Defence Science Journal, Vol 42, No 4, October 1992, pp 273-280

Computation of Particle-Laden Turbulent Gas Jet Flows Employing the Stochastic Separated Flow Approach

V. Ramanujachari
Institute of Armament Technology, Pune-411 025
and
R. Natarajan
Indian Institute of Technology, Madras-600 036

ABSTRACT

The dispersion of particles in the presence of turbulent gas flow is studied theoretically using a stochastic separated flow model and the results compared with the available experimental data. As the particle loading in the jet is of the order of 0.1–0.4 per cent, the particles are assumed to have negligible effect on the mean and the turbulent gas phase properties (one-way coupling). The particle-turbulent eddy interactions are calculated by paying attention to the energy containing eddies, characterised by the integral length scale. The fluctuating velocities are sampled randomly from Gaussian distribution, and the particle trajectories are obtained using a procedure similar to random-walk computation. A large number of particle trajectories are averaged to obtain the statistical nature of the turbulent gas-particle jet. It is seen that the particles with less inertia, which are characterised by the Stokes number, tend to diffuse more. The turbulent diffusivities of the particles are in agreement with the available experimental data, when the time-averaged velocities of gas and particles are the same, obtained by the stochastic separated flow model.

NOMENCLATURE

C	particle concentration
C_m	centre line particle concentration
d	particle diameter
D	jet exit diameter
L	Eulerian spatial integral length scale
L_m	Eulerian integral scale at the centre line of the jet
$\bar{L}_m$	L_m/D
	radial coordinate
$\bar{r}$	r/D
r_p	particle displacement
$\bar{r}_p$	r_p/D
t	time
$\bar{t}$	tU_0/D
t_L	Lagrangian integral time scale of fluid
t_t	transit time
u'	fluid fluctuating velocity (Eulerian)
U	longitudinal average fluid velocity
$\bar{U}$	U/U_0
U_1, U_2	uniform random numbers
U_m	time-averaged centre line velocity of fluid
U_0	jet exit velocity
U_p	longitudinal time-averaged particle velocity
U_{pm}	time-averaged centre line velocity of the particle
v	instantaneous gas velocity
v'	fluid fluctuating velocity (Lagrangian)
V	lateral (radial) time-averaged fluid velocity
v_{pi}	particle velocity in general
V_p	lateral time-averaged particle velocity
x_{pi}	general particle coordinate

Received 6 August 1991, revised 12 December 1991

Y_1, Y_2 normal random variates
Z distance from jet nozzle
$\bar{Z}$ Z/D

Greek letters

ε_{pr} turbulent diffusivity of particles
$\bar{\varepsilon}_{pr}$ ε_{pr}/U_0D
ε_f turbulent diffusivity of fluid lumps
η r/Z
η_1 $(\bar{r}-0.5)/Z$
μ fluid viscosity
ρ, ρ_p mass densities of fluid and particles
ψ Stokes number
< > time-averaged quantity
τ particle relaxation time

1. INTRODUCTION

Interaction of the gas-phase turbulence with the particles is important in many industrial and energy-related processes. The dispersion of droplets from injectors plays a vital role in establishing the flame pattern and in the efficiency of liquid fuel combustors. The dispersion of coal particles by turbulence in the furnaces fueled by pulverised coal is important for their combustion efficiency. The droplet dispersion due to turbulence is an essential phenomenon in such diverse applications as rocket engine exhausts, spraying and waste disposal plumes and aerosol production.

Many propulsive systems employed in defence applications, such as diesel engines for tanks, gas turbines for aeroengines, liquid fuel ramjets for missiles, and liquid propellant rocket engines, involve the introduction of the liquid fuel in the form of a spray, which in most cases undergoes heat, mass and momentum transport phenomena under turbulent flow conditions. The evolution of the spray and efficiency of conversion of chemical energy of the fuel into shaft power or thrust are important areas of concern for the design and analysis of these propulsive devices. A methodology of computation of particle-laden turbulent gas jet flows has been provided, employing the stochastic separated flow approach, which has currently been identified as the most appropriate for such systems.

Two major models can be distinguished for the study of gas-particle flows[1,2] : (i) locally homogeneous flow (LHF) models, where the gas and the particle phases are assumed to be in dynamic and thermodynamic equilibrium; and (ii) separated flow (SF) models, where the effects of finite rates of transport between the phases are considered. The SF models are further classified into (i) deterministic separated flow (DSF) models, where the interactions between the particles and the gas turbulence are ignored, and (ii) stochastic separated flow (SSF) models, where the finite interphase transport rates and the particle-gas turbulence interactions are considered using random-walk computations for the particle phase. Both the finite interphase transport rates and the dispersed phase/turbulence interactions are important in many practical dispersed flows. Therefore, neither the LHF model nor the DSF model is sufficiently complete for the flow field computation. Hence, the SSF method is developed to circumvent these limitations. The SSF model accounts for the effects of turbulent fluctuations on the interface quantities, the turbulent dispersion of particles due to the gas velocity fluctuation, and the gas turbulence modulation caused by the particles.

Two approaches have been pursued for the particle-laden jet flows. When the mass loading of the particles at the nozzle exit is less than 0.1, the gas flow field is assumed to be unaffected by the presence of the particles. This approach is referred to as one-way coupling[3-5]. When the mass loading ratio is high, there is two-way coupling between the gas and the particles[6-10], where the particles act as sources of mass, momentum, and energy for the gas, and the gas controls the motion of the particles. The initial work on these problems concentrated on turbulent dispersion at the small-particle limit[11]. This implies a linear interphase transport (Stokes flow), and that the particles should remain within a single fluid element (eddy) during their motion. Later, the ideas were extended to the turbulent modulation and crossing trajectory effects, representing the fact that the particles and the turbulent eddies follow different trajectories and interact only for a time[12]. Hutchinson *et al*[13] reported one of the earliest studies along these lines, proposing many of the ideas used in current SSF models. Yuu *et al* adopted[3] a procedure similar to the one followed by Hutchinson *et al*[13] to analyse the turbulent diffusivities in the particle-laden jets.

The present analysis is an extension of the above treatment for the particle-laden free-jet flows. It takes into account the one-way coupling, continuous phase to particle phase, with interaction time governed only by eddy lifetime. The eddy properties are calculated using the existing empirical correlations for jet properties. As the particles move through the flow, they are assumed to interact with a succession of turbulent eddies, simulating actual conditions by a random-walk computation. The mean dispersion properties are obtained by averaging over a statistically significant number of particle trajectories.

2. TURBULENT GAS-PARTICLE JET

The traditional approach of modelling particle dispersion in turbulent flows is to treat the process as the Fickian diffusion process in which the diffusional mass flux is proportional to the concentration gradient. The basic problem is to derive an expression for diffusion coefficient. Batchelor[14] noted that when the probability distribution of the displacement of particles has a Gaussian form, the diffusivity can be interpreted as

$$\varepsilon_{pr} = \frac{1}{2}\frac{d}{dt} < \left(r_p - <r_p>\right)^2 > \qquad (1)$$

It was also reported that the diffusivity initially increases with time and becomes constant eventually. Hinze[11] analysed the particle movements in the turbulent flow whose intensity and time-averaged velocity are constant, by using the equation

$$\frac{dv_{pi}}{dt} + \frac{36\mu}{(2\rho_p+\rho)d^2}(v_{pi}-v_i) = \frac{3\rho}{2\rho_p+\rho}\frac{dv_i}{dt} \qquad \frac{18}{(2\rho_p+\rho)d}\sqrt{\frac{3\mu}{\pi}}\int_{t_0}^{t}\frac{\frac{dv_i}{dt'}-\frac{dv_{pi}}{dt'}}{\sqrt{t-t'}}\,dt' \qquad (2)$$

Further the relationship between the particle diffusivity ε_{pr} and the fluid lump diffusivity ε_f was also provided. Assuming the similarity of distribution of the axial (U) and the radial (V) velocities, and the particle concentration (C) in a free-jet and in the region where the particle and the gas-averaged velocities are equal, Hinze gave the relationship as

$$\varepsilon_{pr} \quad \varepsilon_m \ln\frac{U}{U_m} / \ln\frac{C}{C_m} \qquad (3)$$

The experimental value obtained in the main region of the jet is given by

$$\varepsilon_m = 0.013\ U_0\ D \qquad (4)$$

The above relationship obtained by Hinze is not applicable in the developing regions of the jet flow, when the time-averaged particle velocities are not equal to the time-averaged gas velocities. The objective of the present work is to compute the diffusivities in the developing regions of the two-phase turbulent jet, invoking the Lagrangian and the Eulerian considerations for the particle and the gas flows, respectively.

3. ISOTHERMAL JET FLOW

To compute the particle trajectories and the properties of the two-phase turbulent jet, the time-averaged velocities, fluctuating velocities and integral fluid length scales are needed for the gas phase. As the properties are needed at each point in the jet, the experimental values fitted into polynomials are used for mathematical simplicity. Figure 1 shows the coordinate system for the free-jet flow. The empirical formulae for the gas phase provided by Yuu *et al*[3] are reproduced below.

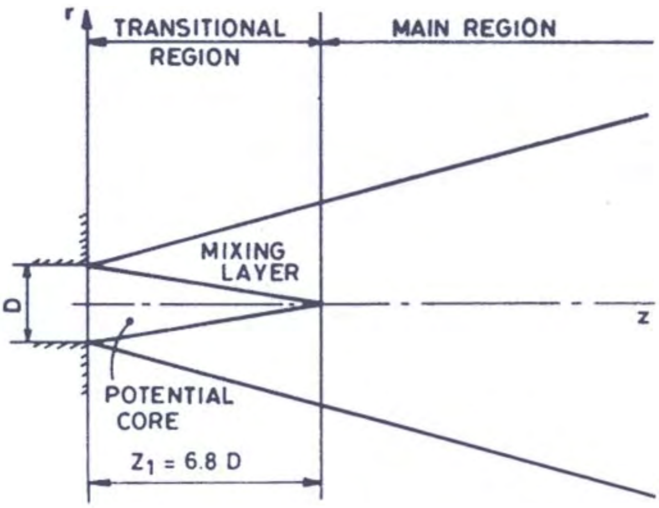

Figure 1. The jet model.

3. Time-Averaged Fluid Velocity

(a) Potential core

$$\bar{U} = 1\ ;\ \bar{V} = 0 \qquad (5)$$

(b) Mixing layer

$$\bar{U} = 92.6\left(\frac{\bar{r}-0}{\bar{z}}\ \ 3.6\right) + 340\left(\frac{\bar{r}-0.5}{\bar{z}}\ \ 3.6\right. \qquad (6)$$

$$\bar{V} = -4.05\left(\frac{\bar{r}-0.5}{\bar{z}}\right)^2 - 11.7\left(\frac{\bar{r}-0.5}{\bar{z}}\right)^3 + 255\left(\frac{\bar{r}-0.5}{\bar{z}}\right)^4 \quad (7)$$

(c) Main region

$$\bar{U} = (6.8 - 630\eta^2 + 2313\eta^3)/\bar{z} \quad (8)$$

$$\bar{V} = (3.4 - 472\eta^3 + 1851\eta^4)/\bar{z} \quad (9)$$

3.2 Fluid Turbulent Intensity

(a) Mixing layer

$$< U_z'^2 > = 0.0217 \exp(-200\eta_1^2)\, U_o^2 \quad (10)$$

$$< U_r'^2 > = 0.0103 \exp(-217\eta_1^2)\, U_o^2 \quad (11)$$

(b) Main region

$$< U_z'^2 > = 1.91\, U_o^2 \exp(-154\eta^2)/\bar{z}^2 \quad (12)$$

$$< U_r'^2 > = 2.26\, U_o^2 \exp(-178\eta^2)/\bar{z}^2 \quad (13)$$

3.3 Integral Scale Distribution

(a) Centre line distribution

$$\bar{L}_m = \frac{L_m}{D} = 0.0132\,\bar{z} \quad (14)$$

(b) Radial distribution

$$\bar{z} \leq 4 \qquad \bar{L} = \bar{L}_m\,[\exp\{-100(\bar{r}+0.45)/\bar{z}\} + \exp\{-100(\bar{r}-0.45)/\bar{z}\}]^{0.5} \quad (15)$$

$$4 < \bar{z} \leq 10 \qquad \bar{L} = \bar{L}_m \exp\,[-40\eta^2] \quad (16)$$

$$\bar{z} > 10 \qquad \bar{L} = \bar{L}_m \exp\,[-50\eta^2] \quad (17)$$

The above empirical formulae agree well with the experimental values, with a maximum error of 4 per cent.

4. COMPUTATION OF PARTICLE TURBULENT DIFFUSIVITY

4.1 Particle Motion

The particle trajectories are determined using the Lagrangian formulation of the governing equations. Since $\rho_p/\rho > 10^3$, the particles cannot follow the fluid motion due to inertia. For the measurements to be considered, it is reasonable to neglect the virtual mass, the Bassett forces, the Magnus forces, and the gravity force in Eqn (2). Under these assumptions, the conservation of momentum yields.

$$\frac{dv_{pi}}{dt} + \frac{18\mu}{\rho_p d^2} v_{pi} - \frac{18\mu}{\rho_p d^2} v_i = 0 \quad (18)$$

The position of the particle is determined by

$$\frac{dx_{pi}}{dt} = v_{pi} \quad (19)$$

The Eqns (18) and (19) can be rewritten for an axi-symmetric jet flow in the non-dimensional form as

$$\psi \frac{d^2\bar{r}}{d\bar{t}^2} + \frac{d\bar{r}}{d\bar{t}} - \bar{v}_r = 0$$

$$\psi \frac{d^2\bar{z}}{d\bar{t}^2} + \frac{d\bar{z}}{d\bar{t}} - \bar{v}_z = 0 \quad (21)$$

The dimensionless parameter ψ, called the Stokes number, expresses the effect of the particle inertia in the flow.

The instantaneous fluid velocity components in Eqns (20) and (21) can be written as

$$\bar{v}_r = \bar{V} + \bar{v}_r'$$

$$\bar{v}_z = \bar{U} + \bar{v}_z'$$

The fluctuating velocity components $\bar{v}_z'$ and v_r' vary randomly in space and time. The fluid time-averaged velocities $\bar{U}$ and $\bar{V}$ vary in space. Hence, the Eqns (20) and (21) become stochastic non-linear equations, which can be solved by numerical methods using a proper model, invoking random processes.

4.2 Particle Trajectory Model

Figure 2 shows the path of a particle in the presence of turbulent eddies. A particle is assumed to interact with an eddy for a time, which is the smaller of the eddy lifetime and the transit time required for the particle to cross the eddy. The eddy lifetime and the

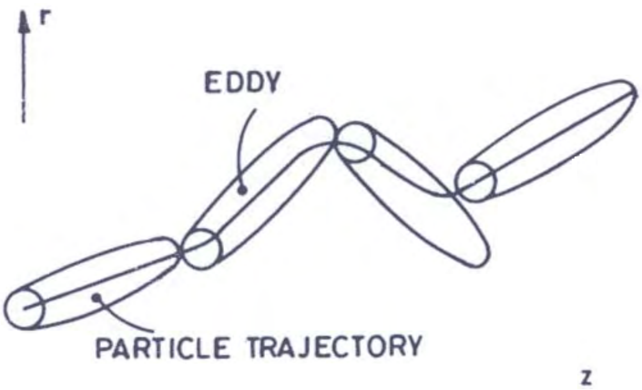

Figure 2. Particle trajectory model.

transit time are determined by assuming that the characteristic size of the randomly sampled eddy is the Lagrangian integral length scale. The eddy lifetime is then estimated as

$$t_L = L / \sqrt{<\bar{u}'^2>} \quad (24)$$

where

$$<\bar{u}'^2> = <\bar{u}_z'^2> + <\bar{u}_r'^2> + <\bar{u}_\theta'^2>$$

$$<\bar{u}_r'^2> = <\bar{u}_\theta'^2>$$

based on the experimental results of Heskestad[15].

The transit time is determined[4] as

$$t_t = -\tau \ln \left(1 - L / \tau(v - v_p)\right) \quad (25)$$

In the present case, as the particles are assumed to be very small (fly ash dust of 15-20 μm) to correspond with the fluid particles, all interaction times are given by t_L, i.e., the particles traverse the eddy for the whole of their lifetime before getting into the next eddy. Therefore, the particle moves, surrounded by the eddy which has a lifetime t_{Li} when $t_i \leq t \leq t_i + t_{Li}$. If the position of the particle has the coordinates ($r = r_i$ and $z = z_i$) and particle velocity components v_{pri} and v_{pzi} at $t = t_i$, the fluid fluctuating components v_r and v_z' at the same point are obtained by a random process.

The following procedure outlines the methodology of computation of the fluid fluctuating velocities. To cater to two coordinate axes, two uniform random numbers (U_1 and U_2) of multiplicative congruential type are generated using any library routine available in the digital computer. As the fluctuating velocities are assumed to follow a Gaussian distribution, the normal random numbers are generated using the Box-Muller Scheme[16]. The normal random numbers (Y_1, Y_2) are given by

$$Y_1 = \cos(2\pi U_2) \sqrt{-2 \ln(U_1)}$$

$$Y_2 = \sin(2\pi U_2) \sqrt{-2 \ln(U_1)}$$

The fluid fluctuating velocities are then set equal to the product of the normal random numbers and the corresponding fluid turbulent intensities given in Eqns (10)-(13). Therefore,

$$v_r' = Y_1 \sqrt{< v_r'^2 >}$$

$$v_z' = Y_2 \sqrt{< v_z'^2 >}$$

It is to be noted that the random numbers Y_1 and Y_2 remain constant for each eddy, unlike the fluid turbulent intensities[8].

The Eqns (5)-(9) give the time-averaged fluid velocity components $\bar{U}$ and $\bar{V}$. Substituting them in Eqns (22) and (23) and solving Eqns (20) and (21) by the second order Runge-Kutta method provide the position of the particles and the corresponding instantaneous particle velocities during the interval t_{Li}. The procedure is repeated until the whole domain of the jet under consideration is traversed.

5. RESULTS AND DISCUSSION

Figure 3 shows the typical trajectories of particles in the turbulent jet flow as a result of the random walk computation described in the previous section. As the turbulent flow field in a jet is homogeneous in time, these particles can be averaged to obtain the gross properties of the two-phase flow.

Figure 4 shows the time-averaged velocities of the particles and the gas as a function of axial distance. The initial conditions at the jet exit are $U_p = U_0$ and $V_p = 0$. About 200 particle trajectories starting at the same time from a particular position ($\bar{r} = 0.1$) at the jet exit, are averaged to obtain the time-averaged particle velocities at a point calculated by Eqns (20) and (21).

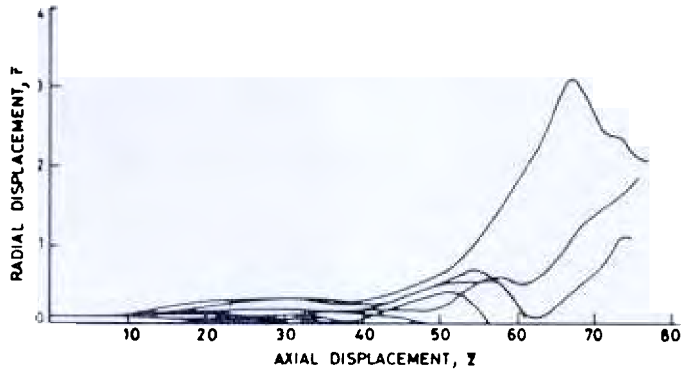

Figure 3. Particle trajectories (random process).

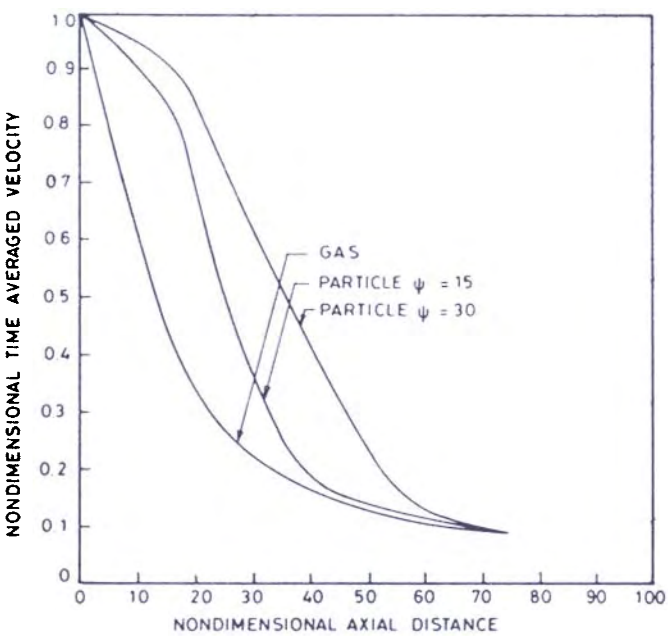

Figure 4. Time-averaged velocity profiles for particle and gas.

The time-averaged (ensemble-averaged) particle velocities normalised by the jet exit velocity shown in Fig. 4, indicate that the particles keep their initial velocities due to their inertia in comparison with the gas phase, i.e., larger the Stokes number smaller is the velocity decay of the particles along the axis. The particle-averaged velocities are nearly equal to the averaged gas phase velocities far away from the jet exit, viz $\bar{z} \simeq 42$ for $\psi = 15$ and $\bar{z} \simeq 65$ for $\psi = 30$.

Figure 5 shows the dimensionless particle turbulent diffusivity along the axis of the jet. It is calculated from the modified form of Eqn (1) as

$$\bar{\varepsilon}_{pr} = <\bar{r}_p \bar{v}_p> - <\bar{r}_p> <\bar{v}_p> \quad (30)$$

The time-averaged particle displacement $<\bar{r}_p>$ and the particle velocity $<\bar{v}_p>$ are obtained by averaging over a large number of particle trajectories (about 2000) at an arbitrary time. Once the time and averaged displacement are known, the axial coordinate gets fixed up automatically. The variable parameter $\bar{r}_0$ in this figure is the dimensionless radial distance in the jet from where a large number of particles get started at the same time. Examination of Fig. 5 reveals that the particle diffusivity in the jet decreases with increase in Stokes number. This is expected as the particle inertia increases with increase in Stokes number, which does not allow the particles to disperse easily in the flow.

Figure 5 shows that the diffusivity increases with the axial distance initially, and becomes nearly constant when the time-averaged velocities of the particles and the gas become equal. This gives an indication that the diffusivity becomes maximum when the two-phase flow is fully developed ($U_p \simeq U$). Also, the distance needed for the gas-particle flow to develop fully is more than

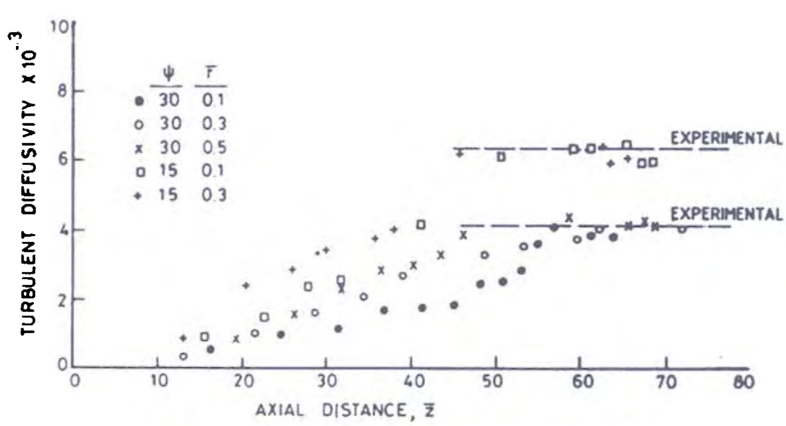

Figure 5. Variation of turbulent diffusivity of particles.

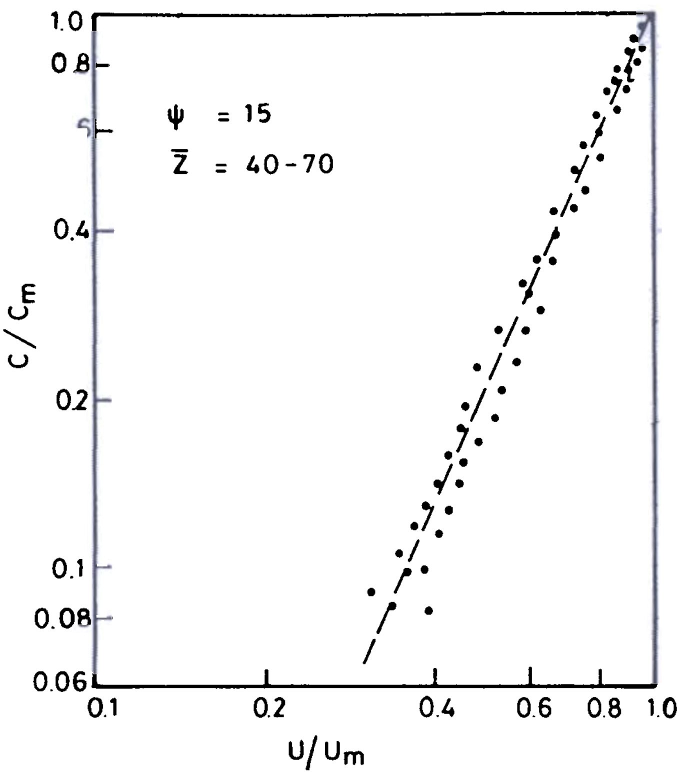

Figure 6. Experimental results in $U_p = U$ region[3]

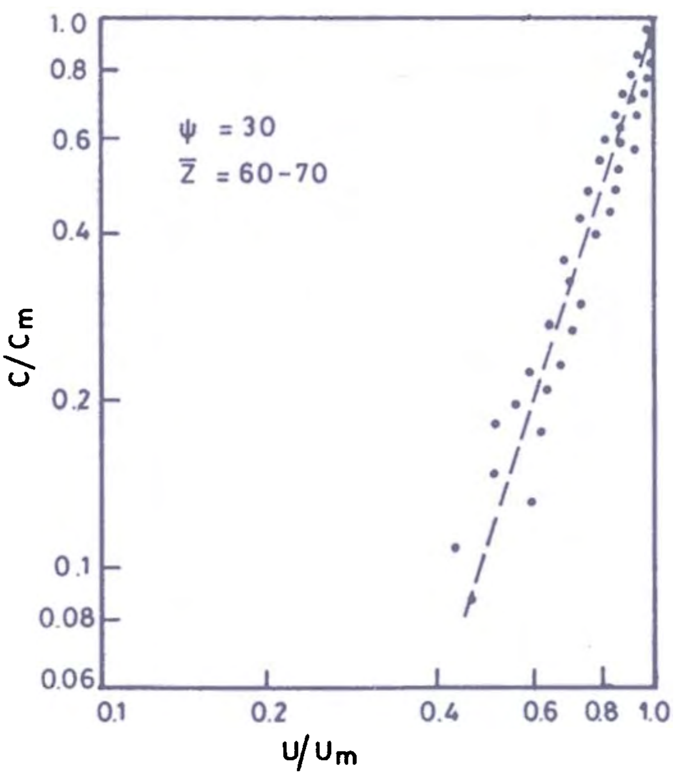

Figure 7. Experimental results in $U_p \neq U$ region[3]

that for the pure gas flow development. Figures 6 and 7 show the experimental data obtained by Yuu *et al*[3] on the particle concentration with respect to the axial velocity. The data correspond to the fully developed gas-particle flow region. The diffusivity data can be extracted from these figures using Eqns (3) and (4). The data obtained through experiments are in agreement with the results obtained based on the random walk computations, as shown in Fig. 5.

6. CONCLUSIONS

The dilute, dispersed flow in the two-phase jet is computed by the SSF model where both the finite interphase transport rates and the dispersed-phase/ turbulence interactions are considered using random walk computations for the dispersed flow.

The particle inertia and the turbulent eddies are characterised by the Stokes number and the Lagrangian integral scale, respectively. The Lagrangian particle motion in the presence of Eulerian turbulent gas motion provides the turbulent diffusivities in the whole region of the jet flow. With this model, the region of fully developed two-phase flow could be located for various Stokes numbers precisely. The experimental particle turbulent diffusivities extracted from the concentration and the velocity data of Yuu *et al*[3] in the fully developed gas-particle flow region, are in agreement with the stochastic dispersion model.

ACKNOWLEDGEMENTS

One of the authors (VR) thanks Shri M Ganapathi, Scientist, Institute of Armament Technology, Pune, for his assistance in the preparation of the computer code.

REFERENCES

Faeth, G.M. Evaporation and combustion of sprays. *Prog. Energy and Comb. Sci.*, 1983, **9**, 1-76.

2 Faeth, G.M. Mixing, transport and combustion in sprays. *Prog. Energy and Comb. Sci.*, 1987, **13**, 293-345.

3. Yuu, S.; Yasukouchi, N.; Hirosawa, Y. & Jotaki, T. Particle turbulent diffusion in a dust laden round jet. *AIChE J.*, 1978, **24**(3), 509-19.

4 Brown, D.J. & Hutchinson, P. The interaction of solid or liquid particles and turbulent fluid flow fields. *J. Fluids Engineering*, 1979, **101**, 265-69.

5 Shuen, J.S.; Chen, L.D. & Faeth, G. M. Evaluation of a stochastic model of particle

dispersion in a turbulent round jet. *AIChE J.*, 1983, **29**(1), 167-70.

Mostafa, A.A. & Elghobashi, S.E. A two-equation turbulence model for jet flows laden with vaporising droplets. *Int. J. Multiphase Flow*, 1985, **11**(4), 515-23.

7. Kirwan, J.E.; Lee, T.A.; Schroering, G.N.; Krier, H.; Peters, J.E.; Renie, J.P. & Kim, K. Experimental and theoretical study of a monodisperse spray. *J. Propulsion and Power*, 1988, **4**(4), 299-307.

8. Asheim, J.P.; Kirwan, J.E. & Peters, J.E. Modelling of a hollow cone liquid spray including droplet collisions. *J. Propulsion and Power*, 1988, **4**(5), 391-98.

9. Mostafa, A.A. & Mongia, H.C. On the interaction of particles and turbulent fluid flow. *Int. J. Heat & Mass Transfer*, 1988, **31**(10), 2063-75.

10. Mostafa, A.A.; Mongia, H.C.; McDonell, V.G. & Samuelsen, G.S. Evolution of particle-laden jet flows. *AIAA Journal*. 1989, **27**(2), 167-83.

11. Hinze, J.O. Turbulence, Ed. 2. McGraw-Hill, New York, 1975.

12. Gosman, A.D. & Ioannides, E. Aspects of computer simulation of liquid fueled combustors. *Journal of Energy*, 1983, **7**(6), 482-90.

13. Hutchinson, P.; Hewitt, G.P. & Dukler, A.E. Deposition of liquid or solid dispersions from turbulent gas streams : stochastic model. *Chemical Eng. Sci.*, 1971. **26**, 419-39.

14. Batchelor, G.K. Diffusion in free turbulent shear flows. *J. Fluid Mechanics*, 1957, **3**, 57.

15. Heskestad, G. Hot-wire measurements in a plane turbulent jet. *J. Applied Mechanics*, 1965, **87**, 721.

16. Brately, P.; Fox, B.L. & Schrage, L.E. A guide to simulation. Springer-Verlag, New York, 1983.