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Evaluating a Weapon System using Fuzzy Analytical Hierarchy Process

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ABSTRACT

We propose a new algorithm for evaluating a weapon system by fuzzy analytical hierarchy process. Using the symmetric triangular fuzzy number we built a judgement matrix through pair-wise comparison technique. To derive fuzzy eigenvectors, we utilized interval arithmetic, α-cuts together with index of optimism β to estimate the degree of satisfaction. Thus, the required weights of the final evaluation could be obtained. Finally by selecting a weapon system as an example we demonstrated the new algorithm.

1. INTRODUCTION

The analytical hierarchy process (AHP) is a systematic procedure for representing the element of any problem hierarchically[1]. It organizes the basic rationality by breaking down a problem into its smaller constituent parts and then guides decision makers through a series of pair-wise comparison judgements to indicate the relative strength or intensity of impact of the elements in the hierarchy. These judgements are then translated to numbers. The AHP includes procedures and principles used to synthesize the many judgements to derive priorities among criteria and subsequently for alternative solutions.

A weapon system is a large and complex system, with multi-level and multi-factor features. Therefore the determination of weights of criteria in a weapon system is an important and formidable task. We have used symmetric triangular fuzzy number $\tilde{1}$ to $\tilde{9}$ to build a judgement matrix through pair-wise comparison technique[2]. To derive fuzzy eigenvectors we have used interval arithmetic, α-cuts together with index of optimism β to estimate the degree of satisfaction[3]. Thus, the required weights of the final evaluation can be obtained. Finally, by selecting a weapon system as example we have demonstrated the new algorithm.

2. THE HIERARCHICAL STRUCTURE MODEL OF THE ANTI-AIRCRAFT ARTILLERY

Establishing a hierarchical structure model of anti-aircraft artillery (AAA) is a difficult task, and it depends on the properties of AAA. Therefore, the study of evaluating small AAA depends on several characteristics such as the technological advance, large killing capacity, long lifetime, high mobility and good logistic maintenance. Its structure is shown in Fig. 1.

Since each factor plays a different role, the factors of hierarchy have been determined. Thus, we must first determine each factor to arrive at the relative order weight. From Fig. 1, we can obtain fuzzy judgement matrix for each criterion through comparison of the performance scores. In Sec. 5, we demonstrate the structure model by selection of a weapon system.

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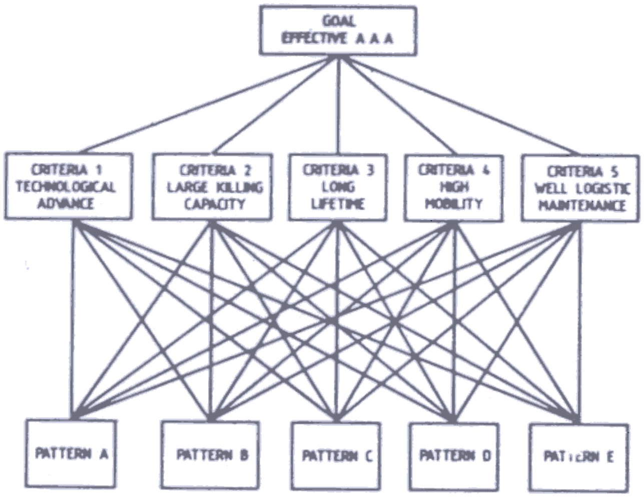

Figure 1. Structure model of evaluating five patterns of AAA.

3. COMPUTATIONAL ASPECTS OF THE FUZZY EIGENVECTOR USING INTERVAL ARITHMETIC AND α-CUTS

A triangular fuzzy number can be defined by a triplet (a_1,a_2,a_3). The membership function is defined as[4,5]:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \le x \le a_3 \\ 0 & x > a_3 \end{cases} \qquad (1)$$

Alternatively, by defining the interval of confidence at level α, we characterize the triangular fuzzy number as[4]:

$$\forall\alpha \in [0,1],\ \bar{A}_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}] =$$

$$[(a_2-a_1)\alpha + a_1, -(a_3-a_2)\alpha + a_3] \qquad (2)$$

Some main operations for positive fuzzy numbers $\tilde{A}$ and $\tilde{B}$ described by the interval of confidence are :

$$\bar{A}_\alpha \oplus \bar{B}_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}] \oplus [b_1^{(\alpha)}, b_3^{(\alpha)}]$$

$$= [a_1^{(\alpha)} + b_1^{(\alpha)}, a_3^{(\alpha)} + b_3^{(\alpha)}]$$

$$\bar{A}_\alpha (-) \bar{B}_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}](-)[b_1^{(\alpha)}, b_3^{(\alpha)}]$$

$$= [a_1^{(\alpha)} - b_3^{(\alpha)}, a_3^{(\alpha)} - b_1^{(\alpha)}]$$

$$\bar{A}_\alpha \otimes \bar{B}_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}] \otimes [b_1^{(\alpha)}, b_3^{(\alpha)}]$$

$$= [a_1^{(\alpha)} b_1^{(\alpha)}, a_3^{(\alpha)} b_3^{(\alpha)}] \qquad (3)$$

$$\bar{A}_\alpha (+) \bar{B}_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}](+)[b_1^{(\alpha)}, b_3^{(\alpha)}] = [\frac{a_1^{(\alpha)}}{b_3^{(\alpha)}}, \frac{a_3^{(\alpha)}}{b_1^{(\alpha)}}]$$

The computational technique is based on the following fuzzy numbers defined in Table 1. A fuzzy number $\tilde{x}$ expresses the meaning of 'about x' (Fig. 2). Here each characteristic function is defined by two parameters of the symmetric triangular fuzzy number. Saaty's method[6] utilizes a technique called pair-wise comparison. Here we use a symmetric triangular fuzzy number to improve the scaling scheme used in the Saaty's method, and an interval arithmetic method to calculate fuzzy eigenvector. The new method is summarized in Table 1.

Table 1. Parameters that define the characteristic function of the fuzzy numbers used

Fuzzy number	
$\tilde{1}$	$a=1, c=2$, and $a \le x \le a+c$
$\tilde{3}$	$a=3, c=2$, and $a-c \le x \le a+c$
$\tilde{5}$	$a=5, c=2$, and $a-c \le x \le a+c$
$\tilde{7}$	$a=7, c=2$, and $a-c \le x \le a+c$
$\tilde{9}$	$a=9, c=2$, and $a-c \le x \le a$

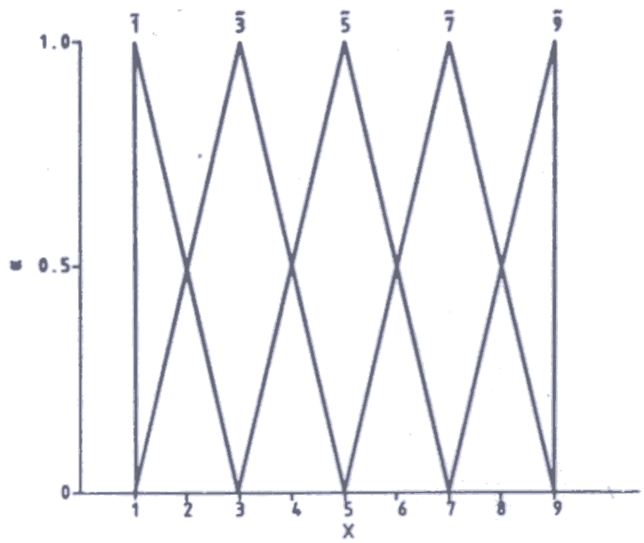

Figure 2. Membership function for fuzzy number $\tilde{x}$.

Step 1

Each element $\tilde{a}_{ij}$ in the pair-wise comparison matrix $\tilde{A}= [\tilde{a}_{ij}]$ is a fuzzy number whose characteristic function can be defined from Table 1. Since each element of the matrix is a fuzzy number, the matrix is here called a fuzzy matrix.

Step 2

A fuzzy eigenvalue $\tilde{\lambda}$ is a fuzzy number solution to

$$\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x} \tag{4}$$

where $\tilde{A}$ is a $n \times n$ fuzzy matrix containing fuzzy numbers $\tilde{a}_{ij}$ and $\tilde{x}$, is a non-zero $n\times1$ fuzzy vector containing fuzzy number. In Eqn (4) we employ regular fuzzy multiplication and addition. Therefore, Eqn (4) is equivalent to

$$(\tilde{a}_{i1} \otimes \tilde{x}_1) \oplus \cdot \quad \cdot \oplus (\tilde{a}_{in} \otimes \tilde{x}_n) = \tilde{\lambda} \otimes \tilde{x}_i \tag{5}$$

for $1 \leq i \leq$, where $\tilde{A} = [\tilde{a}_{ij}]$, $\tilde{x}^t = (\tilde{x}_1, \ldots, \tilde{x}_n)$ is the transposition of the column vector $\tilde{x}$, $\tilde{a}_{ij}$ and $\tilde{x}$ are fuzzy numbers, and $\otimes$, $\oplus$ denote fuzzy multiplication and addition respectively.

Step 3

Fuzzy multiplication and addition are easily performed using interval arithmetic and α-cuts. We may now define for $0 < \alpha \leq$, and all i, j

$$(\tilde{a}_{ij})_\alpha = [a_{ijl}^{(\alpha)}, a_{iju}^{(\alpha)}], \quad (\tilde{x}_i)_\alpha = [x_{il}^{(\alpha)}, x_{iu}^{(\alpha)}], \quad \tilde{\lambda}_\alpha = [\lambda_l^{(\alpha)}, \lambda_u^{(\alpha)}] \tag{6}$$

Substitute Eqn (6) into Eqn (5); we have for $1 \leq i \leq$

$$[a_{i1l}^{(\alpha)}x_{1l}^{(\alpha)}, a_{i1u}^{(\alpha)}x_{1u}^{(\alpha)}] \oplus \cdot \cdot \cdot \oplus [a_{inl}^{(\alpha)}x_{nl}^{(\alpha)}, a_{inu}^{(\alpha)}x_{nu}^{(\alpha)}] = [\lambda_l^{(\alpha)}x_{il}^{(\alpha)}, \lambda_u^{(\alpha)}x_{iu}^{(\alpha)}]. \tag{7}$$

$$a_{i1l}^{(\alpha)}x_{il}^{(\alpha)} + \cdot \quad \cdot + a_{inl}^{(\alpha)}x_{nl}^{(\alpha)} = \lambda_l^{(\alpha)}x_{il}^{(\alpha)}$$

$$a_{i1u}^{(\alpha)}x_{iu}^{(\alpha)} + \qquad + a_{imu}^{(\alpha)}x_{nu}^{(\alpha)} = \lambda_u^{(\alpha)}x_{iu}^{(\alpha)} \tag{8}$$

Step 4

When α has been fixed, we can use the index of optimism β to estimate of the degree of satisfaction (i.e., $\hat{a}_{ij} = (1-\beta)\, a_{ijl}^{(\alpha)} + \beta\, a_{iju}^{(\alpha)}$). The index of optimism β indicates the degree of optimism of a decision maker. A larger β indicates a higher degree of optimism. Then solving Eqn (8), the required weights (the max λ corresponds to the eigenvector) can be obtained.

4. CONSISTENCY TEST

We demand a pair-wise comparison matrix perfect consistency in measurement. But perfect consistency in measurement even with the finest instruments is difficult to attain in practice, and what we need is a way of evaluating how bad is a particular problem. According to Saaty[6], for the consistency index we have

$$C.I. = \frac{\lambda_{max}-n}{n-1}, \tag{9}$$

where n is the number of elements being compared, and λ_{max} is the largest eigenvalue of the judgement matrix. When complete consistency occurs, $\lambda_{max} = n \Rightarrow C.I. = 0$.

For testing, we define a random consistency ratio

$$C.R. = \frac{C.I.}{R.I.} \tag{10}$$

where $R.I.$ is average random consistency index. The average random consistency indices for different random matrices[7] are :

Size of matrix	1	2	3	4	5
Random consistency	0	0	0.58	0.90	1.12

Size of matrix	6	7	8	9	10
Random consistency	1.24	1.32	1.41	1.45	1.49

The value of $C.R.$ should be around 10 per cent or less to be acceptable (i.e. $C.R. < 0.10$). In some cases, 20 per cent may be tolerated as limit. If the $C.R.$ is not within this range, the participants should study the problem and revise their judgements.

Substituting Eqn (9) in Eqn (10), we have

$$C.R. = \frac{\lambda_{max} - n}{(n-1)R.I.}$$

$$C.R. < 0.1 \quad \Rightarrow \quad \lambda_{max} < n + \frac{(n-1)R.I.}{10}$$
$$C.R. < 0.2 \quad \Rightarrow \quad \lambda_{max} < n + \frac{(n-1)R.I.}{5} \qquad 1)$$

How do we justify consistency of a fuzzy judgement matrix ? Since the crisp set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree a is called the a-level set :

$$A_{\alpha} = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\} \quad \text{where } \alpha \in [0, 1]$$

Therefore, we derive λ_{max} for every a-cuts judgement matrix and only assess those λ_{max} satisfying Eqn (11) for a-cuts judgement matrix, (i.e. if a-cuts judgement matrix is inconsistent then we can ignore it. We assess only consistent judgement matrix for all a-cuts).

5. NUMERICAL EXAMPLE

A country's air defence system of mixed formation is based on AAA of small calibre in low air defence and super-low air defence. In general, the effective range of a projectile of small AAA is nearly 3000 m (for a land object nearly 4000 m). Many small AAAs are in the range of 30-40 mm calibre. Now, we use fuzzy AHP method to evaluate some small AAAs.

In general, we evaluate a weapon system through battle's effect, economy and level of advance. But practical items of evaluation are usually determined by the conditions and controlled data of the stage of production and design. Here, we evaluate five AAAs of designed patterns (Table 2). We proposed 5 criteria for judgement, every criterion including some items. We invited experts to judge every item by giveng relative scores.

5.1 Technological Advance

The items of technological advance include initial velocity fire rate and firing range. If the initial velocity is greater than is greater than 1000 m/s, then its score is 1; If the initial velocity is lower than 1000 m/s, then its score is 0.5. If the fire rate of 35 mm small AAA is greater than 500 (fire no./min), or if fire rate of 37 mm small AAA is greater than 400 (fire no./min), or if the fire rate of 40 mm small AAA is greater than 350 (fire no./min), then its score is 1; otherwise, its score is 0.5. If the firing range is greater than 4000 m, then is score is 1; otherwise, its score is 0.5. Table 3 shows the scores for the five alternatives on these items.

By using the total score, decision makers can make pair-wise comparison judgements to express the relative importance of elements in the hierarchy. From Table 3, the scale is 0.5, and from Table 4 we can obtain fuzzy judgement matrix as Eqn (13).

5.2 Large Kill Capacity

The items are kill capacity consist of calibre effect of using pre-set broken shell and kill rate. Scores are shown in Table 5. For its fuzzy judgement matrix, see Eqn. (14) [without its process].

5.3 Long Lifetime of Mechanism

The terms comprise recoil distance, recoil, parts charged of 1500 fire and fire rate. Scores are shown in Table 6. For its fuzzy judgement matrix, see Eqn (15).

Table 2. Nine judgement items for five designed patterns

Pattern	A	B	C	D	E
Calibre (mm)	37	37	35	40	37
Initial velocity (m/s)	>1000	>1000	1175	1060	>1000
Fire rate (fire no/min)	350-400	400-450	550	300	500
Firing range (m)	4000	4000	4000	>4000	4000
Recoil distance (mm)	140-145	140	60	230	~70
Recoil (N)	50000	75000	15000	27000	20000
Breech block type	Cuneate	Spiral	Fish bolt	Cuneate	Fish bolt
Artillery's total weight (kg)	4000	>5000	5850	4800	>6000
Kill rate	0.72	0.72	0.66	0.91	0.72

Table 3. Criterion one for five patterns

Artillery's pattern	A	B	C	D	E
Initial velocity	1	1	1	1	1
Fire rate	0.5	1	1	0.5	1
Firing range	1	1	1	1	1
Total score	2.5	3	3	2.5	3

Table 4. Using fuzzy number to express the relative importance

Fuzzy number a_{ij}	Pair-wise comparison
	$P_i - P_j = 1 \times$ scale
$\tilde{3}$	$P_i - P_j = 2 \times$ scale
$\tilde{5}$	$P_i - P_j = 3 \times$ scale
	$P_i - P_j = 4 \times$ scale
$\tilde{9}$	$P_i - P_j = 5 \times$ scale

Note : $a_{ij} = \frac{1}{a_{ji}}$, $a_{ij} = 1$, where $i = A, B, C, D, E$ and $j = A, B, C$ D, E; $\hat{P}_A$: A's total score; P'_B : B's total score

$$
\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
A & 1 & \tilde{3}^{-1} & \tilde{3}^{-1} & \tilde{1} & \tilde{3}^{-1} \\
B & \tilde{3} & 1 & [illegible] & \tilde{3} & [illegible] \\
C_1 = C & \tilde{3} & [illegible]^{-1} & 1 & \tilde{3} & \tilde{1} \\
D & [illegible]^{-1} & \tilde{3}^{-1} & \tilde{3}^{-1} & & \tilde{3}^{-1} \\
E & \tilde{3} & [illegible]^{-1} & \tilde{1}^{-1} & \tilde{3} & 1
\end{array} \qquad (13)
$$

Table 5. Criterion two for five patterns

Artillery's pattern	A	B	C	D	E
Calibre	1	1	1	1	1
Effect of using pre-set broken shell	1	1	0.5	1	1
Kill rate	1	1	1	1	1

Table 6. Criterion three for five patterns

	A	B		D	E
Recoil distance			0.5	0.5	0.5
Recoil					
Fire rate					

$$
\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
A & 1 & & \tilde{3} & \tilde{1} & \tilde{1} \\
B & \tilde{1}^{-1} & & \tilde{3} & \tilde{1} & \tilde{1} \\
C_2 = C & \tilde{3}^{-1} & \tilde{3}^{-1} & 1 & \tilde{3}^{-1} & \tilde{3}^{-1} \\
D & \tilde{1}^{-1} & \tilde{1}^{-1} & \tilde{3} & 1 & \tilde{1} \\
E & \tilde{1}^{-1} & \tilde{1}^{-1} & \tilde{3} & \tilde{1}^{-1} &
\end{array}
$$

$$
\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
A & & \tilde{7} & \tilde{3} & \tilde{3} & \tilde{3} \\
B & \tilde{7}^{-1} & & \tilde{5}^{-1} & 5^{-1} & \tilde{5}^{-1} \\
C_3 = C & \tilde{3}^{-1} & 5 & 1 & 1 & \\
D & \tilde{3}^{-1} & \tilde{5} & \tilde{1}^{-1} & & \\
E & \tilde{3}^{-1} & \tilde{5} & \tilde{1}^{-1} & \tilde{1}^{-1} &
\end{array}
$$

5.4 High Mobility

The items include artillery's total weight, and time required to enter into battle condition. Score results are shown in Table 7. For its fuzzy judgement matrix, see Eqn (16).

Table 7. Criterion four for five patterns

Artillery's pattern	A	B	C	D	E
Artillery's total weight	1	0.5	0.5	1	0.5
Time required to enter into battle condition	1	0.5	1	1	1
Total score	2	1	1.5	2	1.5

$$
\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
A & 1 & \tilde{5} & 3 & \tilde{1} & \tilde{3} \\
B & \tilde{5}^{-1} & & \tilde{3}^{-1} & \tilde{5}^{-1} & \tilde{3}^{-1} \\
C_4 = C & \tilde{3}^{-1} & 3 & & \tilde{3}^{-1} & 1 \\
D & \tilde{1}^{-1} & \tilde{5} & \tilde{3} & & \tilde{3} \\
E & \tilde{3}^{-1} & \tilde{3} & \tilde{1}^{-1} & \tilde{3}^{-1} & 1
\end{array} \qquad (16)
$$

5.5 Easy Logistic Maintenance

The item contain availability of the parts of AAA, separability of breechblock, complexity of loading mechanism, complexity of structure, availability of fixed-type shells and capability for using old shells. Score results are given in Table 8. Its fuzzy judgement matrix is given in Eqn (17).

Table 8. Criterion five for five patterns

Artillery's pattern	A	B	C	D	
Availability of parts of AAA	0.5	1	1	1	0.5
Separability of breechblock	1	0.5	1	1	1
Complexity of loading machanism	1	0.5	1	1	1
Complexity of structure	1	0.5	1	1	
Availability of fixed type shells	1	0	0	0	0
Capability of using old shells	1	1	0	0	
Total score	5.5	3.5	4	4	4.5

$$C_5 = \begin{array}{c|ccccc|} & A & B & C & D & E \\ \hline A & & \tilde{9} & 3 & \tilde{3} & \\ B & \tilde{9}^{-1} & & \tilde{3}^{-1} & 3^{-1} & \tilde{5}^{-1} \\ C & \tilde{3}^{-1} & \tilde{3} & 1 & & \tilde{3}^{-1} \\ D & \tilde{3}^{-1} & \tilde{3} & \tilde{1} & & 3 \\ E & \tilde{5}^{-1} & 5 & \tilde{3} & 3 & \end{array} \qquad 7)$$

If the importance order of air defence system is technological advance, large kill capacity, high mobility, long lifetime, easy logistic maintenance, then we obtain fuzzy judgement matrix such as Eqn (18) :

$$O = \begin{array}{c|ccccc|} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \hline C_1 & 1 & \tilde{3} & \tilde{7} & \tilde{5} & \tilde{9} \\ C_2 & \tilde{3}^{-1} & 1 & \tilde{5} & \tilde{3} & 7 \\ C_3 & \tilde{7}^{-1} & \tilde{5}^{-1} & 1 & \tilde{3}^{-1} & \tilde{3} \\ C_4 & \tilde{5}^{-1} & \tilde{3}^{-1} & \tilde{3} & 1 & \tilde{5} \\ C_5 & \tilde{9}^{-1} & \tilde{7}^{-1} & \tilde{3}^{-1} & \tilde{5}^{-1} & \end{array} \qquad (18)$$

By Eqn (2), we have

$$\begin{aligned} {}_\alpha &= [1, 3-2\alpha] & \tilde{1}_\alpha^{-1} &= [\tfrac{1}{3-2\alpha}, 1] \\ \tilde{3}_\alpha &= [1+2\alpha, 5-2\alpha] & \tilde{3}_\alpha^{-1} &= [\tfrac{1}{5-2\alpha}, \tfrac{1}{1+2\alpha}] \\ \tilde{5}_\alpha &= [3+2\alpha, 7-2\alpha] & \tilde{5}_\alpha^{-1} &= [\tfrac{1}{7-2\alpha}, \tfrac{1}{3+2\alpha}] \\ \tilde{7}_\alpha &= [5+2\alpha, 9-2\alpha & \tilde{7}_\alpha^{-1} &= [\tfrac{1}{9-2\alpha}, \tfrac{1}{5+2\alpha} \\ 9_\alpha &= [7+2\alpha, 9] & \tilde{9}_\alpha^{-1} &= [\tfrac{1}{9}, \tfrac{1}{7+2\alpha}] \end{aligned} \qquad (19)$$

Substitute Eqn (19) in Eqn (13)-(18), then use Eqn (8) and [Step 4] to derive λ_{max} for all α-cuts judgement matrix. The results are given in Table 9.

Checking Table 9 by Eqn (11), we can obtain :

$C.R. < 0.1 \Rightarrow \lambda_{max} < 5.448$, when $\alpha \geq 0.57$

(satisfy acceptable consistency)

$C.R. < 0.2 \Rightarrow \lambda_{max} < 5.896$, when $\alpha \geq 0.18$

(satisfy maximal tolerance)

As an example, let $\alpha = 0.57$. Using Eqn (19), we can use index of optimism $\beta = 0.5$ (i.e, $\hat{a}_{ij} = (\tilde{a}_{ijl}^{(\alpha)} + (\tilde{a}_{iju}^{(\alpha)})/2$) to estimate the degree of satisfaction, then C_1, C_2, C_3, C_4, C_5, and O become as shown in Eqn (20).

By package MATHCAD, we can calculate the maximal eigenvalue of Eqn (20), which corresponds to eigenvector. Normalizing eigenvectors of the C_1, C_2, C_3, C_4, C_5 and O, we can obtain its correspondence weights as shown in Table 10.

Table 9. The λ_{max} for all α-cuts judgement matrix

	C_1	C_2	C_3	C_4	C_5	O
0.05	6.0720	5.9580	5.7666	5.9708	6.1689	5.7269
0.10	5.9436	5.8542	5.6869	5.8511	6.0480	5.6573
0.17	5.7881	5.7263	5.5893	5.7089	5.9058	5.5763
0.18	5.0558	5.0599	5.1095	5.0956	5.3284	5.2569
0.25	5.6374	5.5995	5.4936	5.5744	5.7732	5.5014
0.30	5.5551	5.5285	5.4407	5.5023	5.7032	5.4621
0.35	5.4803	5.4631	5:3923	5.4378	5.6412	5.4275
0.40	5.4123	5.4024	5.3479	5.3800	5.5862	5.3970
0.45	5.3503	5.3461	5.3072	5.3282	5.5375	5.3701
0.50	5.2938	5.2938	5.2698	5.2817	5.4943	5.3464
0.56	5.2327	5.2361	5.2291	5.2323	5.4491	5.3217
0.57	5.2232	5.2270	5.2228	5.2247	5.4422	5.3180
0.65	5.1539	5.1595	5.1762	5.1701	5.3932	5.2915
0.70	5.1165	5.1223	5.1509	5.1412	5.3678	5.2779
0.75	5.0838	5.0890	5.1287	5.1165	5.3462	5.2664
0.83	5.0414	5.0448	5.0996	5.0851	5.3194	5.2521
0.84	5.0370	5.0401	5.0966	5.0819	5.3167	5.2507
0.90	5.0153	5.0169	5.0817	5.0663	5.3036	5.2435
0.95	5.0040	5.0045	5.0739	5.0584	5.2969	5.2397
1.00	5.0000	5.0000	5.0711	5.0556	5.2943	5.2375

$$C_1 = \begin{bmatrix} 1 & 0.3632 & 0.3632 & .43 & 0.3632 \\ 3 & 1 & 1.43 & 3 & 1.43 \\ 3 & 0.7688 & 1 & 3 & 1.43 \\ 0.7688 & 0.3632 & 0.3632 & 1 & 0.3632 \\ 3 & 0.7688 & 0.7688 & 3 & 1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 1.43 & 3 & 1.43 & 1.43 \\ 0.7688 & 1 & 3 & 1.43 & 1.43 \\ 0.3632 & 0.3632 & 1 & 0.3632 & 0.3632 \\ 0.7688 & 0.7688 & 3 & 1 & 1.43 \\ 0.7688 & 0.7688 & 3 & 0.7688 & 1 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 1 & 7 & 3 & 3 & 3 \\ 0.145 & 1 & 0.2061 & 0.20613 & 0.2061 \\ 0.3632 & 5 & 1 & 1.43 & 1.43 \\ 0.3632 & 5 & 0.7688 & 1 & 1.43 \\ 0.3632 & 5 & 0.7688 & 0.7688 & 1 \end{bmatrix},$$

$$C_4 = \begin{bmatrix} 1 & 5 & 3 & 1.43 & 3 \\ 0.2061 & 1 & 0.3632 & 0.2061 & 0.3632 \\ 0.3632 & 3 & 1 & 0.3632 & 1.43 \\ 0.7688 & 5 & 3 & 1 & 3 \\ 0.3632 & 3 & 0.7688 & 0.3632 & 1 \end{bmatrix},$$

$$C_5 = \begin{bmatrix} 1 & 8.57 & 3 & 3 & 5 \\ 0.117 & 1 & 0.3632 & 0.3632 & 0.2061 \\ 0.3632 & 3 & 1 & 1.43 & 0.3632 \\ 0.3632 & 3 & 0.7688 & 1 & 0.3632 \\ 0.2061 & 5 & 3 & 3 & 1 \end{bmatrix}$$

$$O = \begin{bmatrix} 1 & 3 & 7 & 5 & 8.57 \\ 0.3632 & 1 & 5 & 3 & 7 \\ 0.145 & 0.2061 & 1 & 0.3632 & 3 \\ 0.2061 & 0.3632 & 3 & 1 & 5 \\ 0.117 & 0.145 & 0.3632 & 0.2061 & 1 \end{bmatrix} \tag{20}$$

From Table 10, we can calculate the total weight W. Thus

$$\underline{W} = \underline{C}_{ij} \times \underline{O}_{i}.$$

$$\begin{bmatrix} .0997 & .2748 & .4378 & .3614 & .4964 \\ .3050 & .2426 & .0405 & .0559 & .0427 \\ .2694 & .0791 & .1959 & .1400 & .1233 \\ .0880 & .2143 & .1730 & .3191 & .1094 \\ .2379 & .1892 & .1528 & .1237 & .2282 \end{bmatrix} \times \begin{bmatrix} .5096 \\ .2636 \\ .0644 \\ .1308 \\ .0343 \end{bmatrix}$$

$$= [\ .2154\ \ .2300\ \ 1926\ \ 1577\ \ .2043\ ]' \tag{21}$$

Table 10. C_1, C_2, C_3, C_4, C_5 and O with correspondece to weights

	Criterion	C_1	C_2	C_3	C_4	C_5
Experts' opinions	O	0.5069	0.2636	0.0644	0.1308	
	A	0.0997	0.2748	0.4378	0.3614	0.4964
	B	0.3050	0.2426	0.0405	0.0559	0.0427
Pattern	C	0.2694	0.0791	0.1959	0.1400	0.1233
	D	0.0880	0.2143	0.1730	0.3191	0.1094
	E	0.2379	0.1892	0.1528	0.1237	0.2282

By Eqn (21), pattern B is the best air defence system of mixed formation based on AAA of small calibre in low air defence and super-low air defence.

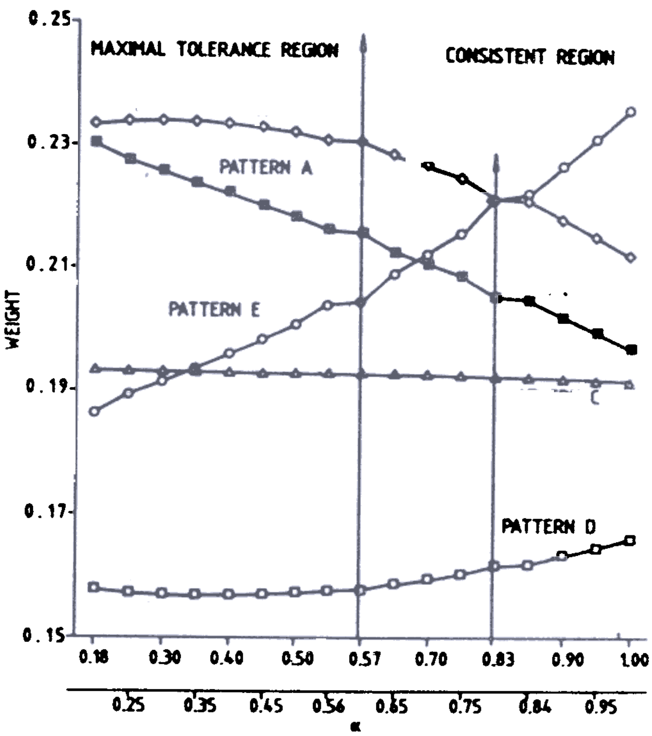

Figure 3. Total arrangement order weights of all levels.

Similarly, let a = 0.18, 0.25, 0.30 ..., 0.95, 1, then we can obtain the whole result as shown in Fig. 3. From Fig. 3, in the consistency case, when a E [0.57, 0.83], the pattern B is the best AAA. If $C.R. < 0.2$ can be used, then a E [0.18, 0.83]; the pattern B is also the best AAA. But, when a E [0.83, 1], pattern E is the best AAA.

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