Defence Science Journal, Vol 42, No 1, January 1992, pp. 53-57


# Convective MHD Oscillatory Flow Past a Uniformly Moving Infinite Vertical Plate

N. Ahmed

*Department of Mathematics, Cotton College, Guwahati-781 001*

and

U.N.Das

*Department of Mathematics, Gauhati University, Guwahati–781 014*

**ABSTRACT**

The convective magnetohydrodynamic flow past a uniformly moving infinite vertical plate, with the magnetic field and the suction velocity applied normal to the plate has been analysed. Presence of heat source or sink has also been considered. The findings are expected to throw light on some problems of defence applications in the areas of aeronautical designs and also flow and heat transfer problems of a chemically reacting fluid.

## 1. INTRODUCTION

Unsteady free and forced convective magnetohydrodynamic (MHD) flow past a vertical porous plate has been studied widely because of their importance in aeronautics, missile aerodynamics, etc. When the difference between the wave temperature and the ambient fluid temperature is quite appreciable, it causes free convective currents to flow in the boundary layer and consequently the skin friction and rate of heat transfer at the walls are affected. The transverse magnetic field and suction or injection at the walls also influence the flow pattern and hence the skin friction and the rate of heat transfer at the walls to a large extent.

Stuart[1] studied the oscillatory flow over an infinite porous plate with constant suction at the plate. Earlier a systematic study of the effects of the free stream oscillation on the laminar skin friction and heat transfer on semi-infinite plate and cylindrical bodies was carried out by Lighthill[2]. Soon after this work there has been a host of papers in the literature using Lighthill's technique. Soundalgekar[3] studied the flow past an infinite vertical plate oscillating in its own plane and with wall temperature.

Messiha[4] studied two-dimensional incompressible fluid flow problem along an infinite flat plate with no heat transfer between the fluid and the plate when the suction velocity normal to the plate as well as the external flow varies periodically with time. This work was an extension of Stuart's[1] problem of constant suction to periodic suction velocity.

Vajravelu[5] made a systematic analysis of convective steady flow and heat transfer of a viscous heat generating fluid past a uniformly moving, infinite, vertical porous plate taking into account the source and sink effects.

This paper aims to extend Vajravelu's[5] works to unsteady and MHD case by considering a uniform transverse magnetic field. Vajravelu solved the problem numerically adopting Runge-Kutta-Gill method. In this study a perturbation technique was adopted to solve the problem giving stress on analytical solution.

Received 28 June 1990, revised 26 February 1991

## 2. BASIC EQUATIONS

Let us consider the combined free and forced convective motion of a viscous incompressible electrically conducting fluid past a flat infinite and uniformly moving porous plate by making the assumptions (i) all the fluid properties except density in the buoyancy force term are constant; (ii) the magnetic dissipation term in the energy equation is negligible; and (iii) the Eckert number $E$ and the magnetic Reynold number are small so that the induced magnetic field can be neglected.

The $x$-axis is taken along the upward vertical plate and $y$-axis perpendicular to it into the fluid region. All quantities except the pressure $p$ are independent of $x$. The velocity vector $\vec{q}$ and the applied magnetic field $\vec{B}$ may be taken as $\vec{q} = \hat{\imath}u + \hat{\jmath}v$, $\vec{B} = B_0\,\hat{\jmath}$ where $\hat{\imath}$, $\hat{\jmath}$ are the unit vectors along $x$-axis and $y$-axis respectively. With the foregoing assumptions and closely following Soundalgekar[6] and Messiha[4], the equations governing the flow and heat transfer of the problem become

$$\rho\left[\frac{\partial \bar{u}}{\partial \bar{t}} - v_0(1+\varepsilon A e^{i\bar{\omega}\bar{t}})\frac{\partial \bar{u}}{\partial \bar{y}}\right] =$$

$$\rho\frac{d\bar{U}}{d\bar{t}} + \mu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \sigma B_0^2(\bar{U}-\bar{u}) + g\rho\beta(\bar{T}-\bar{T}_\infty) \qquad (1)$$

$$\rho C_p\left[\qquad \partial \bar{y}\right.$$

$$K\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + Q(\bar{T}_\infty - \bar{T}) \qquad (2)$$

where $\rho$ is the density; $v_0$ is the mean suction velocity; $\sigma$ the electrical conductivity, $g$, the acceleration due to gravity; $\varepsilon$ is a small reference parameter, $C_p$, the specific heat at constant pressure; $K$, the thermal conductivity. $A$ is a positive real constant such that $\varepsilon A < 1$; $\mu$, the coefficient of viscosity; $\beta$ is the coefficient of volume expansion given by $\rho = \rho_\infty[1-\beta(\bar{T}-\bar{T}_\infty)]$ and the other symbols have the usual meanings.

The relevant boundary conditions are

$$\bar{y}=0 \quad \bar{u} \quad \bar{V}; \quad \bar{T} \quad \bar{T}_w \qquad (3)$$

$$\bar{y}\to\infty \quad \bar{u}\to\bar{U}; \quad \bar{T}\to\bar{T}_\infty \qquad (4)$$

We introduce the following nondimensional quantities:

$$y = \bar{y}v_0/\nu; \quad u = \bar{u}/U_0; \quad v = \bar{U}/U_0; \quad V = \bar{V}/U_0;$$

$$t = v_0^2\bar{t}/\nu; \quad \omega = \bar{\omega}\nu/v_0^2; \quad M = \sigma B_0^2\nu/\rho v_0^2;$$

$$P = \mu C_p/K; \quad \theta = (\bar{T}-\bar{T}_\infty)/\bar{T}_\infty; \; G = \nu g\beta\bar{T}_\infty/U_0 v_0^2;$$

$$E = U_0^2/C_p\bar{T}_\infty; \quad a = Q\nu^2/Kv_0^2 \qquad (5)$$

where $U_0$ is the mean stream velocity.

The nondimensional forms of the Eqns ( and (2) are

$$\frac{\partial u}{\partial t} - (1+\varepsilon A e^{i\omega t})\frac{\partial u}{\partial y} \qquad \frac{\partial^2 u}{\partial y^2} + M(U-u) + G\theta \qquad (6)$$

$$P\left[\quad \partial t \quad -(1+\varepsilon A e^{i\omega t})\frac{\partial \theta}{\partial y}\right.$$

$$\frac{\partial^2\theta}{\partial y^2} + EP\left(\frac{\partial u}{\partial y}\right)^2 - a\theta \qquad (7)$$

Now Eqns (6) and (7) are to be solved subject to the boundary conditions

$$y=0 \quad u \quad V \quad \theta = m = \frac{\bar{T}_w - \bar{T}_\infty}{\bar{T}_\infty} \qquad (8)$$

$$y\to\infty \quad u\to U, \; \theta\to 0 \qquad (9)$$

## 3. METHODS OF SOLUTION

To solve the Eqns (6) and (7) subject to the boundary conditions given in Eqns (8) and (9), velocity profile $u$, stream velocity $U$, temperature profile $\theta$ and plate temperature $m$ are broken into the following :

$$u = \quad - f_1(y \quad e \qquad y)]$$

$$\theta \quad \theta_0(y \qquad \theta_1(y)$$

$$m$$

Substituting Eqns (10) and (11) in Eqns (6–9) and equating the harmonic terms and neglecting $\varepsilon$ , we get the following equations:

$$f_1'' + f_1' - Mf_1 = G\theta_0 \quad (14)$$

$$f_2' + f_2' - (i\omega + M)f_2 = G\theta_1 - Af_1' \quad (15)$$

$$\theta_0'' + P\theta_0' - a\theta_0 = -EPf_1'^2 \quad (16)$$

$$\theta_1'' + P\theta_1 - (iP\omega + a)\theta_1 = -AP\theta_0' - 2EPf_1'f_2' \quad (17)$$

subject to the boundary conditions

$$y = 0 \quad f_1 = \quad - V, \quad f_2 = \quad \theta_0 = 1 \quad \theta_1 \quad (18)$$

$$y \to \infty \quad = f_2 = 0 \quad \theta_0 = \theta_1 = 0 \quad (19)$$

The Eqns (14–17) are still coupled for the variables $f_1, f_2, \theta_0$ and $\theta_1$. To solve them, it is to be noted that $E < 1$ for all incompressible fluids and assumed that

$$\left.\begin{aligned} \theta_0(y) &= \theta_{00}(y) + E\theta_{01}(y) \\ \theta_1(y) &= \theta_{10}(y) + E\theta_{11}(y) \\ f_1(y) &= f_{10}(y) + Ef_{11}(y) \\ f_2(y) &= f_{20}(y) + Ef_{21}(y) \end{aligned}\right\} \quad (20)$$

Substituting values of Eqn (20) in Eqns (14)–(17), equating the coefficients of $E^0$ and $E^1$ in each equation and neglecting $E^2$, the following equations are obtained.

$$f_{10}'' + f_{10}' - Mf_{10} = G\theta_{00} \quad (21)$$

$$f_{11}'' + f_{11}' - Mf_{11} = G\theta_{01} \quad (22)$$

$$f_{20}'' + f_{20}' \quad (i\omega + M)f_{20} = - Af_{10}' + G\theta_{10} \quad (23)$$

$$f_{21}'' + f_{21}' - (i\omega + M)f_{21} \quad - Af_{11}' + G\theta_{11} \quad (24)$$

$$\theta_{00}'' + P\theta_{00}' \quad a\theta_{00} \quad 0 \quad (25)$$

$$\theta_{10}'' + P\theta_{10}' - (ip\omega + a)\theta_{10} = -AP\theta_{00}' \quad (26)$$

$$\theta_{01}'' + P\theta_{01}' \quad (ip\omega + a)\theta_{01} = -AP\theta_{00}' \quad (27)$$

$$\theta_{11}'' + P\theta_{11}' - (ip\omega + a)\theta_{11} = -AP\theta_{01}' - 2\,Pf_{10}'\,f_{20}' \quad (28)$$

subject to the boundary conditions

$$y = 0 : \theta_{00} = 1, \theta_{01} = 0, \theta_{10} = 1, \theta_{11} = 0$$
$$f_{10} = 1, f_{11} = 0, f_{20} = \quad, f_{21} = 0$$

$$\left.\begin{aligned} y \to \infty \quad & \theta_{00} = \theta_{01} = \theta_{10} = \theta_{11} = 0 \\ & f_{10} = f_{11} = f_{20} = f_{21} = 0 \end{aligned}\right\}$$

The Eqns (21–28) are solved subject to the boundary conditions in Eqns (29) and (30) in the order of Eqns (25), (27), (21), (23), (26), (22), (28) and (24). The solutions are not presented here for the sake of brevity.

## 4. SKIN FRICTIONS AND HEAT TRANSFERS

The nondimensional skin friction $\tau_{yt}$ is given by

$$\tau_{yt} = \frac{\partial u}{\partial y} = - f_{10}'' \quad Ef_{11}' - \varepsilon e^{i\omega t}[f_{20}' + Ef_{21}'$$

The skin friction at the plate $y = 0$ is given by

$$\tau_0 \quad \frac{\partial u}{\quad}\Big|_{y=0} = - f_{10}'(0) - Ef_{11}'(0) - \varepsilon e^{i\omega t} [f_{20}'(0) + Ef_{21}'b(0)]$$

Splitting Eqn.(32) into real and imaginary parts, we get

$$\tau_0 = \tau_0^0 + \varepsilon |\,B\,| \cos(\omega t + a_0)$$

where $B = B_r + iB_i = f_{20}'(0) - Ef_{21}'(0)$

$$\tau_0^0 = -f_{10}'(0) - Ef_{11}'(0)$$
$$|\,B\,| = [B_r^2 + B_i^2]^{1/2}$$
$$\tan a_0 = B_i/B_r$$
$$B_r = R_e(B)$$
$$B_i = I_m(B) \quad (33$$

Expressions for $B_r$, $B_i$ are not presented here for the sake of brevity. The nondimensional heat transfer at the plate $y = 0$ is given by

$$Nu_0 = \theta'_0(0) + \varepsilon e^{i\omega t}\theta_1'(0)$$

Splitting Eqn (34) into the real and imaginary parts, we get

$$Nu_0 = Nu_0^{\,0} + \varepsilon \ |H| \cos(\omega t + \beta_0) \tag{35}$$

where

$$Nu_0^{\,0} = \theta_0'(0),\ H = \theta_1'(0) = H_r + iH_i,$$

$$\tan\beta_0 = H_i/H_r, \quad |H| = [H_r^2 + H_i^2]^{1/2} \tag{36}$$

## 5. RESULTS AND DISCUSSION

The graphs for the skin friction amplitude $|B|$, skin friction phase $\tan\alpha_0$, the heat transfer amplitude $|H|$ and the heat transfer phase $\tan\beta_0$ given by the Eqns (33) and (36) respectively are presented in the Figs. 1–4.

From the curves I, II, III in Fig. 1, it is observed that the skin friction amplitude $|B|$ increases with the Prandtl number $P$ for small values of $\omega$, but for larger values of $\omega$ this property is found to be reversed. From curves I and IV, it is seen that $|B|$ increases with the heat source parameter $a$.

Figure 1 also indicates that $|B|$ decreases sharply, when $\omega$ is increased for small values of $\omega$ but for larger values $|B|$ increases steadily with $\omega$ from a minimum value.

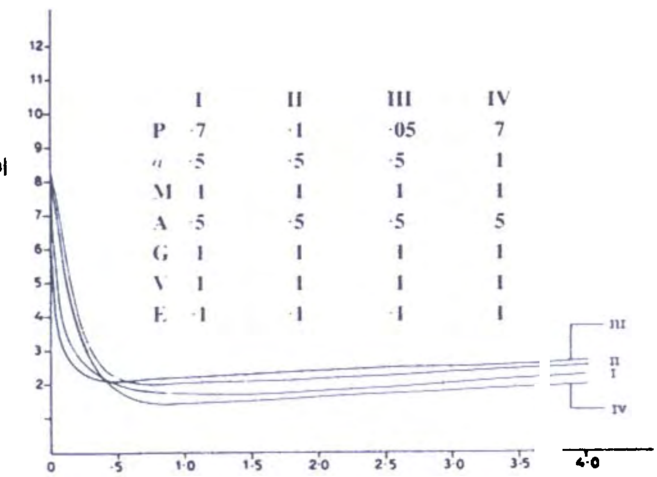


Figure 1 The skin friction amplitude $|B|$ versus frequency parameter

It is seen from Fig. 2 that the skin friction phase $\tan\alpha_0$ increases with $P$ (curves I, II, III), decreases when $M$ is increased (curves I, IV) and increases with $a$ (curves I, V). Figure 2 also indicates that skin friction phase difference $\alpha_0 \to \pi/2$ as $\omega \to 0$, and for small values of $\omega$ $\tan\alpha_0$ decreases when $\omega$ is increased whereas for larger values of $\omega$, it increases steadily with $\omega$ and approaches a constant value.

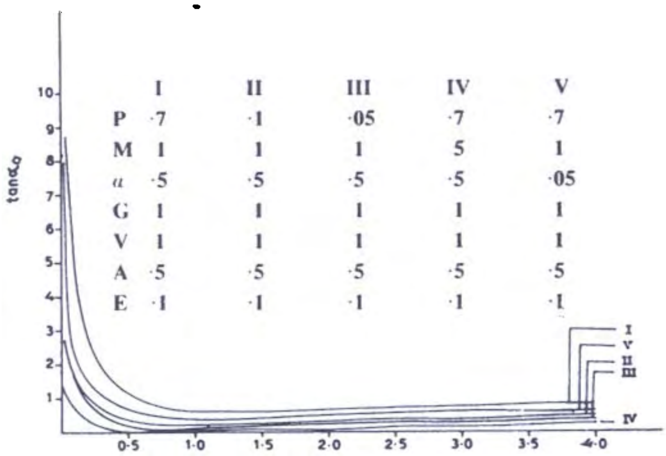


Figure 2. The skin friction phase $\tan\alpha_0$ against

Figure 3 clearly shows that the heat transfer amplitude $|H|$ increases with $P$ (curves I, II, III), $a$ (curves IV, V) and $\omega$. Figure 4 shows that the heat transfer phase $\tan\beta_0$ increases with $\omega$, $P$ (curves I, II, III) and decreases when $a$ is increased (curves IV, V)

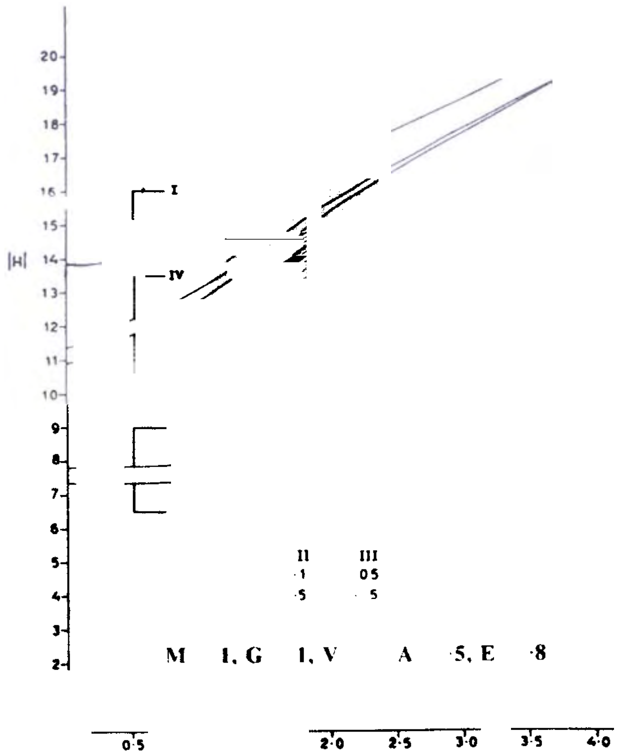


Figure 3. The heat transfer amplitude $|H|$

and also indicates that $\tan \beta_0 \to 0$ as $\omega \to 0_+$, that is $\beta_0 \to 0$ as $\omega \to 0_+$. Hence the heat transfer at the plate $y = 0$ for $\omega = 0$ has the same phase as the suction velocity.

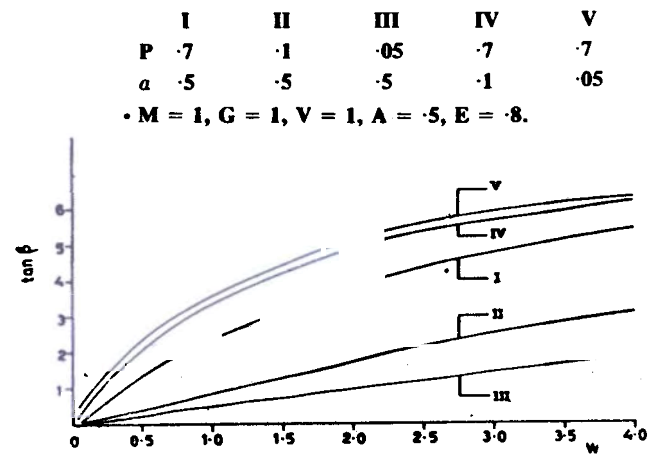


Figure 4. The transfer phase $\tan \beta_0$ Versus $\omega$.

### 5.1 Explanation for Boundary Conditions for the Plate Temperature

Messiha[4] has taken the nondimensional temperature as $\theta = (\hat{T} - \hat{T}_\infty) / \hat{T}_\infty$ and Soundalgekar[6] as $\theta = (\hat{T} - \hat{T}_\infty) / (\hat{T}_\omega - \hat{T}_\infty)$ and their boundary conditions for $\theta$ at the plate are $\delta\theta/\delta y = 0$ and $\theta = 1$ respectively. We have taken here $\theta = (\hat{T} - \hat{T}_\infty) / \hat{T}_\infty$ same as taken by Messiha[4] but a different plate temperature $\theta_\omega = m = 1 + \varepsilon e^{i\omega t}$. *Messiha*[4] has considered only the forced convection case where the velocity component $u$ remains uncoupled from the temperature $\theta$ and the energy equation is easily solved for $\theta$ once $u$ is obtained from the Navier-Stokes equation. Further, wall temperature corresponds to the adiabatic condition at the wall, i.e., there is no heat exchange between the wall and the neighbouring fluid layers.

On the other hand Soundalgeakar[6] and the present study considered free convection flow where the velocity and temperature become coupled, Soundalgekar's[6] boundary condition at the plate for $\theta$ is same as that chosen in the present study except for the small oscillating $\varepsilon e^{i\omega t}$. Here Soundalgekar[6] has considered that steady problem for which the unsteady part $\varepsilon e^{i\omega t}$ was not necessary. Whereas in this paper, this added term facilitates the decomposition of the governing coupled equations into simple component equations corresponding to various powers of $\varepsilon$, that is, to the steady and unsteady parts. Further, from physical point of view also, this is quite reasonable. As in the case of free stream velocity, we have superimposed a small oscillating part over a steady mean wall temperature. It is a familiar procedure in solving oscillating problems by Lighthill's[2] technique. In fact Singh, *et al*[7], while studying the fluctuating boundary layer on a neated horizontal plate, have taken the same condition where the plate temperature oscillates harmonically in time about a non zero mean.

Similarly, the boundary condition $u = V$ at $y = 0$ is because of the uniform motion of the vertical plate paralled to itself. At $y \to \infty$, the condition $u = U = 1 + \varepsilon e^{i\omega t}$ is same as taken by Messiha[4], but different from that considered by Soundalgekar [6]. The definitions of $\theta = (\hat{T} - \hat{T}_\infty) / \hat{T}_\infty$ or $\theta = (\hat{T} - \hat{T}_\infty) / (\hat{T}_\omega - \hat{T}_\infty)$ have no essential difference. By putting $a = 0$ in Eqn (7) and $V = 0$ we recover Messiha's[4] problem and by putting $V = 0$ and omitting the oscillating part we can go back to Soundalgekar's[6] problem.

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