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Heat Transfer to Peristaltic Transport in a Non-Uniform Channel

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ABSTRACT

The problem of heat transfer for the motion of a viscous incompressible fluid induced by travelling sinusoidal waves has been analytically investigated for a two-dimensional non-uniform channel. Assuming that the wavelength of the peristaltic wave, λ, is large in comparison to the mean half width of the channel, d, a perturbation solution in terms of the small parameter, d/λ, has been obtained. Closed form expressions for velocity, temperature and coefficient of heat transfer are presented up to second order. The effects of several pertinent parameters on temperature and heat transfer have been studied and the numerical results obtained are presented.

NOMENCLATURE

a	amplitude of the wave
c	phase speed of the wave
d	mean half width of the channel
δ	wall slope parameter
E	Eckert number
ε	amplitude ratio
K	non-uniform parameter
k'	non-uniformity of the channel
κ	thermal conductivity of the fluid
λ	wavelength of the wave
ν	kinematic viscosity of the fluid
P	Prandtl number
p	pressure of the fluid
$\bar{Q}$	mean volume flux
R	Reynolds number
ρ	density of the fluid
T	temperature of the fluid
t	time parameter
τ	specific heat of the fluid
u	velocity component in x-direction
v	velocity component in y-direction
Z	coefficient of heat transfer

1. INTRODUCTION

Peristaltic pumping is a mechanism for fluid transport which is achieved when progressive waves of area contraction or expansion propagate along the walls of a distensible channel (or tube) containing the fluid. Peristalsis plays a definite role in many physiological processes; the principle of peristaltic pumping is also used in designing biomedical instruments for fluid transport with no internal moving parts. For example, the blood pump for dialysis is designed to prevent contamination of the transported fluid, and the peristaltic transport of a toxic liquid is used in nuclear industry to avoid contamination of the outside environment. Mechanical devices like finger pumps and roller pumps also operate on this principle. Several workers[1-5] have studied peristalsis under different conditions with reference to physiological and mechanical situations.

Peristaltic transport in non-uniform ducts may be of considerable interest as many channels in engineering and physiological problems are known to be of non-uniform cross-section. Gupta and Seshadri[6] and Srivastava and Srivastava[7] have considered peristaltic transport in non-uniform channels. However, the interaction between peristalsis and heat transfer has not received much attention. Thermodynamic aspects of blood become significant in processes like oxygenation and hemodialysis. Victor and Shah[8] considered heat transfer to blood flowing in a tube assuming blood to be a Casson fluid.

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A mathematical model is presented here for studies on interaction between peristalsis and heat transfer for the motion of a viscous incompressible fluid in a two-dimensional non-uniform channel. The momentum and energy equations have been linearized under long wavelength approximation and analytical solutions for the flow variables have been derived.

2. MATHEMATICAL MODEL

The motion of a viscous incompressible fluid through a two-dimensional non-uniform channel with flexible walls is shown in Fig.1. A rectangular Cartesian

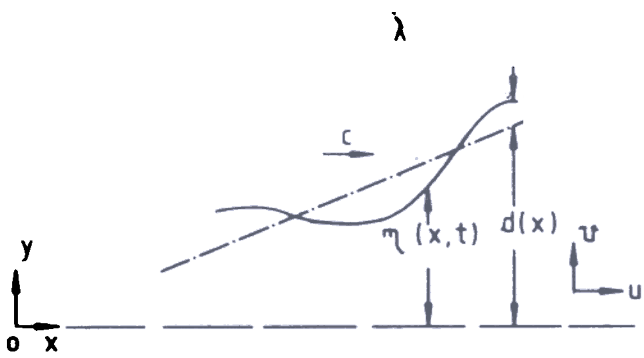

Figure 1. Flow geometry.

coordinate system (x, y) is chosen with x-axis aligned with the centre line of the channel and in the direction of wave propagation on the walls. The geometry of the wall channel is given by

$$H(x, t) = d(x) + a\cos\frac{2\pi}{\lambda}(x-ct) \quad (1)$$

where

$$d(x) = d + k'x \; (k' << 1)$$

The equations governing the flow are (subscripts t, x and y in the following equations denote the partial derivatives with respect to t, x and y respectively).

Momentum equation:

$$u_t + uu_x + vu_y = -\frac{1}{\rho}p_x + \upsilon\nabla^2 u \quad (2)$$

$$v_t + uv_x + vv_y = -\frac{1}{\rho}p_y + \upsilon\nabla^2 v \quad (3)$$

Continuity equation

$$u_x + v_y = 0 \quad (4)$$

Energy equation

$$6(T_t + uT_x + vT_y) = \frac{\kappa}{\rho}\nabla^2 T + \upsilon\phi \quad (5)$$

where

$$\phi = 2\left[(u_x)^2 + (v_y)^2\right] + (v_x + u_y \quad (6)$$

The boundary conditions are

$$u = 0$$
$$v = H_t \quad \text{on } y = \pm H$$
$$T = T_0 \quad (7)$$

Eliminating the pressure term from Eqns (2) and (3) introducing stream function ψ such that

$$u = \psi_y, \; v = -\psi_x \quad (8)$$

and the following non-di nsional variables

$$x' = \frac{x}{\lambda}, y' = \frac{y}{d}, t' = \frac{c}{\lambda}t, \psi' = \frac{\psi}{cd}, \theta = \frac{T-T_0}{T_0} \quad (9)$$

in Eqns (1), (2), (3), (5), (6) and (7), we get (after dropping the primes),

$$\nabla^2\psi_t + \psi_y\nabla^2\psi_x - \psi_x\nabla^2\psi_y = \frac{1}{\delta R}\nabla^4\psi$$

$$R\delta\left[\theta_t + \psi_y\theta_x - \psi_x\theta_y\right] = \frac{1}{P}\nabla^2\theta + E$$

$$\left[4\delta^2(\psi_{xy})^2 + (\psi_{yy} - \delta^2\psi_{xx})^2\right]$$

$$\psi_y = 0$$
$$\psi_x = \mp 2\pi\epsilon\sin 2\pi(x-t) \quad \text{on } y = \pm\eta$$
$$\theta = 0$$

where

$$\eta(x,t) = 1 + Kx + \epsilon\cos 2\pi(x-t)$$

$$\eta(x,t) = \frac{H(x,t)}{d}$$

$$\nabla^2 = \left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$$

$$R = \frac{cd}{\upsilon} \qquad E = \frac{c^2}{6T_0}$$

$$P = \frac{\rho\upsilon 6}{\kappa} \qquad \epsilon = \frac{a}{d}$$

$$\delta = \frac{d}{\lambda} \qquad K = \frac{\lambda k'}{d}$$

3. SOLUTIONS

We seek perturbation solution in terms of the small parameter δ as follows:

$$f = f_0 + \delta f_1 + \delta^2 f_2 + \quad (16)$$

where f represents any flow variable.

Substituting Eqn (16) in Eqns (10) and (11), collecting coefficients of various powers of δ and solving the resultant equations under the relevant boundary conditions, we finally get

$$\psi = \psi_0 + \delta\psi_1 + \delta^2 \psi_2 \quad (17)$$

$$\theta = \theta_0 + \theta_1 + \delta^2\theta_2 \quad (18)$$

where

$$\psi_0 = A_3 y^3 + A_1 y \quad (19)$$

$$\psi_1 = R\left[D_1 y^7 + D_2 y^5 + k_3 y^3 + k_1 y \right] \quad (20)$$

$$\psi_2 = R\left[A_{11} y^{11} + A_9 y^9 + A_7 y^7 + A_5 y^5 + S_3 y^3 + S_1 y \right] \quad (21)$$

$$\theta_0 = -PE\left[3A_3^2 y^4 + c_1 \right] \quad (22)$$

$$\theta_1 = PER\left[H_1 \frac{y^8}{56} + H_2 \frac{y^6}{30} + H_3 \frac{y^4}{12} + H_4 \frac{y^2}{2} + P_1 \right] \quad (23)$$

$$\theta_2 = PE\left[N_1 \frac{y^{12}}{132} + N_2 \frac{y^{10}}{90} + N_3 \frac{y^8}{56} + N_4 \frac{y^6}{30} + N_5 \frac{y^4}{12} + N_6 \frac{Y^2}{2} + B_1 \right] \quad (24)$$

$$A_3 = \frac{c_0 - \frac{1}{2}\epsilon \cos 2\pi (x-t)}{\eta^3}$$

$$A_1 = -3A_3 \eta^2$$

$$D_1 = \frac{A_3 A_{3x}}{70}$$

$$D_2 = \frac{1}{20}(A_{3t} + A_1 A_{3x} - A_3 A_{1x})$$

$$c_1 = -3A_3^2 \eta^4$$

The other constants are not given, as they are too lengthy. Z is given by

$$Z = Z_0 + \delta Z_1$$

where

$$Z_0 = \eta_x \theta_{0y}$$

$$Z_1 = \theta_{0x} + \eta_x \theta_{1y}$$

4. RESULTS AND DISCUSSION

The expressions for temperature and coefficient of heat transfer are given by Eqns (18) and (25). To explicitly see the effects of various parameters on temperature, Eqn (18) has been numerically evaluated and the results are presented in Figs. 2-7. For small values of K, backflow does not occur.

The constant of integration, c_0, which arises during the integration of momentum equation, as shown by Zien and Ostrach[2], is related to $\bar{Q}$. Here, its value is taken to be –0.15 in plotting all the graphs. Further, the value of K is taken to be 0.1 in Figs. 2-5 and 7, which indicates that the channel is of diverging nature. It is seen from the figures that the non-dimensional temperature θ $(= (T-T_0)/T_0)$ is negative at the inlet, i.e., $T < T_0$, but becomes positive downstream for all values of the parameters.

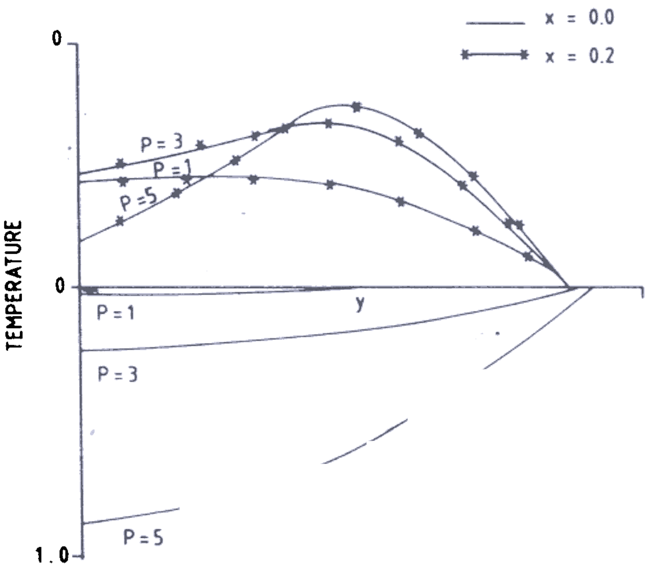

Figure 2. Effect of P on temperature (E = 3, R = 5, δ = 0.1, ε = 0.1, K = 0.1, t = 0.4).

It is obvious from Fig. 2 that the absolute value of the temperature increases as P increases at the inlet, i.e., at x = 0. However, the temperature increases downstream when P increases from 1 to 3, but decreases as P increases to 5 up to a certain value of y and then

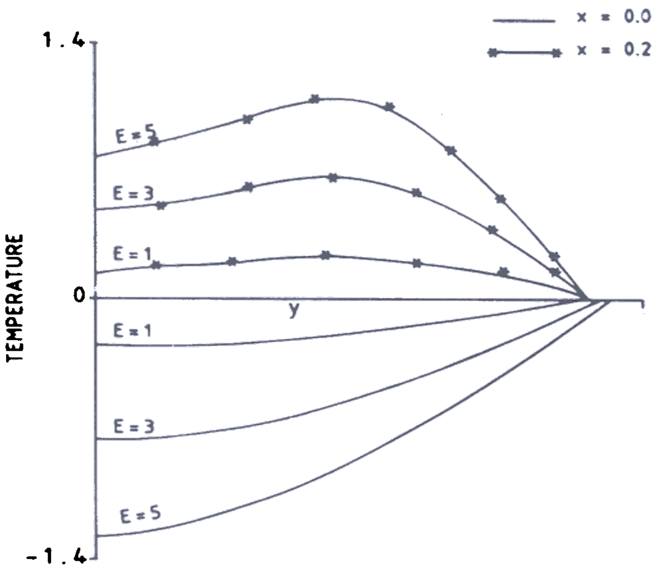

Figure 3. Effect of E on temperature ($P = 3$, $R = 5$, $\delta = 0.1$, $\varepsilon = 0.1$, $K = 0.1$, $t = 0.4$).

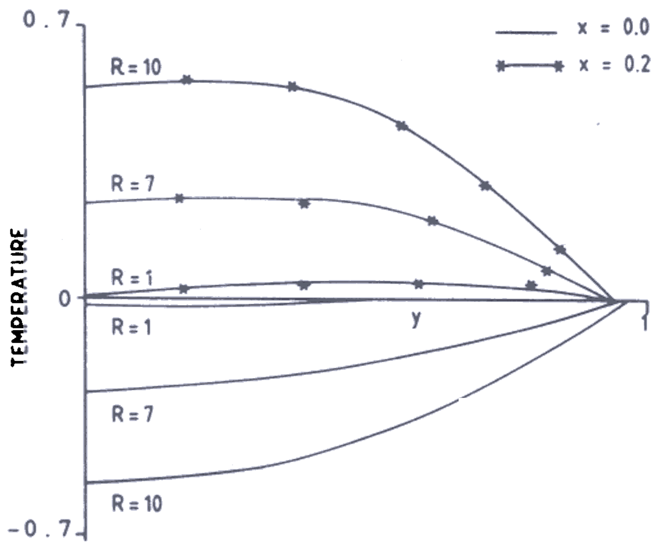

Figure 4. Effect of R on temperature ($P = 1$, $E = 1$, $K = 0.1$, $\varepsilon = 0.1$, $\delta = 0.1$, $t = 0.4$).

increases with P. The absolute value of temperature increases with E as well as R for fixed values of all other parameters (Figs. 3 and 4). As ε increases, the absolute value of temperature increases at the inlet as well as downstream (Fig. 5). The temperature for a divergent channel ($K > 0$) is higher compared to its value for a straight channel, whereas it is lower for a convergent channel ($K < 0$) (Fig. 6). The effect of δ on temperature is depicted in Fig. 7. It is seen that for fixed values of

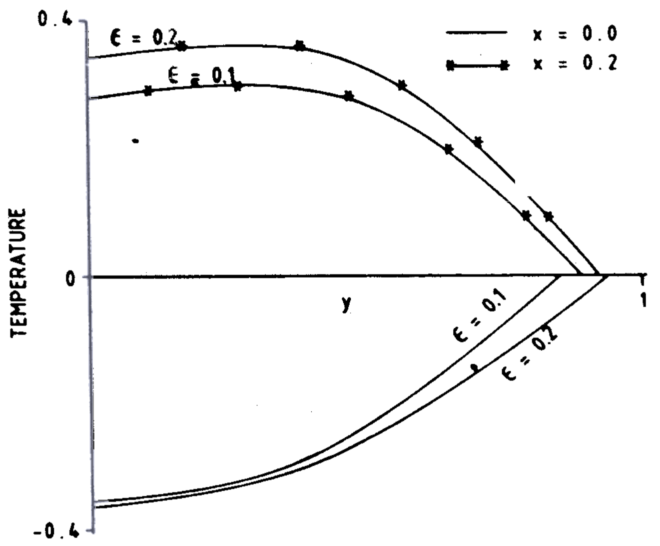

Figure 5. Effect of ε on temperature ($P = 1$, $E = 1$, $R = 5$, $\delta = 0.1$, $K = 0.1$, $t = 0.4$).

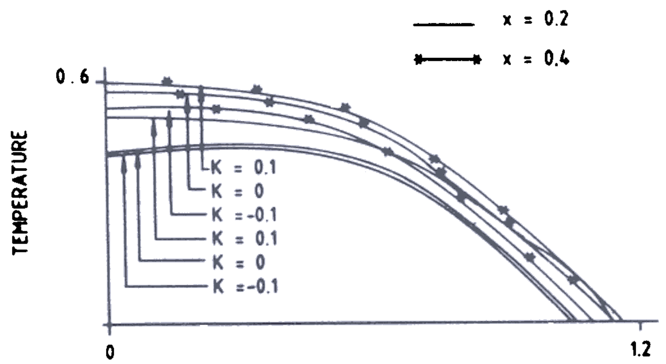

Figure 6. Effect of K on temperature ($P = 1$, $E = 1$, $R = 5$, $\delta = 0.1$, $t = 0.4$, $\varepsilon = 0.1$).

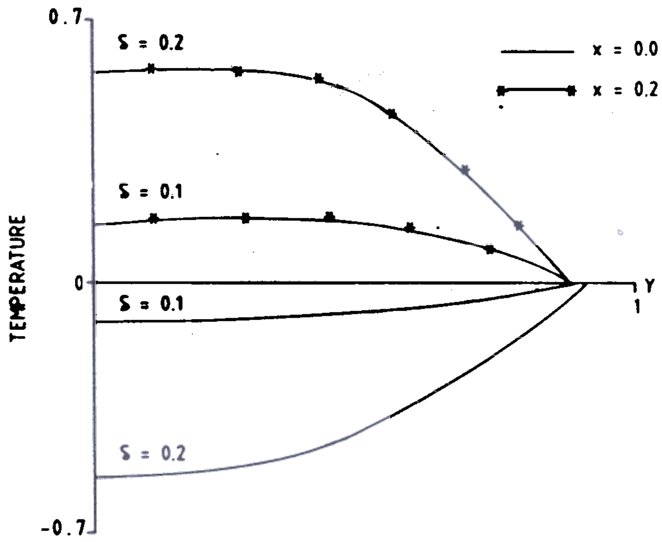

Figure 7. Effect of δ on temperature ($P = 1$, $E = 1$, $R = 5$, $\varepsilon = 0.1$, $K = 0.1$, $t = 0.4$).

all other parameters, the absolute value of temperature increases with δ.

The effect of various parameters on heat transfer, Z, is obvious from the data presented in Tables 1-6. Here also, c_0 is taken to be −0.15. We notice from Table 1 that at the inlet, heat transfer is positive for P = 1, but becomes negative for higher values of P. However, downstream, it is positive for all values of P and

Table 1. Variation of Z with P
(E = 3, R = 5, δ = 0.1, ε = 0.1, K = 0.1, t = 0.4)

X	P		
	1	3	5
0.0	0.543	−2.0978	−18.056
0.2	0.0607	2.7046	12.952

increases with P. Heat transfer is negative at the inlet, but becomes positive downstream and increases with E (Table 2). Z is seen to be positive for certain values of R, and becomes negative for some other values (Table 3). The effects of geometric parameters on heat transfer are shown in Tables 4-6. Data in Table 4 show that Z is negative for all values of ε at the inlet, whereas it becomes positive downstream as ε increases. The value of Z decreases and becomes negative for convergent channels, whereas it increases and becomes

Table 2. Variation of Z with E
(P = 3, R = 5, δ = 0.1, ε = 0.1, K = 0.1, t = 0.4)

X		E	
		3	5
0.0	−0.6992	−2.0978	−3.4964
0.2	0.9015	2.7046	4.5077

Table 3. Variation of Z with R
(P = 3, E = 3, δ = 0.1, ε = 0.1, K = 0.1, t = 0.4)

X		R	
		5	9
0.0	1.5872	−2.0978	−17.433
0.2	−0.1025	2.7046	16.3839

Table 4. Variation of Z with ε
(P = 3, E = 3, R = 5, δ = 0.1, K = 0.1, t = 0.4)

X			
	0.0	0.1	0.2
0.0	−0.2246	−2.0978	−33.2740
0.2	−0.2116	2.7046	− 2.9356

Table 5. Variation of Z with K
(P = 3, E = 3, R = 5, δ = 0.1, ε = 0.1, t = 0.4)

X	K		
	−0.1	0	0.1
0.1	−1.9448	0.013	1.532
0.2	−2.5144	0.0235	2.7046

Table 6. Variation of Z with δ
(P = 3, E = 3, R = 5, ε = 0.1, K = 0.1, t = 0.4)

X	δ		
	0.0	0.1	0.2
0.0	.2838	−2.0978	−16.2703
0.2	−0.1386	2.7046	11.2106

positive for divergent channels (Table 5). Further, heat transfer becomes negative for certain values of δ (Table 6).

For fixed values of all other parameters, the value of $\bar{Q}$ (= −2 c_0) (Zien and Ostrach[2]) for which heat transfer changes its sign from positive to negative is numerically determined and it is found to be approximately 0.038.

The results of the present study will hopefully enable a better understanding of the interaction between peristalsis and heat transfer in nuclear industry, with particular reference to Defence.

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