Defence Science Journal, Vol 43, No 4, October 1993, pp 419-428
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# Unsteady Hele-Shaw Flow of a Conducting Rivlin-Ericksen Fluid

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**ABSTRACT**

This paper aims to study the unsteady Hele-Shaw flow of conducting Rivlin-Ericksen fluid under the influence of a uniform transverse magnetic field. The study has been carried out when the pressure gradient is (i) proportional to $e^{int}$, (ii) zero for $t < 0$ and equal to a constant for $t \geqslant 0$, and (iii) proportional to $e^{-nt}$. It is interesting to note that the time for the motion to become steady when started from rest (case ii) is of the order $\sim 4\, s^2 / \pi^2$ where $s$ is the viscoelastic parameter. The rheological equation of the Rivlin-Ericksen fluid is described in section 2. The expressions for velocity components $u$ and $v$ of the fluid in $x$ and $y$ directions are derived in section 3. The effects of magnetic field and viscoelasticity are discussed in section 4.

## 1. INTRODUCTION

The mechanical behaviour of a large number of materials of industrial importance such as the synthetic fibres, high polymer solutions and many other highly viscous fluids cannot be explained fully by the classical linear stress-strain relations. Attempt in the last two decades, has been made to formulate more general non-linear theories which could take into account the observed behaviour of the non-Newtonian and elastic viscous fluids. Consequently, a large number of flow problems have been solved by using such general theories, either for some theoretical interest or from the point of view of comparing the experimental results with the predictions of various theories. As the general stress-strain relations are expressed by highly complicated non-linear differential equations, to work out solutions for such a class of fluids even for slow flows is not an easy task. Oldroyd[1] has considered a class of steady state problems which include the simple shearing flow, Couette flow, Poiseuille flow, flow through pipes of arbitrary cross section, and flow between two rotating cones, using his own theory of elastico-viscous fluids. He found that his theory could exhibit at least qualitatively almost all rheological behaviour of non-Newtonian fluids. Generalising the stress-strain velocity relations of classical hydrodynamics the rheological behaviour of the non-Newtonian liquids has been studied by Rivlin[2]. On account of the non-linear nature of the equation of state of even the simplest elastico-viscous fluid, it is almost impossible to obtain the exact solutions of the equations of motion and one has to resort to the approximate methods. In most of the investigations of elastico-viscous fluids, the flow has been considered slow and parameters characterising the elastic properties of the fluid have been assumed small. In fact the increase emergence of non-Newtonian fluids such as the molten plastics, pulps, emulsions, aqueous solutions of polyacrylamid and polyisobutylene, etc., as important raw materials and chemical products in a large variety of industrial processes has stimulated a considerable attention in recent years to the study of non-Newtonian fluids and their related transport processes. Lee and Fung[3], Buckmaster[4], Lamb[5], and Thompson[6] have discussed the steady Hele-Shaw flows of viscous incompressible fluids assuming the pressure gradient to be constant. Swaminathan[7] has devoted to the study of unsteady Hele-Shaw flow of viscous incompressible liquid and has discussed the cases when the pressure gradient is (i) proportional to $e^{int}$, (ii) zero for $t < 0$ and equal to a constant for $t \geqslant 0$.

Received 19 March 1991, re-revised 12 October 1992

## 2. RHEOLOGICAL EQUATIONS OF STATE

The constitutive equation for a Rivlin-Ericksen is

$$T = - PI + \phi_1 d + \phi_2 b + \phi_3 d^2 \quad (1)$$

where

$T \| T_{ij} \|$, $T_{ij}$ is the stress tensor

$I = \| \delta_{ij} \|$, $\delta_{ij}$ is the Kroneckar delta

$d = \| d_{ij} \|$,

$d_{ij} = 1/2\ (w_{i,j} + w_{j,i})$ (deformation rate tensor)

$b = \| b_{ij} \|$, $b_{ij} = a_{i,j} + a_{j,i} +$

$2W_{m,i}\ W_{m,j}$ (acceleration gradient tensor)

with

$a_i = \dfrac{\partial w_i}{\partial t} + w_{ij}\, w_j$ (acceleration vector)

The $\phi_1$, $\phi_2$, $\phi_3$ are coefficients of viscosity, viscoelasticity and cross-viscosity respectively and these are in general functions of temperature, material properties and invariants of $d$, $b$, $d^2$. For many liquids and aqueous solutions of polyacrylamide and polybutylene $\phi_1$, $\phi_2$, $\phi_3$ may be taken as constant.

## 3. FORMULATION AND SOLUTION OF THE PROBLEM

The flow of a conducting viscoelastic second grade fluid (Rivlin-Ericksen) confined between two parallel plates $z = \pm\, h$ past a circular cylinder $x^2 + y^2 = a^2$, $-h \leqslant z \leqslant h$, under the influence of a uniform transverse magnetic field has been considered (Fig. 1). Assume

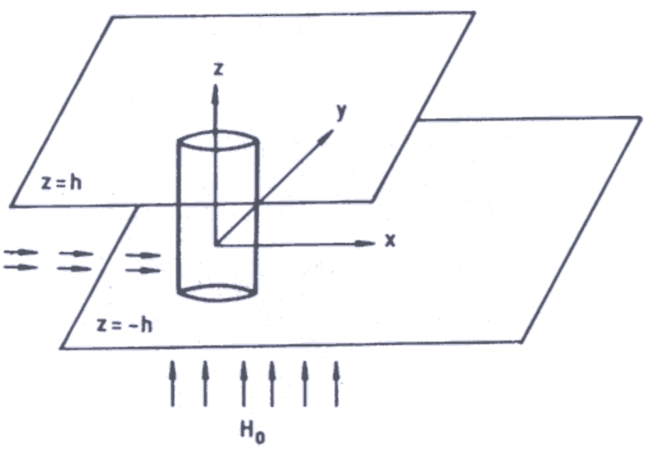


**Figure 1. Flow past a circular cylinder.**

that the fluid is of small electrical conductivity with the magnetic Reynolds number much less than unity so that the induced magnetic field can be neglected in comparison with the applied magnetic field (Sparrow and Cess[8]). In the absence of any input electric field, the equations governing the motion of the conducting Rivlin-Ericksen fluid in the Hele-Shaw cell under the influence of a uniform transverse magnetic field are

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + (\nu + \beta\frac{\partial}{\partial t}\ \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\sigma \mu_e^2 H_0^2}{\rho}\, u$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + (\nu + \beta\frac{\partial}{\partial t})\ \frac{\partial^2 v}{\partial z^2}$$

$$\frac{\sigma \mu_e^2 H_0^2}{\rho}\, v$$

where $u$ and $v$ are the components of the fluid velocity in the $x$ and $y$ directions respectively, $t$ the time, $\rho$ the constant fluid density, $p$ the fluid pressure, $\nu$ the coefficient of kinematic viscosity, $\beta$ the kinematic viscoelasticity, $\sigma$ the electrical conductivity of the fluid, $\mu_e$ the magnetic permeability and $H_0$ is the intensity of the magnetic field introduced in the $z$ direction.

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

If $\beta \to 0$ the above equations reduce to the case of incompressible viscous conducting fluid, then

$$u = 0,\ v = 0 \text{ at } z = \pm\, h \quad (5)$$

The following non-dimensional quantities introduced :

$$z^* = \frac{U_z}{\nu},\ t^* = \frac{tU^2}{\nu},\ u^* = \frac{u}{U},$$

$$v^* = \frac{u}{U},\ y^* = \frac{U_y}{\nu},\ p^* = \frac{P}{U^2}$$

$$x^* = \frac{ux}{\nu} \quad (6)$$

where $U$ is a characteristic velocity.

In view of Eqn (6), the Eqns 2 to 4 reduces to (after neglecting superscripts)

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \left(1 + s\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial_z^2} - Mu \tag{7}$$

$$\frac{\partial y}{\partial t} = -\frac{\partial p}{\partial x} + (1 + s\frac{\partial}{\partial t}\frac{\partial^2 y}{\partial^2 x^2} - MY \tag{8}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

where $s = \frac{\beta U^2}{\nu^2}$ (viscoelastic parameter)

$M = \frac{\sigma\mu_e^2 H_0^2}{\rho U^2}$ (magnetic parameter)

The non-dimensional boundary conditions are

$$u = 0,\ v = 0 \text{ at } z = \pm R \tag{10}$$

where $R = \frac{Uh}{\nu}$

Using Eqn (9), the Eqns (7) and (8) gives

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \tag{11}$$

Hence $p$ is a function of $x$, $y$, $z$ and $t$
Suppose that

$$u = f(t,z)\frac{\partial \phi}{\partial x} \text{ and } v = f(t,z)\frac{\partial \phi}{\partial y} \tag{12}$$

where $\phi$ is a function of $x$ and $y$
Substituting Eqn (12) in Eqn (9)

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{13}$$

From Eqns (7), (8) and (12) the following is obtained

$$P_x = \phi_x (f_{zz} + sf_{tzz} - f_t - Mf) \tag{14}$$

where $f_{zz} = \frac{\partial^2 f}{\partial z^2}$, $f_{tzz} = \frac{\partial^3 f}{\partial t \partial z^2}$,

$f_t = \frac{\partial f}{\partial t}$, $p_x = \frac{\partial p}{\partial x}$, $\phi_x = \frac{\partial \phi}{\partial x}$

and

$$p_y = \phi_y (f_{zz} + sf_{tzz} - f_t - Mf) \tag{15}$$

Integrating Eqns (14) and (15) it was obtained

$$p = \phi (f_{zz} + sf_{tzz} - f_t - Mf) + \psi(t)$$

where $\psi(t)$ is an arbitrary function of time

### 3.1 Periodic Motion

Let the pressure gradient be proportional to $e^{int}$

Let

$$f_{zz} + sf_{tzz} - f_t - Mf = -Ae^{int}$$

where $A$ is a given constant

$$f(t, z) = e^{int} F(z) \tag{18}$$

Then $F$ satisfies the equation

$$\frac{d^2 F}{dz^2} - \frac{(in + M)}{(1 + isn)} F = \frac{-A}{(1 + isn)}$$

Now the boundary conditions are

$$F = 0 \text{ at } z = \pm R$$

Solving Eqn (19) and using Eqn (20) it is obtained

$$\begin{aligned} F = A\,[\,\{&\cos h\,(cR\cos B)\cos(cR\sin B) \\ &+ i\sin h\,(cR\cos B)\sin(cR\sin B)\} \\ &\cos h\,(cz\cos B)\cos(cz\sin B) \\ &+ i\sin h\,(cz\cos B)\sin(cz\sin B)\}\,] \\ &\div (in - M) \quad \cos h\,(cR\cos B) \\ &\cos(cR\sin B) + i\sin h\,(cR\cos B) \\ &\sin(cR\sin \end{aligned} \tag{21}$$

where

$$c = \frac{\sqrt{M^2 + n^2}}{\{(M + sn^2) + n^2(1 - sM)^2\}^{1/4}} \tag{22}$$

$$\operatorname{Tan} 2B = \frac{(1 - sM)}{(M + sn^2)}$$

the function $\phi(x, y)$ can be calculated by solving Eqn (13) subject to the condition

$$U\cos\theta + v\sin\theta = 0$$

$$\frac{\partial \phi}{\partial r} = 0, \text{ when } r = a \tag{23}$$

where $x = r\cos\theta$, $y = r\sin\theta$ and $\frac{\partial \phi}{\partial x} \rightarrow$

$$1, \quad \frac{\partial\phi}{\partial y} \rightarrow 0 \text{ as } /x/, /y/ \rightarrow \infty$$

$$\text{Hence } \phi(x,y) = (r + a^2/_r) \cos\theta \tag{24}$$

From Eqns (12), (17), (21) and (24) it is obtained that

$$u_{real} = [A \quad (M \cos nt - n \sin nt) \\ (a_1 a_2 + b_1 b_2 - a_2^2 - b_2^2) \\ - (M \sin nt + n \cos nt)(a_1 b_1 - a_1 b_2)\} \\ \div (M^2 + n^2)(a_2^2 + b_2^2)] \\ \times \left\{ 1 - \frac{a^2(x^2 - y^2)}{(x^2 + y^2)^2} \right\} \tag{25}$$

$$v_{real} = A \quad (M \cos nt - n \sin nt) \\ (a_1 a_2 + b_1 b_2 - a_2^2 - b_2^2) \\ - (M \sin nt + n \cos nt) \\ (a_2 b_1 - a_1 b_2)\} \\ \div (m^2 + n^2)(a_2^2 + b_2^2) \\ \times \left\{ \frac{-2a^2 xy}{(x^2 + y^2)^2} \right\} \tag{26}$$

where

$a_1 = \cos h\,(c\,z \cos B) \cos (cz \sin B)$

$b_1 = \sin h\,(cz \cos B) \sin (cz \sin B)$

$a_2 = \cos h\,(cR \cos B) \cos (cR \sin B)$

$b_2 = \sin h\,(cR \cos B) \sin (cR \sin B)$

and $u_{real}$ and $v_{real}$ denote the real parts of $u$ and $v$. When $n = 0$, we obtain

$$u = A \quad M (a_1 a_2 + b_1 b_2 - a_2^2 - b_2^2)\} \\ \div M^2 (a_2^2 + b_2^2)] \times \left\{ 1 - \frac{a^2(x^2 - y^2)}{(x^2 + y^2)^2} \right\} \tag{27}$$

$$v = [A \{ M (a_1 a_2 + b_1 b_2 - a_2^2 - b_2^2)\} \\ \div M^2 (a_2^2 + b_2^2)] \times \left\{ \frac{2a^2 xy}{(x^2 + y^2)^2} \right\} \tag{28}$$

which is the solution for steady flow in the Hele-Shaw cell.

If $M = 0$, $s = 0$ then $c = n^{1/2}$ and $\beta = \pi/4$

In this case the velocity components $u$ and $v$ for unsteady flow of viscous incompressible fluid are :

$$u = \{A (\sin nt + \cos nt)(a_2^2 + b_2^2 - a_1 a_2 - b_1 b_2 \\ \div n (a_2^2 + b_2^2)\} \times \left\{ 1 - \frac{a^2(x^2 - y^2)}{(x^2 + y^2)^2} \right\} \tag{29}$$

$$v = \{A (\sin nt + \cos nt)(a_1 a_2 + b_1 b_2 \\ - a_2^2 - b_2^2) \div \{ n (a_2^2 + b_2^2)\} \times \\ \left\{ \frac{2a^2 xy}{(x^2 y^2)^2} \right\} \tag{30}$$

The above results agree with those obtained by Swaminathan[7].

### 3.2 Impulsive Motion

The motion starts from a state of rest. In the Eqn (16) it is assumed that

$$f_{zz} + s f_{tzt} - f_t - Mf = -AH(t) \tag{31}$$

where

$$H(t) = 0 \text{ when } t < 0 \tag{32}$$

$$= 1 \text{ for } t \geq 0 \tag{33}$$

The Laplace transform is defined as

$$\bar{f} = \int_0^\infty f e^{-pt}\, dt \tag{34}$$

with the inversion

$$f = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \bar{f} e^{pt}\, dp \tag{35}$$

Using Eqn (34), The Eqn (31) is transformed to

$$\frac{d^2 \bar{f}}{dz^2} - \frac{(p+M)}{(1+sp)} \bar{f} = \frac{-A}{p(1+sp)} \tag{36}$$

The transformed boundary conditions are :

$$\bar{f}\,(p \pm R) = 0 \tag{37}$$

Solving Eqn (36) and using Eqn (37)

$$f(p,z) = \frac{A}{p^2}$$

$$\left[ 1 - \frac{\cos h \sqrt{(p+M)/(1+ps)}\, z}{\cos h \sqrt{(p+M)/(1+ps)}\, R} \right] \quad (38)$$

Inverting Eqn (38) using the inversion Eqn (35)

$$f(t,z) = \left[ \frac{A(R^2 - z^2)}{2} - \frac{A\pi}{R^2} \sum_{n=}^{\infty} \frac{1}{a^2_n} \right.$$

$$f(t,z) = \left[ \frac{A(R^2 - z^2)}{2} - \frac{A\pi}{R^2} \sum_{n=1}^{\infty} \frac{1}{a^2_n} \right.$$

$$\{ -1)^n (2n+1)(1 - sa_n)^2 e^{-a_n t}$$

$$\cos h \sqrt{a_n/(1 - sa_n)}\, z \quad ] \quad (39)$$

where $a_n = \dfrac{\pi^2 (2n+1)^2 + 4R^2 M}{4R^2 + \pi^2 (2n+1)^2 s}$

n = 0 or any integer

Using Eqns (12), (24) and (39) it was obtained

$$U = [A \frac{(R^2 - z^2)}{2} - \frac{A\pi}{R^2} \sum_{n=1}^{\infty} \frac{1}{a^2_n} \{ (1-\nu)^n$$

$$(2n+1)(1 - sa_n)^2 e^{-a_n t}$$

$$\cos h \sqrt{a_n/(1-sa_n)}\, z \ \} ] \times$$

$$\left\{ 1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right\} \quad (40)$$

$$V = \{ A \frac{(R^2 - z^2)}{2} - \frac{A\pi}{R^2} \sum_{n=1}^{\infty} \frac{1}{a^2_n} \{ (1-\nu)^n$$

$$(2n+1)(1 - sa_n)^2 e^{-a_n t}$$

$$\cos h \sqrt{a_n/(1-sa_n)}\, z \quad \times$$

$$\left\{ \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right\} \quad (41)$$

The time after which the motion starting from rest becomes steady may be obtained by requiring that the exponential terms in Eqns (40) and (41) be sufficiently small. Thus, setting $(\pi^2 t/4\, s^2) \simeq 10$, the exponent of e is $-10$ and $e^{-10} = 0.000045$.

### 3.3 Pressure Gradient is Proportional to $e^{-nt}$

Let

$$f_{zz} + s f_{tzz} - f_t - Mf = -A\, e^{-nt}$$

where $n$ is a positive constant and $A$ is a given constant.

Suppose $f(t, z) = e^{-nt} F(z)$

Now the Eqn (42) reduces to

$$\frac{\partial^2 F}{\partial z^2} + \frac{(n - M)}{(1 - sn)} F = \frac{-A}{(1 - sn)} \quad (44)$$

the boundary conditions are

$$F = 0 \text{ at } z = \pm R \quad (45)$$

Solving Eqn (44) using Eqn (45) we got

$$F(z) = \frac{A}{(1 - sn)}$$

$$\left[ \frac{\cos h \sqrt{(n - M)/1 - sn)}\, z}{\cos h \sqrt{(n - M)/(1 - sn)}\, R} - 1 \right] \quad (46)$$

Using Eqns (12), (24), (43) and (46);

$$u = \frac{A}{\ } e^{-nt}$$

$$\left[ \frac{\cos h \sqrt{(n - M)/(1 - sn)}\, z}{\cos h \sqrt{(n - M)/(1 - sn)}\, R} - 1 \right]$$

$$\times \left[ 1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right]$$

$$v = \frac{A}{\ } e^{-nt}$$

$$\left[ 1 - \frac{\cos h \sqrt{(n - M)/(1 - sn)}\, z}{\cos h \sqrt{(n - M)/(1 - sn)}\, R} \right] \frac{2a^2 xy}{(x^2 + y^2)^2}$$

#### 3.3.1 $M \quad 0, s \neq 0, n \neq 0$

The velocity components in the case of unsteady Hele-Shaw flow of viscoelastic fluid are

$$u = \frac{A}{\ } e^{-nt} \left[ \frac{\cos h \sqrt{(n/1 - sn)}\, z}{\cos h \sqrt{(n/(1 - sn)}\, R} - 1 \right]$$

$$\left\{ 1 - \frac{a^2 (x^2-y^2)}{\quad} \right\} \tag{49}$$

$$v = \frac{A}{\quad} e^{-nt} \left[ 1- \frac{\cos h \sqrt{(n/(1-sn))}\, z}{\cos h \sqrt{(n/(1-sn))}\, R} \right] \frac{2a^2xy}{(x^2+y^2)^2} \tag{50}$$

*3.3.2* $M \neq 0, s = 0, n \neq 0$

The velocity components in the case of unsteady Hele-Shaw flow of viscous incompressible conducting fluid under the influence of a uniform transverse magnetic field are :

$$u = \frac{A}{n-M} e^{-nt} \left[ \frac{\cos h \sqrt{n-M}\, z}{\cos h \sqrt{n-M}\, R} - 1 \right] \times \left\{ 1 - \frac{a^2 (x^2-y^2)}{(x^2+y^2)^2} \right\} \tag{51}$$

$$v = \frac{A}{\quad} e^{-nt} \left[ 1- \frac{\cos h \sqrt{n-M}\, z}{\cos h \sqrt{n-M}\, R} \right] \frac{2a^2xy}{(x^2+y^2)^2} \tag{52}$$

*3.3.3* $M \neq 0, s \neq 0, n = 0$

The velocity components in the case of steady Hele-Shaw flow of viscoelastic fluid under the influence of a uniform transverse magnetic field are

$$u = \frac{A}{M} \left[ 1- \frac{\cos \sqrt{M}\, z}{\cos \sqrt{M}\, R} \right] \times \left\{ 1 - \frac{a^2 (x^2-y^2)}{(x^2+y^2)^2} \right\} \tag{53}$$

$$v = \frac{A}{M} \left[ \frac{\cos \sqrt{M}\, z}{\cos \sqrt{M}\, R} - 1 \right] \frac{2a^2xy}{(x^2+y^2)^2} \tag{54}$$

*3.3.4* $M = 0, s = 0, n \neq 0$

The velocity components in the case of unsteady Hele-Shaw flow of viscous incompressible fluid are

$$u = \frac{A}{\quad} e^{-nt} \left[ \frac{\cos h \sqrt{n}\, z}{\cos h \sqrt{n}\, R} - 1 \right] \times \left\{ 1 - \frac{a^2 (x^2-y^2)}{(x^2+y^2)^2} \right\} \tag{55}$$

$$v = \frac{A}{\quad} e \left[ 1- \frac{\cos h \sqrt{n}\, z}{\cos h \sqrt{n}\, R} - 1 \right] \frac{2a^2xy}{(x^2+y^2)^2} \tag{56}$$

*3.3.5* $M = 0, s \neq 0, n = 0$

The velocity components in the case of steady Hele-Shaw flow of viscoelastic fluid are

$$u = \frac{A (R^2 - z^2)}{2} \left[ 1- \frac{a^2(x^2-y^2)}{(x^2+y^2)^2} \right] \tag{57}$$

$$v = \frac{A (R^2 - z^2)\, a^2\, xy}{(x^2 + y^2)^2} \tag{58}$$

It is interesting to note that these are the velocity components even in the case of steady Hele-Shaw flow of viscous incompressible fluid (Eqn (39)).

*3.3.6* $M \neq 0, s = 0, n = 0$

The velocity components in the case of steady Hele-Shaw flow of viscous incompressible conducting fluid are

$$u = \frac{A}{M} \left[ 1- \frac{\cos \sqrt{M}\, z}{\cos \sqrt{M}\, R} \right] \times \left\{ 1 - \frac{a^2 (x^2-y^2)}{(x^2+y^2)^2} \right\} \tag{59}$$

$$v = \frac{A}{M} \left[ 1- \frac{\cos \sqrt{M}\, z}{\cos \sqrt{M}\, R} - 1 \right] \times \left\{ \frac{2a^2\, xy}{(x^2+y^2)^2} \right\} \tag{60}$$

*3.3.7* $M = 0, s = 0, n = 0$

The velocity components in the case of steady Hele-Shaw flow of viscous incompressible fluid are

$$u = \frac{A\,(R^2 - z^2)}{2}\left\{ 1 - \frac{a^2(x^2-y^2)}{(x^2+y^2)^2} \right. \qquad (61)$$

$$v = \frac{A\,(R^2 - z^2)\,a^2\,xy}{(x^2 + y^2)^2} \qquad (62)$$

## 4. CONCLUSIONS

Figures 2 and 3 bring out the effects of magnetic and viscoelastic parameters $M$ and $s$, respectively on velocity component $u$ in case (i) discussed in section 3. It is observed that the velocity component $u$ increases as $M$ increases whereas it decreases with the increase in $s$. It is also noted that $u$ decreases as $z$ increases.

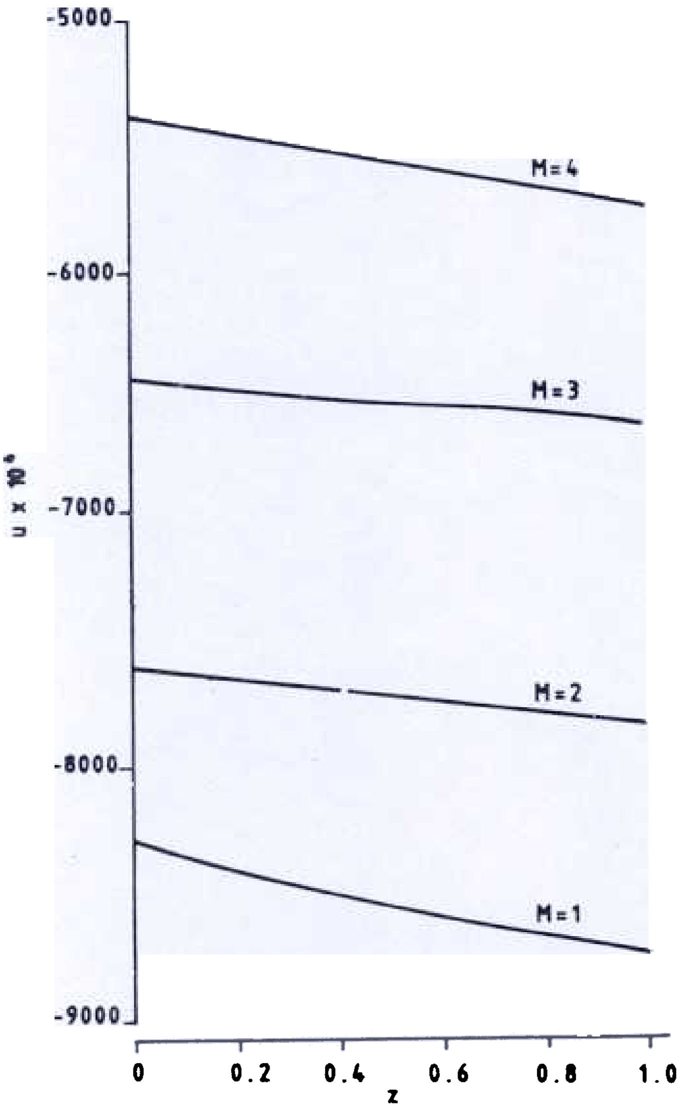


Figure 2. Velocity distribution $u$ against $z$ for different values of magnetic parameter $M$ (case i).

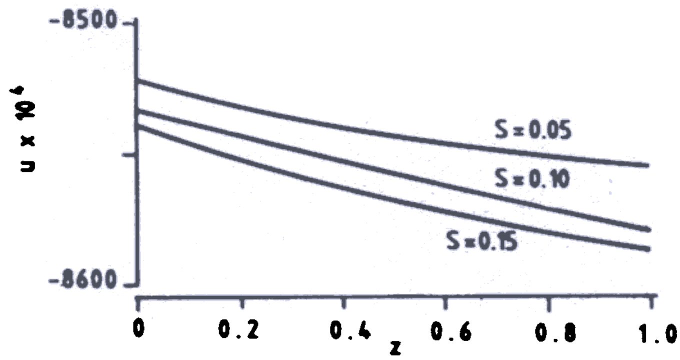


Figure 3. Velocity distribution $u$ against $z$ for different values of viscoelastic parameter $s$ (case i).

Figures 4 and 5 depict the effects of $M$ and $s$ on velocity component $v$ in case (i). It is observed that the velocity component $v$ increases with the increase in $z$. An

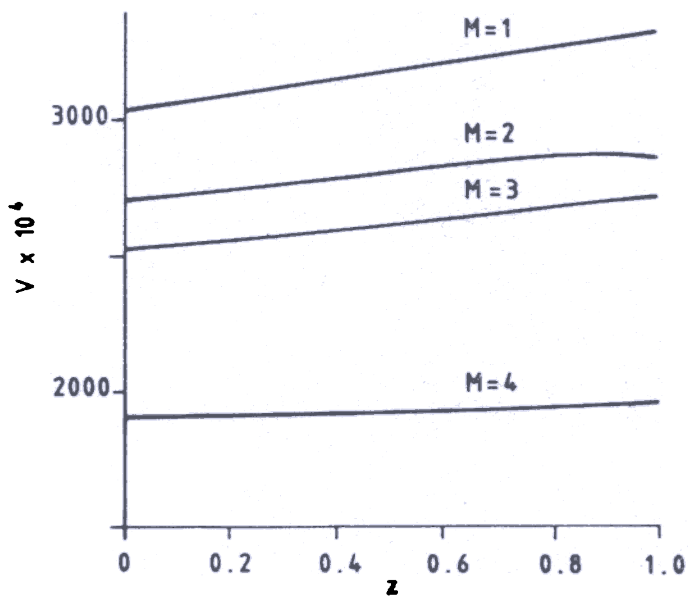


Figure 4. Velocity distribution $v$ against $z$ for different values of magnetic parameter $M$ (case i).

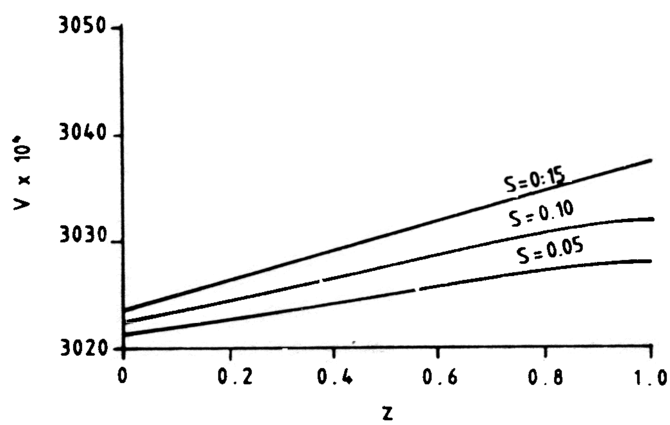


Figure 5. Velocity distribution $v$ against $z$ for different values of viscoelastic parameter $s$ (case i).

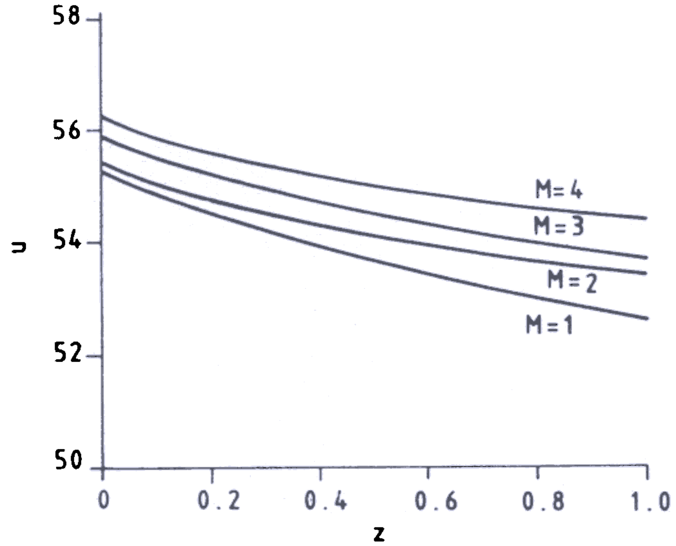


Figure 6. Velocity distribution $u$ against $z$ for different values of magnetic parameter $M$ (case i).

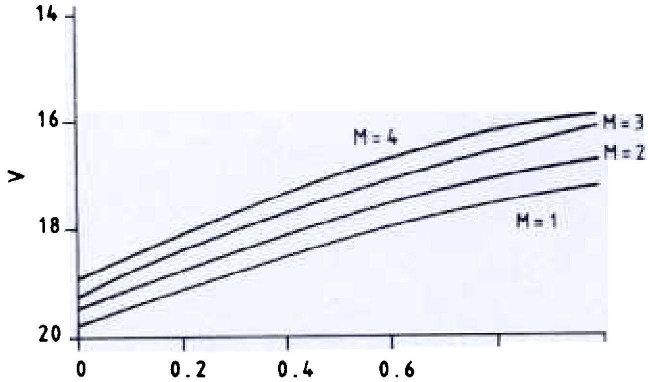


Figure 8. Velocity distrib tion $u$ against $z$ for different values of magnetic parameter $M$ (case ii).

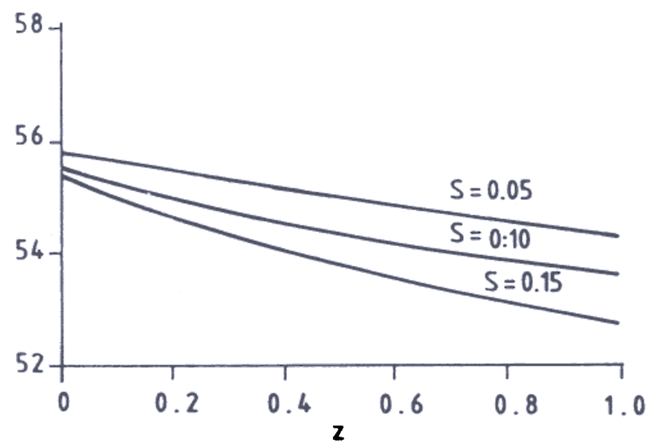


Figure 7. Velocity distribution $u$ against $z$ for different values of viscoelastic parameter $s$ (case ii).

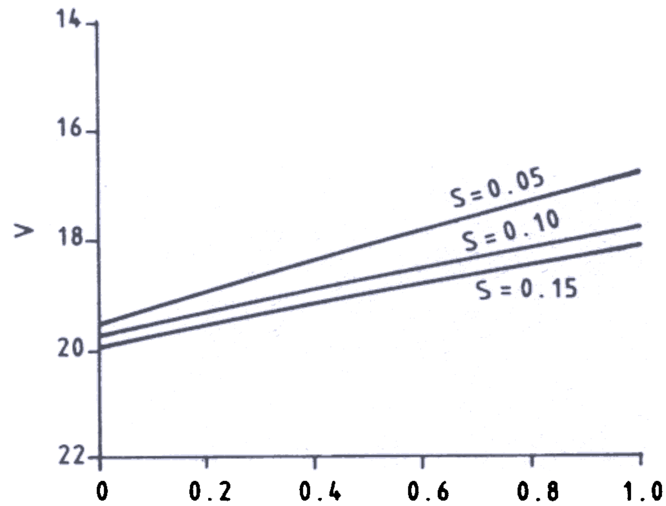


Figure 9. Velocity distribution $u$ against $z$ for different values of viscoelastic parameter $s$ (case ii).

increase in $M$ leads to a decrease in the velocity component $v$ whereas the velocity component $v$ increases with the increase in $s$. Figures 6 and 7 show the velocity component $u$ against $z$ for different values of $M$ and $s$ respectively in case (ii) discussed in section 3. An increase in $M$ leads to an increase in $u$ and also $u$ increases with the increase in $s$. Further it is noticed that $u$ decreases as $z$ increases. Figures 8 and 9 show the effects of magnetic and viscoelastic parameters $M$ and $s$ on the velocity component $v$ in case (ii) discussed in section 3. An increase in $z$ results in an increase in $v$. An increase in $M$ leads to an increase in $v$, and $v$ also decreases with the increase in $s$. In Figs. 10 and 11, $u$ against $z$ has been drawn for different values of $M$ and $s$ respectively, and is discussed in section 3. It is noticed that $u$ increases with the increase in $z$. Further it is noticed that $u$ decreases with the increase in $M$ or $s$. Figures 12 and 13 have been drawn to investigate the effects of magnetic and viscoelastic parameters $M$ and $s$ respectively on the velocity component $v$ in case (iii) which was discussed earlier in section 3. An increase in $z$ results in a decrease in $v$. The velocity component $v$ increases with the increase in $M$ whereas the opposite trend is observed when $s$ increases.

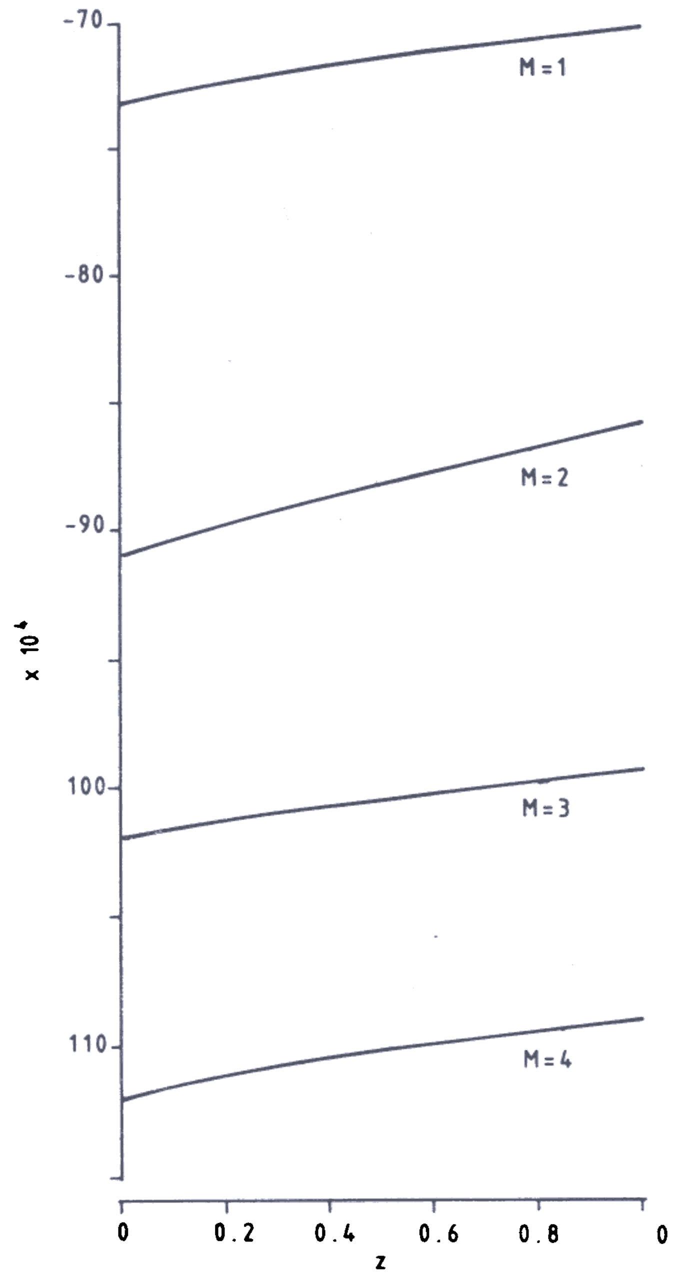


**Figure 10.** Velocity distribution $u$ against $z$ for different values of magnetic parameter $M$ (case ii).

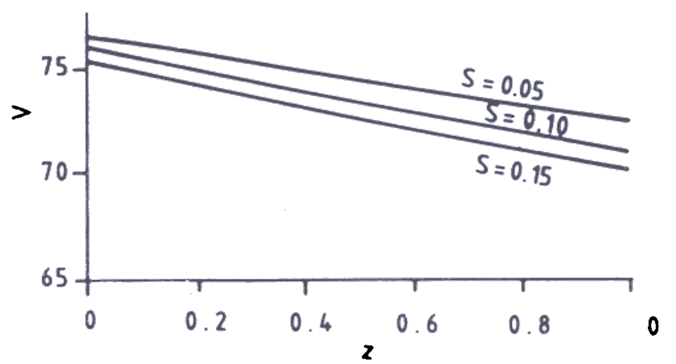


**Figure 11** Velocity distribution $u$ against $z$ for different values of viscoelastic parameter $s$ (case iii).

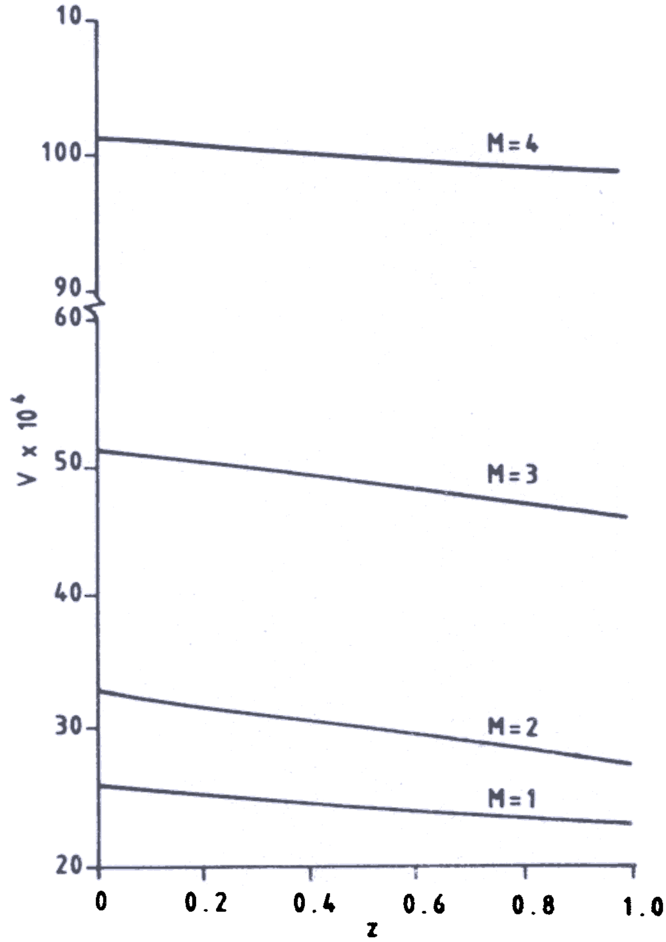


**Figure 12.** Velocity distribution $u$ against $z$ for different values of magnetic parameter $M$ (case ii).

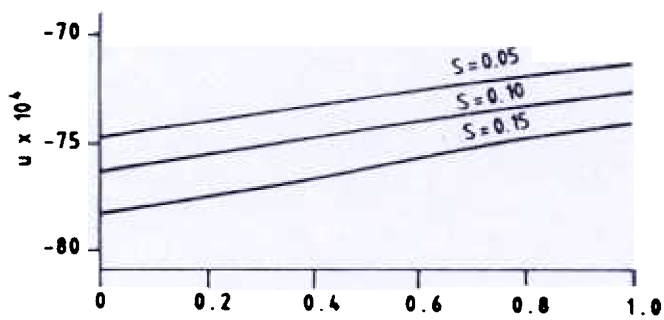


**Figure 13.** Velocity distribution $u$ against $z$ for different values of viscoelastic parameter $s$ (case iii).

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