Defence Science Journal, Vol 45, No. 4, October 1995, pp 333-340

Mathematical Model for Damage Assessment of Composite Panels Subjected to Blast Loading from Conventional Warhead

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ABSTRACT

A mathematical model has been developed to assess the damage to composite panels subjected to blast loading due to detonation of a conventional warhead in its vicinity. The theory is based on large dynamic deformation incorporating the effects of transverse shear deformation. A damage criterion based on Tsai-Hill/Hoffman failure criterion has been used. The results have been compared with those obtained by classical theory.

1. INTRODUCTION

The blast wave resulting from the detonation of a conventional warhead is a potential threat for a military aircraft in a battlefield. Among the various aircraft components, its structure is the largest of potentially vulnerable items. A mathematical model for damage to aircraft skin panels due to blast loading was developed by Singh and Singh[1], in which the damage to isotropic panels was considered and a threshold distance for permanent damage was obtained using a modified form of von-Mises criterion.

Since many of the modern fighter aircraft (e.g., LCA) contain components comprising composite materials, the assessment of damage for such components would be useful for aircraft designers. Recently, a number of papers[2-5] have appeared on this subject. In particular, the paper by Librescu and Nosier[3] is worth noting, in which the response of laminated composite flat panels to sonic boom and explosive blast loading is obtained theoretically.

In the present paper, a mathematical model has been presented to assess the damage to composite panels subjected to blast loading due to detonation of a warhead in its vicinity. This paper emphasises the vulnerability aspect of the problem rather than the response aspects as in the above mentioned papers. The vulnerability for a given structural element and a given warhead can be quantitatively expressed in terms of a threshold distance for permanent damage, i.e., the maximum damage up to which the structure is likely to suffer a permanent damage for a given warhead, based on Tsai-Hill/Hoffman failure criterion.

2. MATHEMATICAL MODEL

2.1 Panel with Single Ply with Fibre Direction taken as x-axis

When the distance of the panel from the point of explosion is sufficiently large, the dynamic response lies within elastic regime and is governed by the following system of partial differential equations :

$$L_1(\omega, \alpha, \beta) = \alpha + \frac{\partial \omega}{\partial x} \quad \left(\frac{6k}{5G_{xz}\, h}\right)\left[D_x \frac{\partial^2 \alpha}{\partial x^2} + D_{xy} \frac{\partial^2 \alpha}{\partial y^2} - \frac{\rho h^3}{12} f \frac{\partial^2 \alpha}{\partial t^2} + (\bar{H} \quad D_{xy}) \frac{\partial^2 \beta}{\partial x\, \partial y}\right] = 0 \tag{1}$$

Received 09 June 1993, revised 02 January 1994

$$L_2(\omega, \alpha, \beta) = \beta + \frac{\partial \omega}{} \quad \frac{6k}{5G_{yz}\, h} \Bigg| \Bigg[D_y \frac{\partial^2 \beta}{} + D_x \frac{\partial^2 \beta}{2} - \frac{\rho h^3}{12} f \frac{\partial^2 \beta}{\partial t^2} + (\bar{H} - D_{xy}) \frac{\partial^2 \alpha}{\partial x\, \partial y} \Bigg] = 0 \qquad (2)$$

$$L_3(\omega, \alpha, \beta) = G_{xz}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial \alpha}{\partial x}\right) + G_{yz}\left(\frac{\partial^2 \omega}{\partial y^2} \quad \frac{\partial \beta}{}\right) + \left(\frac{6k}{5h}\right) \bar{\alpha}^2 D_x \left(\frac{\partial^2 \omega}{\partial x^2} + \gamma \frac{\partial^2 \omega}{\partial y^2}\right) \left(\frac{6\rho k}{5}\right) \frac{\partial^2 \omega}{\partial t^2} + p(x, y, t) = 0 \qquad (3)$$

where α and β are rotations, ω is lateral deflection, h is the thickness of the panel, f and k are tracing constants identifying the effects of rotatory inertia and transverse shear deformation respectively; D_x, D_y and D_{xy} are the orthotropic stiffnesses defined as

$$D_x = \frac{E_x h^3}{12(1 - \nu_{xy}\nu_{yz})}, D_y = \frac{E_y h^3}{12(1 - \nu_{xy}\nu_{yx})}$$

$$D_{xy} = \frac{G_{xy}\, h^3}{12}, \bar{H} = D_x\, \nu_{yx} + 2D_{xy}$$

$$\bar{\alpha}^2 = \frac{6}{h^2 A} \iint_{\text{panel area}} \left[\left(\frac{\partial \omega}{\partial x}\right) + \frac{D_y}{D_x}\left(\quad \right)^2\right] dx\, dy$$

where E_x, E_y are Young's moduli in fibre and transverse directions respectively, G_{xy}, G_{xz}, G_{yz} are orthotropic shear moduli, ν_{xy}, ν_{yx} are Poisson's ratios, ρ is the material density, A is the panel area and $p(x,y,t)$ is the externally applied load taken to be the normally reflected blast pulse (assumed to be uniform over a panel of small dimensions) and is given by[6]

$$p(x, y, t) = p_r(1 - t/t_d)\, e^{-\alpha t/t_d} \qquad (4)$$

$$p_r = \frac{[8(p_0/p_a) + 7][(p_0/p_a) + 1]}{[(p_0/p_a) + 7]} \qquad (5)$$

normal incidence, incident blast overpressure and ambient pressure respectively.

The solution of the problem lies in determining the panel deflection ω and the rotations α and β subject to the prescribed boundary conditions. The structural element under consideration is a panel bounded by stiffners on all sides. The panel has been considered to have the following dimensions : length a, width b and thickness h, with the origin of coordinate axes at a corner of the panel. The boundary conditons are[6]:

(i) Simply supported panel (SS)

$$\beta = 0, \omega = 0, \frac{\partial^2 \omega}{\partial x^2} \cdot \nu_{yx} \frac{\partial^2 \omega}{\partial y^2} = 0, \text{ at } x = 0, a$$

$$\alpha = 0, \omega = 0, \frac{\partial^2 \omega}{\partial y^2} + \nu_{xy} \frac{\partial^2 \omega}{\partial x^2} = 0, \text{ at } y = 0, b \qquad (6)$$

(ii) Clamped in panel (CL)

$$\omega = 0,\ \alpha = 0,\ \beta = 0, \text{ at } x = 0,\ a \text{ and } y = 0,\ b \qquad (7)$$

In accordance with the conditions occurring in aircraft structure, the panel has been taken to be rigidly framed, hence the edges of the panel have been taken to be immovably constrained. An approximate solution has been attempted using Galerkin method. The general form of solution assumed is[7].

$$\omega = W\phi_{\omega m}(x)\phi_{\omega n}(y)\tau(t)$$

$$\alpha = \Gamma\phi_{\alpha m}(x)\, \phi_{\alpha n}(y)\tau(t)$$

$$\beta = \Lambda\phi_{\beta m}(x)\phi_{\beta n}(y)\tau(t) \qquad (8)$$

where in the SS case :

$$\phi^S_{\omega m}(x) = \sin \lambda^S_m x,\ \phi^S_{\omega n}(y) = \sin \delta^S_n y$$

$$\phi_{\alpha m}(x) = \cos \lambda^S_m x,\ \phi_{\alpha n}(y) = \sin \delta^S_n y$$

$$\phi_{\beta m}(x) = \sin \lambda^S_m x,\ \phi_{\beta n}(y) = \cos \delta^S_n y \qquad (9)$$

with $\lambda^s_m = \frac{m\pi}{a}$, $\delta^s_n = \frac{n\pi}{b}$ and in the CL case, for odd

$$\phi^C_{\omega m}(x) = \cos \mu_m\left(\frac{x}{a} - \frac{1}{2}\right) + \eta_m \cos h\, \mu_m\left(\frac{x}{a} - \frac{1}{2}\right),$$

$$\phi^C_{\omega n}(y) = \cos\mu_n\left(\frac{y}{b} - \frac{1}{2}\right) + \eta_n \cos h\,\mu_n\left(\frac{y}{b} \quad \frac{1}{2}\right)$$

$$\phi^C_{\alpha m}(x) = \sin\mu_m\left(\frac{x}{a} - \frac{1}{2} \quad - \eta_m \sin h\,\mu_m\left(\frac{x}{a} \quad \frac{1}{2}\right)\right.$$

$$\phi^C_{\alpha m}(y) = \cos\mu_n\left(\frac{y}{b} - \frac{1}{2}\right) - \eta_n \cos h\,\mu_n\left(\frac{y}{b} \quad \frac{1}{2}\right)$$

$$\phi^C_{\beta m}(x) = \cos\mu_m\left(\frac{x}{a} \quad \frac{1}{2}\right) + \eta_m \cos h\,\mu_m\left(\frac{x}{a} \quad \frac{1}{2}\right)$$

$$\phi^C_{\beta n}(y) = \sin\mu_n\left(\frac{y}{b} \quad \frac{}{2}\right) - \eta_n \sin h\,\mu_n\left(\frac{y}{b} \quad \frac{}{2} \right. \qquad 10)$$

where μ_m are the roots of the equation

$$\tan\left(\frac{\mu_m}{2}\right) \qquad \left(\frac{\mu_m}{2}\right) = 0$$

and

$$\eta_m = \frac{\sin\left(\frac{\mu_m}{2}\right)}{\sin h\left(\frac{\mu_m}{2}\right)} \qquad (1$$

Similar relation holds for μ_n. Changing sin and sinh to cos and cosh, respectively and replacing the minus sign in functions for $\phi^C_{\alpha m}(x)$, etc, by plus sign, the corresponding functions for even values of m are obtained.

Now, employing Galerkin's method, the following algebraic equations are obtained from the Eqns (1) and (2) (for both SS and CL cases) :

$$a_{11}W + a_{12}\Gamma + a_{13}\Lambda = 0 \qquad 12)$$

$$a_{21}W + a_{22}\Gamma + a_{23}\Lambda = 0 \qquad 13)$$

whence Γ and Λ can be expressed as

$$\Gamma = q_1 W = \frac{(a_{23}a_{11} - a_{21}a_{13})}{(a_{22}a_{13} - a_{23}a_{12})} W \qquad (14)$$

$$\Lambda = q_2 W = \frac{(a_{12}a_{21} \quad a_{11}a_{22})}{\quad a_{23}a_{12})} W \qquad (15)$$

Now employing again Galerkin's procedure in the Eqn (3), and using (14) and (15), the equation of motion is finally obtained in the following form :

$$A\ddot{\tau} + B\tau + C\tau^3 = P(t) \qquad (16)$$

where for the SS case

$$a^S_{11} = \lambda^S_m, a^S_{12} = \; + \frac{6}{5G_{xz}h}(D_x\lambda^{S^2}_m + D_{xy}\delta^{S^2}_n)$$

$$a^S_{13} = \frac{6}{5G_{xz}h}(\bar{H} - D_{xy})\lambda^S_m\delta^S_n$$

$$a^S_{21} = \delta^S_n, a^S_{22} = \frac{6}{5G_{xz}h}(\bar{H} - D_{xy})\lambda^S_m\delta^S_n$$

$$a^S_{23} = \; + \frac{6}{5G_{yz}h}(D_y\delta^{S^2}_n + D_{xy}\lambda^{S^2}_m$$

$$\bar{\alpha}^S = \frac{3}{2}\left(\frac{W}{h}\right)^2(\lambda^{S^2}_m + \gamma\delta^S_n \quad \tau^2$$

and

$$A^S = \frac{6\rho}{5}, B^S = G_{xz}(\lambda^{S^2}_m + \lambda^S_m q^S_1) + G_{yz}(\delta^{S^2}_n + \delta^S_n q^S_2)$$

$$C^S = \frac{18}{10}\left(\frac{W}{h}\right)^2 \frac{D_x}{h}(\lambda^{S^2}_m + \gamma\delta^S_n \quad {}^2, P^S(t) = \frac{16}{\cdot}\, p(t)$$

and in the CL case

$$\lambda^C_m = \frac{\mu_m}{a}, \delta^C_n = \frac{\mu_n}{b}, \text{ and } \qquad - \lambda^C_m d_{21}$$

$$a^C_{12} = d_{21} + \frac{6D_x}{5G_{xz}h}\lambda^{C^2}_m d_{11} + \frac{6D_{xy}}{5G_{xz}h}\delta^{C^2}_n d_{22}$$

$$a^C_{13} = \frac{6}{5G_{xz}h}(\bar{H} - D_{xy})\lambda^C_m\delta^C_n d_{22}$$

$$a^C_{21} = \quad \delta^C_n d_{12}, a^C_{22} = \frac{6}{5G_{yz}h}(\bar{H} \quad D_{xy})\lambda^C_m\delta^C_n d_2$$

$$a_{23} = d_{22} + \frac{6}{5G_{yz}h}(D_y\delta^{C^2}_n d_{21} + D_{xy}\lambda^{C^2}_m d_{22}$$

$$\bar{\alpha}^{C^2} = \frac{3}{2}\left(\frac{W}{h}\right)^2(d_{21}\lambda^{C^2}_m + d_{12}\delta_{12}\delta^C_n$$

$$P^C(t) = \frac{16}{\mu_m\mu_n W}\sin\left(\frac{\mu_m}{2}\right)\sin\left(\frac{\mu_n}{2} \quad p(t\right.$$

and

$$A^C = \frac{6\rho}{5}, B^C = G_{xz}d_{21}(\lambda^C_m \quad \lambda^C_m q^C_1$$

$$+ G_{yz}d_{12}(\delta^{C^2}_n \quad q^C_2\delta^C_n)$$

$$C^C = \frac{18}{10}\left(\frac{W}{h}\right)^2\frac{D_x}{h}(d_{21}\lambda^{C^2}_m + \gamma$$

$$d_{11} = (1 + \eta^2_m)(1 + \eta^2_n)$$

$$d_{12} = (1 + \eta_m^2)\left(\quad \eta_n^2 + \frac{2}{l'} \sin \mu_n \right)$$

$$d_{21} = \left(- \eta_m^2 + \frac{2}{l'} \sin \mu_m \right)(1 + \eta_n^2)$$

$$d_{22} = \left(- \eta_m^2 + \frac{2}{l'} \sin \mu_m \right)\left(- \eta_n^2 + \frac{}{l'} \sin \mu_n \right)$$

The non-linear ordinary differential Eqn (16) was solved numerically by Runge-Kutta-Gill method, assuming the deflection and the velocity of the panel to be zero initially.

2.2 Calculation of Stresses and Failure Criterion

For large deflection of panels, bending and stretching are coupled, hence the stresses consist of the sum total of bending and membrane stresses :

$$(\sigma \quad \sigma \quad \sigma_{xy}) = \frac{1}{h}(N_x, N_y, N_{xy})$$
$$\frac{12z}{h^3}(M_x, M_y, M \qquad (17)$$

where N_x, N_y, N_{xy} are the membrane stresses and M_x, M_y, M_{xy} are the bending moments.

The magnitude of the stresses would depend upon the intensity of the blast pulse, which in turn depends upon the distance between the structural element and the point of explosion. For a given explosive (whose strength is known in terms of TNT equivalent), the threshold distance for permanent damage is defined here as the distance at which the explosion of a given mass of TNT is just sufficient to cause the failure of the panel as characterised by Tsai-Hill/Hoffman criterion of failure[8].

$$\frac{\sigma_1^2}{\quad} - \frac{\sigma_1\sigma_2}{F_{1T}F_{1C}} + \frac{\sigma_2^2}{F_{2T}F_{2C}} + \left(\frac{}{F_{1T}} \quad \frac{1}{F_{1C}} \right)\sigma_1$$
$$+ \left(\frac{1}{F_{2T}} - \frac{1}{F_{2C}} \right)\sigma_2 + \frac{\sigma_{12}^2}{F_{12}} \leq 0 \qquad (18)$$

where σ_1, σ_2, σ_{12} are the stresses in the fibre direction, transverse direction and shear stress, respectively; F_{IT} and F_{IC} are ultimate strengths in tension and compression respectively in the fibre direction; F_{2T} and F_{2C} are ultimate strengths in tension and compression respectively in the transverse direction and F_{12} is the ultimate shear strength. Since the *x*- and *y*-axes have been taken to be parallel to the fibre and transverse directions respectively, hence

$$\sigma_1 = \sigma_x, \ \sigma_2 = \sigma_y, \ \sigma_{12} = \sigma_{xy}$$

2.3 Generalisation for a Laminate Panel Consisting of Several Plies at Different Orientations

2.3.1 Moduli for Orthotropic Laminate with Several Plies

Thus the formulation developed here was limited to a panel consisting of a single ply and a coordinate system so chosen that *x-y* axes coincide with the fibre and transverse directions, respectively. But most of the composite panels in applications are in laminate form comprising several plies at various orientations. The formulation described above can be generalised easily for laminates especially for orthotropic laminates, the most widely used ones in applications. We consider the ply pattern : 0°/θ°/–θ° so as to produce an orthotropic panel, where θ is the angle made by the fibre with the reference axis.

The stress-strain law under plane stress conditions for a single ply with the axes parallel and transverse to the fibre direction (i.e. θ = 0) are :

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} Q_{11}(0) & Q_{12}(0) & 0 \\ Q_{12}(0) & Q_{22}(0) & 0 \\ 0 & 0 & Q_{66}(0) \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} \qquad (19)$$

where

$$Q_{11}(0) = E_x/(1 - \nu_{xy}\nu_{yx}), \ Q_{12}(0) = \nu_{yx}E_x/(1 - \nu_{xy}\nu_{yx}$$

$$Q_{22}(0) = E_y/(1 - \nu_{xy}\nu_{yx}), \ Q_{66}(0) = G_{xy}$$

For a single ply in which fibres make an angle θ with a reference axis fixed in the laminate, the stress-strain law is given by :

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} Q_{11}(\theta) & Q_{12}(\theta) & Q_{16}(\theta) \\ Q_{12}(\theta) & Q_{22}(\theta) & Q_{26}(\theta) \\ Q_{16}(\theta) & Q_{26}(\theta) & Q_{66}(\theta) \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \qquad (20)$$

where $Q_{ij}(\theta)$ are related to $Q_{ij}(0)$ by the following equations :

$$\begin{bmatrix} Q_{11}(\theta) \\ Q_{12}(\theta) \\ Q_{22}(\theta) \\ Q_{16}(\theta) \\ Q_{26}(\theta) \\ Q_{66}(\theta) \end{bmatrix} = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 & 4c^2s^2 \\ c^2s^2 & c^4+s^4 & c^2s^2 & -4c^2s^2 \\ s^4 & 2c^2s^2 & c^4 & 4c^2s^2 \\ c^3s & -cs(c^2-s^2) & -cs^3 & -2cs(c^2-s^2) \\ cs^3 & cs(c^2-s^2) & -c^3s & 2cs(c^2-s^2) \\ c^2s^2 & -2c^2s^2 & c^2s^2 & (c^2-s^2)^2 \end{bmatrix} \begin{matrix} Q_{11}(0) \\ Q_{12}(0) \\ Q_{22}(0) \\ Q_{66}(0) \end{matrix} \qquad (21)$$

where $c = \cos\theta$ and $s = \sin\theta$.

Now, consider a laminate comprising n plies and let the angle between the fibre direction in the k-th ply and x-axis be θ_k. In terms of the average stresses (averaged through the thickness), the stress-strain law for the laminate becomes :

$$\begin{matrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{matrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \qquad (22)$$

where

$$A_{ij} = \left(\sum_{k=1}^{n} Q_{ij}(\theta_k)\right)/n \qquad (23)$$

For orthotropic laminates, having structural axes as the axes of orthotropy, $A_{16} = A_{26} = 0$ (as in the case with a laminate consisting of 0° and matched pairs of ±θ° plies only). The moduli for orthotropic laminate can be obtained as[8]

$$E_x = A_{11} - A_{12}^2/A_{22},\ E_y = A_{22} - A_{12}^2/A_{11},$$

$$\nu_{xy} = A_{12}/A_{22},\ \nu_{yx} = A_{12}/A_{11},\ G_{xy} = A_{66}$$

Once the moduli are determined, the dynamic response of the laminate panel may be obtained as discussed in 2.1.

2.3.2 Calculation of Stresses and Failure Criterion for Individual Plies in a Laminate

Once the lay-up of plies in a laminate is known, the procedure described in 2.1 would give the deflection and hence the strains may be computed which are the same for all the plies. However, if the above procedure is used to obtain the stresses, then only average stresses are obtained. To determine the actual stresses in a particular ply, the strains obtained thus have to be substituted into the Eqn. (20). The actual stress distribution may be very different from the average one, as transverse direct stresses and shear stresses may develop even though no such stresses are applied to the laminate.

Thus, it is more reasonable to develop a failure criterion based on the actual stresses in the individual plies than the average stresses as it is quite possible that only a few plies at a particular orientation may be damaged under a given blast load while the majority of the plies may remain unaffected.

3. COMPARISON WITH CLASSICAL THEORY

Here, the methodology based on classical theory is described briefly. The details are given in[9]. The governing system of partial differential equations in this case is :

$$\frac{1}{E_y}\frac{\partial^4 F}{\partial x^4} + \left(\frac{}{E_3}\right)\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{}{E_x}\frac{\partial^4 F}{\partial y^4} = \left(\frac{\partial^2 \omega}{\partial x \partial y}\right) \frac{\partial^2 \omega}{\partial x^2}\frac{\partial^2 \omega}{\partial y^2} \qquad (25)$$

$$D_x \frac{\partial^4 \omega}{\partial x^4} + 2D_3 \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 \omega}{\partial y^4} + \rho h \frac{\partial^2 \omega}{\partial t^2} = p(x,y,t) + h\left[\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 \omega}{\partial x^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 \omega}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 \omega}{\partial y^2}\right] \qquad (26)$$

where F is the Airy's stress function, and

$$1/E_3 = 1/G_{xy} - 2\nu_{xy}/E_x,\ D_3 = D_x\nu_{yx} + 2G_{xy}h^3/12.$$

(a) SS panel : In this case, the solution is assumed in the form :

$$\omega(x,y,t) = hf(t)\cos\frac{\pi x}{a}\cos\frac{\pi y}{b} \qquad (27)$$

$$F(x, y, t) = F^*(x, y) f^2(t) \quad (28)$$

Using the conditions of rigid framing of edges together with the Eqns. (25) and (27), the following equation is obtained :

$$F^*(x, y) = \frac{h^2 \pi^2}{16(1 - \nu_{xy}\nu_{yx})} \left[\left(\frac{\nu_{yx} E_x}{a^2} + \frac{E_y}{b^2} \right) x^2 + \left(\frac{E_x}{a^2} + \frac{\nu_{xy} E_y}{b^2} \right) y^2 \right]$$

$$\frac{h^2}{32} \left[E_y \frac{a^2}{b^2} \cos \frac{2\pi x}{a} + E_x \frac{b^2}{a^2} \cos \frac{2\pi y}{b} \right] \quad (29)$$

(b) CL panel : In this case, the solution is assumed in the form :

$$\omega(x, y, t) = hf(t) \cos^2 \frac{\pi x}{a} \cos^2 \frac{\pi y}{b} \quad (30)$$

The stress function in this case is obtained as

$$F^*(x, y) = C_1 x^2 + C_2 y^2 + C_3 \cos \frac{2\pi x}{a} + C_4 \cos \frac{2\pi y}{b} + C_5 \cos \frac{2\pi x}{a} \cos \frac{2\pi y}{b} + C_6 \cos \frac{4\pi x}{a} + C_7 \cos \frac{4\pi y}{b} + C_8 \cos \frac{2\pi x}{a} \cos \frac{4\pi y}{b} + C_9 \cos \frac{4\pi x}{a} \cos \frac{2\pi y}{b} \quad (31)$$

where

$$C_1 = \frac{3\pi^2 h^2}{64(1 - \nu_{xy}\nu_{yx})} \left(\frac{\nu_{yx} E_x}{a^2} + \frac{E_y}{b^2} \right)$$

$$C_2 = \frac{3\pi^2 h^2}{64(1 - \nu_{xy}\nu_{yx})} \left(\frac{E_x}{a^2} + \frac{\nu_{xy} E_y}{b^2} \right)$$

$$C_3 = -\frac{E_y h^2}{32} \frac{a^2}{b^2}, \quad C_4 = -\frac{E_x h^2}{32} \frac{b^2}{a^2}$$

$$C_5 = -\frac{h^2}{16} a^2 b^2 E_x E_y E_3 / (b^4 E_3 E_x + a^2 b^2 E_x E_y + a^4 E_y E_3)$$

$$C_6 = -\frac{E_y h^2}{512} \frac{a^2}{b^2}, \quad C_7 = -\frac{E_x h^2}{512} \frac{b^2}{a^2}$$

$$C_8 = -\frac{h^2}{32} a^2 b^2 E_x E_y E_3 / (16 b^2 E_3 E_x + 4a^2 b^2 E_x E_y + a^4 E_y E_3)$$

$$C_9 = -\frac{h^2}{32} E_x E_y E_3 / (16 b^4 E_3 E_x + 4a^2 b^2 E_x E_y + a^4 E_y E_3)$$

Employing now the Galerkin's procedure, the equation of motion is obtained as :

$$A\ddot{f} + Bf + Cf^3 = P(t) \quad (32)$$

where in the SS case

$$A^S = \rho h^2, \quad B^S = h\pi^4 \left(\frac{D_x}{a^4} + \frac{2D_3}{a^2 b^2} + \frac{D_y}{b^4} \right)$$

$$C^S = \frac{h^4 \pi^4}{8(1 - \nu_{xy}\nu_{yx})} \left[\left(\frac{1}{a^4} + \frac{\nu_{yx}}{a^2 b^2} \right) E_x + \left(\frac{\nu_{xy}}{a^2 b^2} + \frac{1}{b^4} \right) E_y \right] + \frac{h^4 \pi^4}{16} \left(\frac{E_x}{a_4} + \frac{E_y}{b^4} \right)$$

$$P^S(t) = \frac{16}{\pi^2} p(x, y, t)$$

and in the CL case

$$A^C = \frac{9}{16} \frac{\rho h^2}{\pi^4}, \quad B^C = h \left(\frac{3}{a^4} D_x + \frac{2D_3}{a^2 b^2} + \frac{3}{b^4} D_y \right)$$

$$C^C = \frac{2h^2}{a^2 b^2} \left[\cdot \; 3 - (C_3 + C_4 + C_5 + C_6 + C_7) - \frac{1}{2}(C_8 + C_9) \right]$$

$$P^C(t) = \frac{1}{\pi^4} p(x, y, t)$$

The stresses and the failure criteria are obtained as described in Sec. 2.

4. RESULTS AND DISCUSSION

A laminate consisting of 24 plies arranged symmetrically in the form 0°/θ°/ – θ° is considered here. The values of θ are chosen as 30°, 45°, 60°, 75° and 90°. The moduli for a single ply are taken from[10].

$E_x = 142$ *GPa*, $E_y = 9.0$ *GPa*, $G_{xy} = G_{yz} = G_{xz} = 5.5$ *GPa*, $\nu_{xy} = 0.32$, $\nu_{yx} = 0.02028$, $F_{1T} = 1130$ *MPa*, $F_{1C} = 869$ *MPa*, $F_{2T} = 37.2$ *MPa*, $F_{2C} = 145$ *MPa*, $F_{12} = 51$ *MPa*.

The panel is assumed to have the following dimensions : length 10.0 cm, width 8.0 cm and thickness 0.25 cm. Let n_1, n_2, n_3 be the number of plies inclined at 0°, θ°, – θ° to the material *x*-axis.

In Fig. 1, the amplitudes of non-linear oscillation of the panel for SS and CL boundary conditions are plotted for both classical theory and shear deformation theory. It is observed that the amplitude of oscillation as

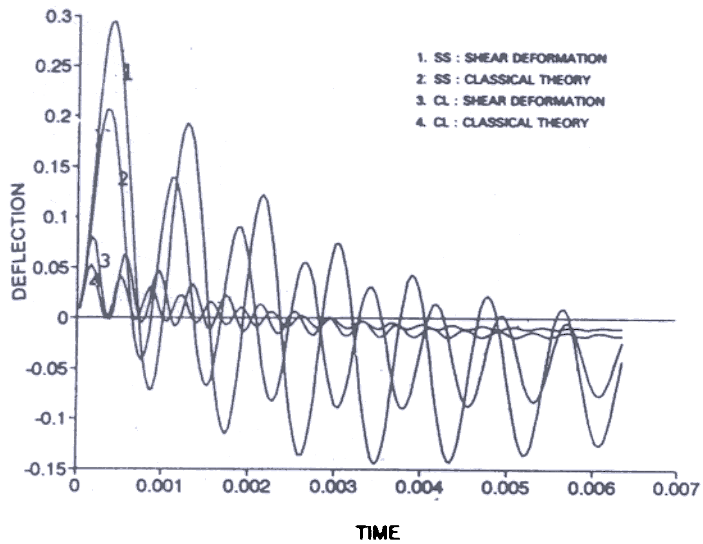

Figure **Non-linear oscillation of the composite panel for different boundary conditions.**

obtained from shear deformation theory is nearly 1.5 times of that given by classical theory. The distance between the panel and the point of explosion is 4 m (so that there is no damage). In this case $n_1 = 24$, $n_2 = n_3 = 0$. Following Bauer[11] and Ref.(1), $(W/h) = 1$. In Fig. 2, the non-linear vibrational behaviour of the clamped panel with $n_1 = 16$, $n_2 = n_3 = 4$ is shown for various values of θ (for shear deformation theory only). It is seen

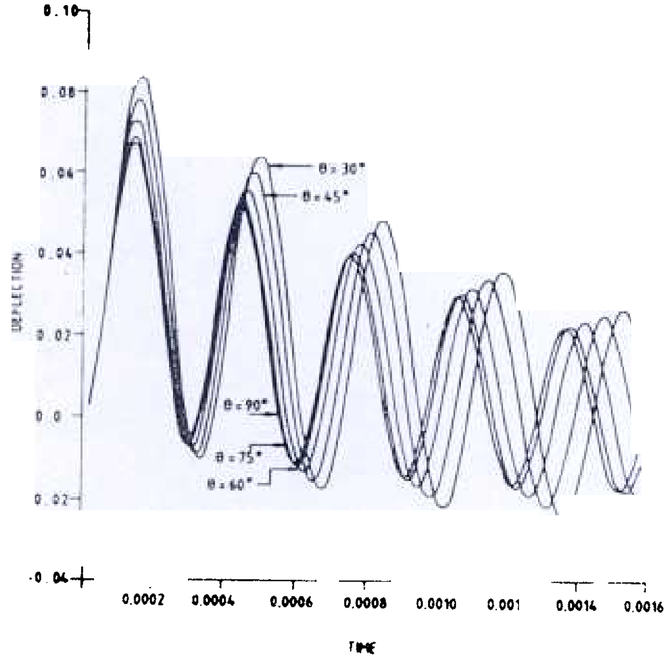

Figure 2. **Non-linear oscillation of the composite panel for different orientations of the plies.**

that with increasing values of θ, there is a decrease in the amplitude of vibration and also there is a gradual reduction in the period of vibration.

Since the aircraft structure can be more closely modelled as consisting of clamped in panels, only the CL case has been considered to study the damage. Damage in a particular ply is assumed to occur when the Tsai-Hill/Hoffman criterion is violated. Since the failure of various plies with different orientations would correspond to various threshold distances, we have taken the distance for which 0° plies would fail as an overall measure of the threshold distance. The dependence of the threshold distance on the number of plies n_2 and n_3 inclined at ±θ is shown in Table 1.

Table 1. Threshold distance for composite laminate for various orientations of plies

No. of plies at angles			Threshold distance in *m* for various values of θ				
0° n_1	θ° n_2	–θ° n_3	30°	45°	60°	75°	90°
24	0	0	3.5	3.5	3.5	3.5	3.5
20	2	2	3.3	3.3	3.3	3.2	3.2
16	4	4	3.3	3.2	3.0	2.9	2.9
12	6	6	3.3	3.1	2.9	2.8	2.7
8	8	8	3.2	3.0	2.8	2.6	2.6
4	10	10	3.2	2.9	2.6	2.5	2.5

It is observed that as θ is increased, the threshold distance decreases, thus showing a resistance to damage. Further, for a given θ, more is the magnitude of n_2 (or n_3) the more is the resistance to damage. Hence, it may be concluded that a larger number of plies inclined at the largest possible angle θ to the central ply would ensure a better safety against the blast attack.

ACKNOWLEDGEMENTS

The authors are thankful to Dr. R Natarajan, Director, CASSA for encouragement and giving permission to publish this paper.

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