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SHORT COMMUNICATION

Empirical Methods to Estimate the Burn Rate Scale up Factor from Sub-scale to Full Scale Solid Rocket Motors

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ABSTRACT

Two empirical methods to estimate the burn time of full scale motors (FSMs) are presented. The static test data of sub-scale motors and other FSMs provide the necessary input for the use of these methods. The applicability of these methods was verified by comparing the predicted values with actual values which were found to match closely. Advantages and disadvantages of each method are discussed.

NOMENCLATURE

a	Pressure coefficient
A_b	Burning surface area
A_t	Nozzle throat area
F	Augmentation factor from BEM to FSM burn rate
C_D	Discharge coefficient
K	Motor design constant
m	Mass flow rate of gases
n	Burn rate index
P_c	Chamber pressure (web average pressure)
P_*	Standardised/normalised chamber pressure
r_b	Burn rate at web average pressure
r_{*FSM}	Burn rate of full scale motor at normalised conditions
r_{BEM}	Burn rate of ballistic evaluation motor (BEM) at normalised conditions
t_b	Burn time (s)
W	Propellant web thickness (mm)
FSMBR	Full scale motor burn rate
Motor X	Flight motor or FSM whose parameters are unknown and which have to be predicted.

1. INTRODUCTION

An approach using a combination of (i) demonstrated performance data obtained by sub-scale motor (BEM) firing or from previous firings of FSM of similar design and (ii) theoretical calculations, in conjunction with established inter-relationships among these parameters, grain design and geometry considerations forms the basis for estimating the burn rate scale-up factor from sub-scale to full scale solid rocket motors (SRMs). This paper describes two methods to estimate burn time of FSM. Of the two, one method can be used to calculate the web average pressure also.

2. APPROACH

Burn rate is generally described by an empirical formula

$$r = aP^n = r'(1+F)$$

where r' = linear burn rate
F = augmentation factor, which takes into account burning conditions that exist in the FSM.

Since the geometry/configuration of BEM and FSM are frozen at the end of the design phase, the factors that cause the enhancement of burn rate from mode of measurement to next (e.g., from BEM burn rate to FSM burn rate) remain the same for all subsequent motors of identical configuration under similar testing conditions. Hence the overall effect of various factors

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can be combined and represented by a single constant called augmentation factor (F).

In effect

(Burn rate of BEM $\times$ F = Burn rate of FSM)

The augmentation factor (F) from BEM burn rate to FSM burn rate is an essential input to the use of any of the two predictive methods, described in this paper.

2.1 Method I

This method requires the following inputs :

(a) Value of n, based on BEM firings,
(b) Augmentation factor from BEM to FSM, and
(c) The value of K of FSM, based on prior static tests

to predict the following parameters of the FSM :

(i) Web average pressure,
(ii) Burn rate at average web pressure, and
(iii) Burn time.

For clarity, the method is described stepwise, viz., steps (i)-(vii). Steps (i), (ii) and (iii) are based on the data derived from static tests of BEM and FSM.

(i) The burn rates (r_b) of BEMs and FSMs that are static-tested at various pressures (P_c) are normalised to standard pressure (P_*) by the relationship.

$$r_b \left[\frac{P_*}{P_c} \right]^n$$

(ii) Normal burn rate of FSM = $F \times$ normal burn rate of BEM.

F is derived from this relationship.

(iii) The motor design constant K is derived as follows:

$$m = A_b \cdot \rho \cdot r_b = C_D \, P_c \, A_t$$

The term $A_b \rho / C_D A_t$ is a constant for an SRM having specific propellant composition, grain port configuration and nozzle and can be denoted as K. Hence,

$$P_c = K r_b \text{ or } K = \frac{P_c}{r_b}$$

From the available static test data of FSMs, P_c and r_b are known; hence K is derived.

From the inputs of (i), (ii) and (iii) the parameters of the unknown motor are derived.

(iv) $r_{*FSM} = F \times r_{*BEM}$

The two terms on the RHS are derived from (i) and (ii)

(v) $P_c = K r_b = K r \left(\frac{P_c}{P_*} \right)^n$

Rearranging and simplifying

$$P_c = \left(\frac{K r}{P_*} \right)^{\frac{1}{(1-n)}}$$

(vi) $r_b = P_c / K$
(vii) Burn time = web/r_b = W/r_b

For convenience and quick use of this method nomographs of the above-mentioned procedure have been generated for small variations in the values of n and r_*.

2.2 Method II

This is a comparative technique, which uses one reference FSM parameter to estimate that of other FSMs. The procedure involves the estimation of deviation of certain parameters at BEM level and use of F value. This method gives the band width of burn time within which the burn time of the FSM under consideration is to fall.

The relation is

$$\frac{\Delta t_b}{t_{b\,ref}} \doteq \frac{\Delta n}{(1-n)^2} \ln \frac{S}{F r_{BEM}} - \frac{1}{(1-n)} \frac{\Delta r_{BEM}}{r_{BEM}}$$

where Δr_{BEM} = difference in burn rates of reference motor and any motor X, evaluated at BEM level, and

$$S = P_* K$$

Hence the burn time band width of any motor is

$$t_b = t_{b\,ref} \pm \Delta t_b$$

and $t_{b\,ref}$ = burn time of reference motor.

3. RESULTS AND DISCUSSION

The basis for prediction revolves around the value of K. The average value of K is estimated from the data of all static-tested FSMs.

The accuracy of Method I is better than that of Method II, since it uses data generated from a large number of prior FSM static tests to establish the values of two critical constants F and K. Method II uses the test data of a single reference FSM as basis. This gives satisfactory correspondence with actual test data in the case of most motors, though not all. To improve the utility of Method I, various steps have been presented in the form of nomographs, incorporating a small bandwidth for input parameters to account for motor-to-motor variation. The values of P_c and t_b predicted by Method I for PS3-492 (static-tested), PS1-FM-01 and PS3-503 (flight motors) are compared with those actually obtained after static test/launch as shown in Table 1.

Table 1. Values of P_c and t_b predicted by Method I and actual test values.

Motor No.	Predicted values		Actual values	
	P_c (MPa)	t_b (s)	P_c (MPa)	t_b (s)
PS3-492	4.539	74.55	4.635	73.61
PS3-503	4.539	74.55	**	75.50
PS1-FM-01	3.620	93.67	**	97.97

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