight | 40 degrees |
| Miss distance | 10 m |
| Target area-plain view | $46 \mathrm{~m}^{2}$ |
| Target area-end view | $3.3 \mathrm{~m}^{2}$ |
| Minimum fragment striking | $\mathbf{6 0 0 0} \mathrm{J}$ |
| energy required (assumed) |  |


| $A_{2}$ <br> $($ deg $)$ | $N$ | $\overline{N_{h}}$ | $E_{f}$ <br> $(\mathrm{~J})$ | $R$ | $\overline{\text { MOE }}$ |
| :---: | ---: | ---: | ---: | ---: | ---: |
| $25-30$ | 5 | 10 | 2596 | 0.43 | 0 |
| $30-35$ | 44 | 72 | 2596 | 0.43 | 0 |
| $35-40$ | 44 | 64 | 2596 | 0.43 | 0 |
| $40-45$ | 5 | 6 | 2660 | 0.44 | 0 |
| $45-50$ | 10 | 10 | 6268 | 1.04 | 10 |
| $50-55$ | 110 | 98 | 6292 | 1.05 | 103 |
| $55-60$ | 232 | 192 | 6702 | 1.12 | 215 |
| $60-65$ | 248 | 192 | 7368 | 1.23 | 235 |
| $65-70$ | 357 | 265 | 7341 | 1.22 | 324 |
| $70-75$ | 491 | 357 | 6989 | 1.16 | 415 |
| $75-80$ | 197 | 141 | 6602 | 1.10 | 155 |
| $80-85$ | 13 | 9 | 6219 | 1.04 | 10 |

Note : $A_{z}$ is the fragment beam angular zone; $E_{f}$ is the strike energy of a fragment; and $R_{e}$ ratio of $E_{f}$ to minimum required energy.
The program for animation of the dynamic situation has been developed using Draft Pack 2-D package of OMC Computers Limited in an 8-bit PC/XT environment. Due to the inherent limitations of the software package and system, the speed of animation obtained is rather low. However, the speed can be improved by using a 16/32-bit computer systems.

## 4. ANALYSIS OF RESULTS

Analysis of warhead performance parameters presented in Table 3 show that the fragment front in the angular zone 70-75 degrees has got maximum MOE for the
given angle of sight ( 40 degrees), missile velocity ( $700 \mathrm{~m} / \mathrm{s}$ ), and target velocity ( $300 \mathrm{~m} / \mathrm{s}$ ). From the analysis of optimum missile orientations shown in Table 4, the warhead orientations should correspond to either 78.2 or -66.8 degrees with respect to the line of sight to achieve the maximum performance at angle of sight 40 degrees.



FRAGMENT - TARGET (A/C) INTERCEPTION PHASE - II

* TARGET VELOCITY $=300 \mathrm{~m} / \mathrm{s}$
* MISSILE VELOCITY $=700 \mathrm{~m} / \mathrm{s}$


FRAGMENT - TARGET (A/C)
INTERCEPTION PHASE - III
Figure 3. Animation depicting non-intercept condition.


Figure 4. Animation depicting intercept condition.

Algorithms used in evaluating intercept geometry and performance were validated through computer graphics. The orientation of missile with respect to line of sight was continuously increased from zero value and the feasibility of fragment-target interception was examined. For a given tine of sight, a small zone of missile orientations give interception with varying MOE. It was observed that the missile orientation giving maximum MOE through computer analysis falls within the zone of missile orientations obtained through animation program. Thus the computer graphics proves to be an important tool in the perception of dynamic conditions of missile target interception.

Table 4. Optimum missile orientations for various angles of sight


Note : $A_{\mathrm{zm}}$ is the fragment beam angular zone for maximum MOE.

## 5. CONCLUSIONS

Optimum fragment-target intercept angles have been arrived through a computer code and the results were validated by the computer animation program using computer graphics. The computer graphics has proved to be an important tool to enhance the perception of the performance studies of the warhead.

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