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Rupture of Human Skin Membrane under Impact of Paraboloidal Projectile: Bullet Wound Ballistics

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ABSTRACT

This paper attempts to study the effect of the impact of a paraboloidal projectile on human skin membrane. The tip of the projectile (i.e., the bullet tip) has been considered to be paraboloidal and is made of lead or steel. The threshold velocity, i.e., the velocity when the skin membrane is about to rupture has been calculated for human beings of various age groups. The threshold velocity for a paraboloidal projectile of certain dimensions has been found, for all age groups, to be less than that of a spherical projectile under similar conditions.

1. INTRODUCTION

The effect of the impact of a spherical projectile has already been studied by Jauhari and Mohanta[1]. The classic breakdown of human skin under the impact of a spherical projectile has been theorised by Jauhari and Bandhopadhyay[2] by taking recourse to the theory of elasticity. As a first approximation, the skin has been treated as homogeneous, isotropic, elastic membrane like a ductile material. On the basis of maximum shear theory, the elastic breakdown was investigated and an expression for the threshold velocity for the elastic break down was derived. As in the yielding of an elastic body, the elastic breakdown of the skin membrane is the first stage in the process leading to its ultimate rupture. Once yielding starts, the inelastic strain increases, eventually leading to the rupture.

It has been assumed that the skin membrane would just rupture when kinetic energy of the bullet projectile per unit volume of the strained skin membrane equals the strain energy per unit volume obtained in a simple tension test of the skin membrane. If m is the mass of the bullet, V_{th} is the threshold velocity just to rupture the skin membrane, Δ_o the volume of the skin strained due to impact, and A the area under the stress-strain curve in a simple tension test, then the threshold velocity for penetration of the skin membrane is given as :

$$V_{th} = \sqrt{2\,\Delta_o A/m} \tag{1}$$

Equation (1) can be used to calculate the threshold velocity for the penetration, provided A, Δ_o and m are known.

2. EXPRESSION FOR Δ_o

The tip of the bullet is taken as the origin, its axis as X-axis and the frame of reference as moving with the bullet. The punch of the bullet is considered up to distance x in the membrane (Fig. 1). The paraboloid is considered as the solid of revolution obtained by revolving about X-axis, the parabola :

$$y^2 = 4Cx, \quad C = b^2/4a \tag{2}$$

A length $2y$ of the unstrained skin will be stressed to a length L given by

$$L/2 = \sqrt{x}.\ \sqrt{x+C} + C\ \log \frac{\sqrt{x+C}+\sqrt{x}}{\sqrt{C}} \tag{3}$$

The percentage elongation of the strained skin is given by

$$\varepsilon = \frac{L-L_o}{L_o} \times 100 \tag{4}$$

Equation (4) gives the percentage elongation so long as the bullet has pressed the skin up to a distance only less than or equal to a. For further depression, the percentage elongation ε will be independent of y which takes the constant value b.

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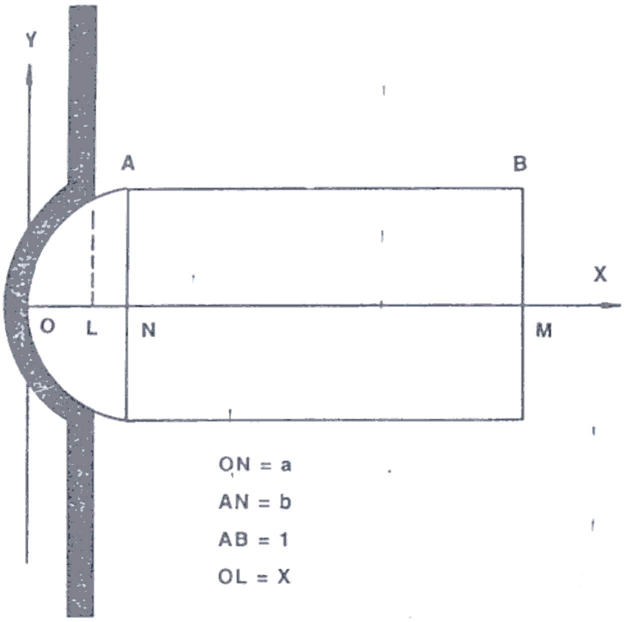

Figure 1. Impact of a bullet on skin membrane

When $x \leq a$,

$$\varepsilon/100 = \frac{\sqrt{x}.\sqrt{x+C} + C \log \frac{\sqrt{x+C} + \sqrt{x}}{\sqrt{C}} - 2\sqrt{C.x}}{2\sqrt{C}}$$

$(x<a)$ (5)

When $x = a$,

$$\varepsilon = \frac{100}{b}\left[\frac{1}{2}\sqrt{4a^2+b^2} + \frac{b^2}{4a}\log\frac{\sqrt{4a^2+b^2}+2a}{b} - b\right] \quad (6)$$

In particular, when $b = a$

$\varepsilon = 48\%$ (approximately)

The expression for Δ_o is obtained by multiplying the area of presentation of the bullet with the thickness of the skin. Thus,

$$\Delta_o = \pi y^2 t_o = \pi \frac{b^2}{a} x\, t_o \quad (7)$$

where t_o is the thickness of the skin. Therefore, the maximum value of Δ_o is obtained when $x = a$ and is given by

$$\max(\Delta_0) = \pi b^2 t_o \quad (8)$$

3. EXPRESSION FOR m

The mass of the bullet consists of two parts, viz., the paraboloidal part and the cylindrical part. Thus,

$$m = \frac{1}{2}\pi b^2 (a + 2l)\rho \quad (9)$$

where ρ is the density of the material of the bullet. Here, the bullet is considered to be of steel or lead for which ρ (steel) = 7.8 g/cc, ρ (lead) = 11 g/cc.

4. EXPRESSION FOR A

The A represents the area under the stress-strain curve of skin in a simple tension test. According to Seely[3] this, in the case of ductile materials, is approximately represented as :

$$A = \frac{1}{2}(S_y - S_u)\,\varepsilon_u \quad (10)$$

where S_y and S_u are the yield point and the ultimate strength of the material, respectively and ε_u is the strain at the rupture point. Stress-strain curves for various age groups are available [4]. The area under these curves has been directly calculated and is given in Table 1.

5. EXPRESSION FOR V_{th}

Finally, expression for the threshold velocity V_{th} is obtained by substituting the values of Δ_o, m and A in Eqn. (1) i.e.,

$$V_{th} = \frac{\sqrt{4 \times t_o A}}{a(a + 2l)\rho} \quad (11)$$

Equation (11) can be used to find the threshold velocity when the other parameters are known. It is further assumed that $l = 2a$, thereby reducing the Eqn. (11) to

$$V_{th} = \sqrt{\frac{4}{5}\frac{x}{a}\frac{t_o}{a}\frac{A}{\rho}} \quad (12)$$

The threshold velocities for various values (x/a), (t_o/a) and A are calculated for different age groups for bullet of steel or lead.

Table 1. A, percentage elongation at rupture and threshold velocity (r = 1/16" and t_o = 5 mm) for different age groups

Age group	A (ft-poundal/ cu.inch)	Elongation at rupture (%)	
15-30 years	822.55	55	113
30-50 years	837.79	33	113
50-80 years	891.10	31	115

6. DISCUSSION

The variations of the threshold velocity for the bullets of lead and steel for different values of t_o/a in the age group 30- 50 years have been shown in Fig. 2. Under identical conditions, a steel bullet has a greater threshold velocity than a lead bullet of same dimensions. Further, V_{th} increase with increase in the value of t_o/a.

Figure 3 shows the variations of the threshold velocity of a steel bullet for different age groups ($t_o/a = 2.0$). It is clear from the curves that more the age, greater is the threshold velocity. This change is more appreciable between the age groups of under three years and 15-30 years than between the age groups of 15-30 years and 50-80 years.

In Fig. 4, comparison between the threshold velocities of a spherical projectile and a paraboloidal bullet, both of steel, has been shown for the age group 30-50 years. If the two projectiles are of same mass, then from Eqn. (1) it is seen that for the two should be equal, other conditions remaining the same. The curves have been drawn with $r = b = a$ and $1 = 2a$. The

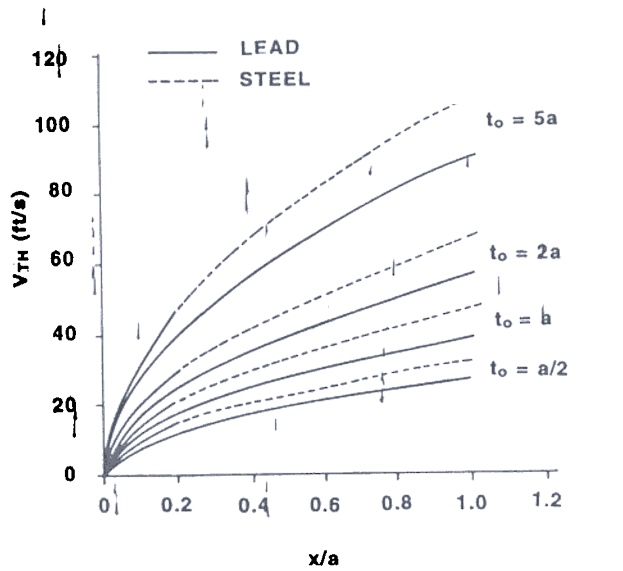

Figure 2. Threshold velocity V_{th} (ft/s) for the age group 30-50 years for different values of skin thickness.

threshold velocity for the bullet has been found to be considerably less than that of spherical projectile. In fact, the radius of curvature of the bullet tip is $a/2$ as compared to a of the spherical projectile, i.e., the bullet has a more sharp tip.

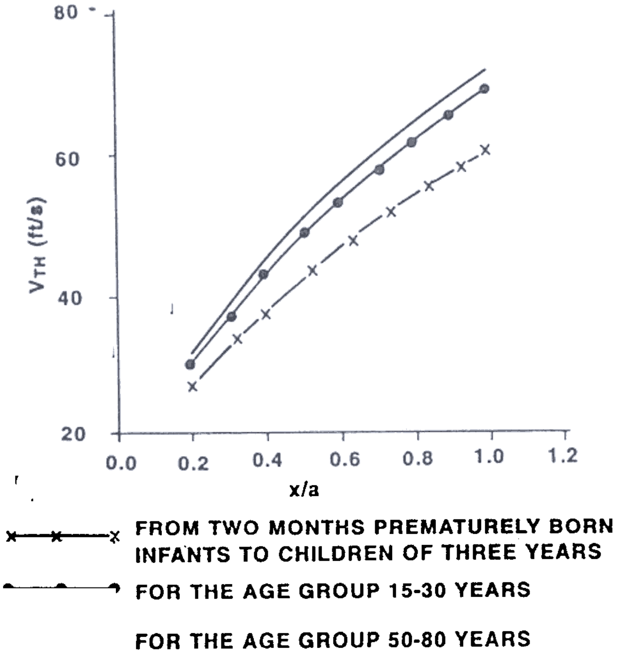

Figure 3. Threshold velocity V_{th} (ft/s) of steel bullet for different age groups with t_o=2a.

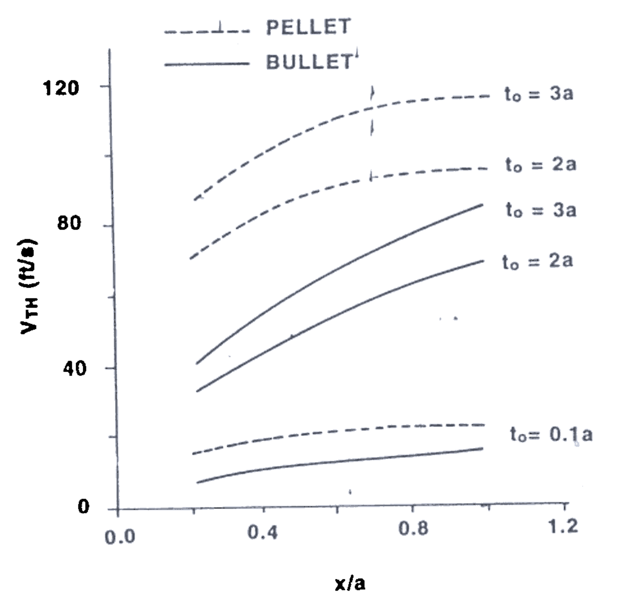

Figure 4. Threshold velocities for spherical and paraboloidal projectiles of same mass for different values of skin thickness.

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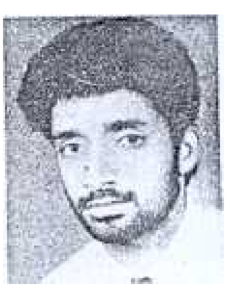

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