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On the Prediction of Damping and Vibratory Behaviour of Free-Free Multi-layered Conical Shells

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ABSTRACT

An analysis is presented for the prediction of resonance frequencies (ω) and the associated system loss factors (η_s) for all families of modes of vibrations in free-free multilayered conical shells, which are frequently used in aircraft, missiles and other allied systems. Because of the spin of a high order, missiles and gun-launched shells are subjected to torsional vibrations (axisymmetric). The governing equations of motion for the axisymmetric vibrations of a general multilayered conical shell have been derived using Hamilton's vibrational principle. The solution is obtained by utilising simple trignometric series modal assumptions in Galerkin's procedure. The correspondence principle of linear viscoelasticity for harmonic motions is used for evaluating the damping effectiveness of shells with elastic and viscoelastic layers. The resonance frequencies and the associated system loss factors for three-, five- and seven-layered conical shells with free edges are evaluated and their variations with geometric parameters are investigated. An increase in the number of layers (N) increases maximum obtainable η_s for most of the modes with proper selection of geometric parameters, but increasing N beyond 5 yields only marginal increase in η_s. Uniformly high values of η_s are obtained for all modes of vibration for high values of thickness ratio parameter. For thick shells, more layers are advisable for relatively high values of η_s for all modes.

NOMENCLATURE

$(E_x)_i$, $(E_\phi)_i$	Young's moduli of ith elastic layer along the X and ϕ directions, in-phase Young's moduli of ith viscoelastic layer along the X and ϕ directions
$(G_{x\phi})_i$, $(G_{xz})_i$ and $(G_{\phi z})_i$	shear moduli of the ith elastic layer, in-phase components of shear moduli of ith viscoelastic layer
L	shell slant length
m	number of terms in the expression for displacement; modal number along the meridional direction
N	total number of layers in multilayered shell
r_i	$R_{oi} + x \sin\alpha + z_i \cos\alpha$
R_{oi}	radius of the middle surface of ith layer at the small end of the shell
t_i	thickness of ith layer
t_1 and t_2	time limits
T	total thickness of the shell
$\overline{T}$	total kinetic energy
u_i	meridional displacement in ith layer of the shell along the X direction
$\overline{U}$	total strain energy
U_i	meridional displacement in ith layer along the X direction in the expression for the assumed approximate solution function
v_i	circumferential displacement in ith layer of the shell along the ϕ direction
V	ratio of thickness of viscoelastic layers to elastic layers for a constant size symmetric multilayered shell
V_i	circumferential displacement in ith layer along the ϕ direction in the expression for the assumed approximate solution function
w	transverse displacement of the multi-

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	layered shell along the Z direction
W	transverse displacement along the Z direction in the expression for the assumed approximate solution function
$\overline{W}$	work done
X	meridional coordinate
z_i	distance of a point from the middle surface of ith layer along the Z coordinate
Z	coordinate along the thickness of the shell
α	cone semi vertex angle
δ	ratio of shear modulus of viscoelastic layers to Young's modulus of elastic layers
η_i	material loss factor in shear for ith layer
η_s	system loss factor
$(\upsilon_{x\phi})_i$, $(\upsilon_{\phi x})_i$	Poisson's ratios of ith layer in the X—ϕ and ϕ—X directions
ρ_i	mass density of ith layer
ϕ	angular circumferential coordinate
ω	resonance frequency (Hz)

Subscripts

$(\dot{\ }) = {}_{,t}$	differentiation with respect to time t
x	differentiation with respect to x
$,\phi$	differentiation with respect to ϕ

1. INTRODUCTION

The growing use of multilayered composite conical shells in a variety of aircraft and missile structures has engendered much interest in their theoretical analysis. Suitable arrangements of elastic and viscoelastic layers in structural configurations are used to control vibration response by dissipation of vibratory energy effected by the deformation of the viscoelastic materials. If in a structure the viscoelastic layer is sandwiched between two elastic layers, it is predominantly the shear deformation in the viscoelastic layer which is responsible for energy dissipation. Nakra[1-3] reviewed the vibration analyses of beams, plates and shells in which layers of viscoelastic materials are used in constrained or unconstrained arrangements. Extensive review work on the vibration of shells has been reported by Hu[4] and Leissa[5]. An exhaustive work on free vibrations of layered conical shell has been reported by Wilkins, *et al*[6]. Siu and Bert[7] presented an analysis for axisymmetric and antisymmetric vibrational modes of free-free conical shells of homogeneous isotropic material and of sandwich construction with specially orthotropic facings and core.

In the present work a general multilayered conical shell consisting of an arbitrary number of alternate stiff elastic and soft viscoelastic layers has been considered. The material of the layers has been taken to be specially orthotropic. The governing equations of motion have been derived by variational principles. The present analysis considers extension, bending, inplane and transverse shear deformation in each of the layers of the shells. The longitudinal, translatory and rotary inertias along with the transverse inertia have been taken into account. The Galerkin method has been applied for

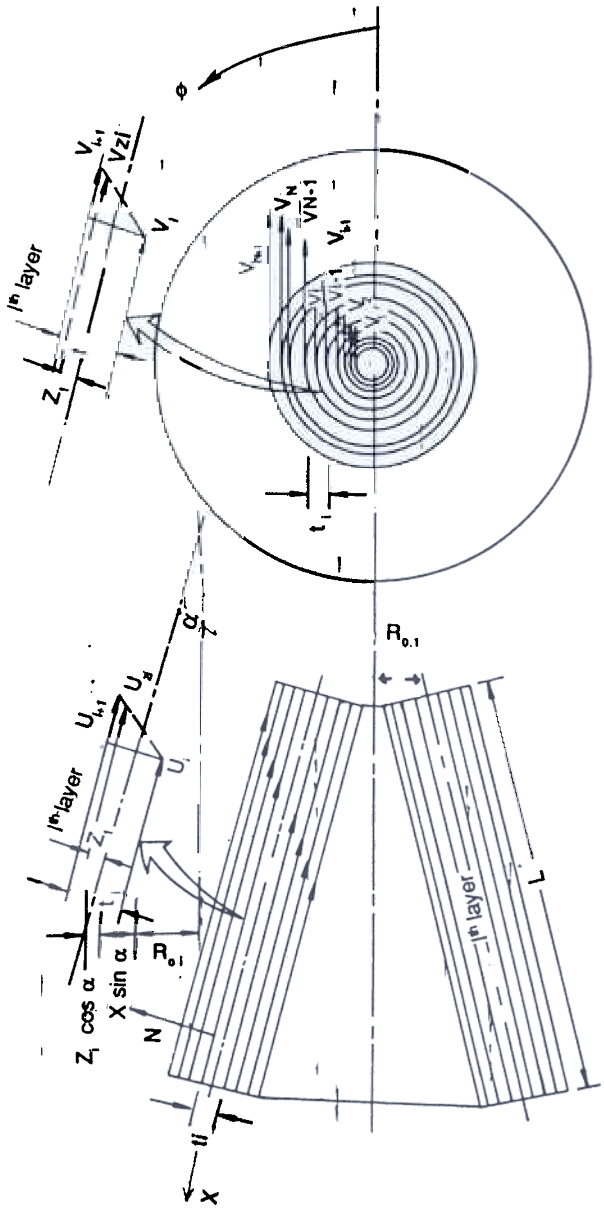

Figure 1. Assumed deformation pattern in the multilayered conical shell.

finding the solution and the damping effectiveness in the form of the system loss factor has been evaluated by applying the correspondence principle of linear viscoelasticity for harmonic motion. A computer program has been developed for computing resonant frequencies and associated system loss factors for a general layered shell with alternate elastic and viscoelastic layers. Variation of the resonance frequencies and the associated system loss factors with geometric parameters—thickness ratio parameter and total thickness parameter for three-, five- and seven-layered shells are presented. The results provide useful data for the proper choice of shell parameters to obtain the maximum possible system loss factors for different modes of vibrations.

2. GOVERNING EQUATIONS OF MOTION

The multilayered shell configuration is shown in Fig. 1. It is assumed that normal section in each layer remains plane and continuous before and after deformation and there is no slip at the interfaces. The deformations take account of bending, extension, inplane shear and transverse shear in all the layers of the shell. The deformations u_{zi}, v_{zi} at a distance z_i from the mid-plane of ith layer along X and ϕ directions respectively are given by

$$u_{zi}=\frac{1}{t_i}\left[u_i\left\{\frac{t_i}{2}-z_i\right\}+u_{i+1}\left\{\frac{t_i}{2}+z_i\right\}\right]$$

$$v_{zi}=\frac{1}{t_i}\left[v_i\left\{\frac{t_i}{2}-z_i\right\}+v_{i+1}\left\{\frac{t_i}{2}+z_i\right\}\right] \tag{1}$$

where u_i, v_i, u_{i+1} and v_{i+1} are displacement at radii R_{oi} and R_{oi+1} of the ith layer along X and ϕ directions respectively and t_i is the thickness of the ith layer. Strains in the ith layer of the shell are given as:

$$(\epsilon_{xx})_i=\frac{1}{t_i}\left[u_{i,x}\left\{\frac{t_i}{2}-z_i\right\}+u_{i+1,x}\left\{\frac{t_i}{2}+z_i\right\}\right]$$

$$(\epsilon_{\phi\phi})_i=\frac{\zeta_i}{t_i}\left[\left(v_{i,\phi}+u_i\sin\alpha\right)\left\{\frac{t_i}{2}-z_i\right\}+\left(v_{i+1,\phi}+u_{i+1}\sin\alpha\right)\left\{\frac{t_i}{2}+z_i\right\}+t_iw\cos\alpha\right]$$

$$(\epsilon_{zz})_i=0$$

$$(\gamma_{\phi z})_i=\frac{\zeta_i}{t_i}\left[t_iw_{,\phi}-v_i\cos\alpha\left\{\frac{t_i}{2}-z_i\right\}-v_{i+1}\cos\alpha\left\{\frac{t_i}{2}+z_i\right\}\right]+\frac{\{v_{i+1}-v_i\}}{t_i}$$

$$(\gamma_{zx})_i=w_{,x}+\frac{\{u_{i+1}-u_i\}}{t_i}$$

$$(\gamma_{x\phi})_i=\frac{\zeta_i}{t_i}\left[\left(u_{i,\phi}-v_i\sin\alpha\right)\left\{\frac{t_i}{2}-z_i\right\}+\left(u_{i+1,\phi}-v_{i+1}\sin\alpha\right)\left\{\frac{t_i}{2}+z_i\right\}\right]+\frac{1}{t_i}\left[v_{i,x}\left\{\frac{t_i}{2}-z_i\right\}+v_{i+1,x}\left\{\frac{t_i}{2}+z_i\right\}\right] \tag{2}$$

where

$$\zeta_i=\frac{1}{r_i}=\left(R_{oi}+x\sin\alpha+z_i\cos\alpha\right)^{-1} \tag{3}$$

Considering material of layers to be specially orthotropic, stress-strain relations are given as:

$$\begin{bmatrix}(\sigma_{xx})_i\\(\sigma_{\phi\phi})_i\\(\tau_{x\phi})_i\end{bmatrix}=\begin{bmatrix}(Q_{11})_i&(Q_{12})_i&0\\(Q_{21})_i&(Q_{22})_i&0\\0&0&(Q_{66})_i\end{bmatrix}\begin{bmatrix}(\varepsilon_{xx})_i\\(\varepsilon_{\phi\phi})_i\\(\gamma_{x\phi})_i\end{bmatrix}$$

$$\begin{bmatrix}(\tau_{\phi z})_i\\(\tau_{xz})_i\end{bmatrix}=\begin{bmatrix}(C_{44})_i&0\\0&(C_{55})_i\end{bmatrix}\begin{bmatrix}(\gamma_{\phi z})_i\\(\gamma_{xz})_i\end{bmatrix} \tag{4}$$

where the specially orthotropic material constants are

$$(Q_{11})_i=\frac{(E_x)_i}{\left\{1-(v_{x\phi})_i(v_{\phi x})_i\right\}}$$

$$(Q_{22})_i=\frac{(E_\phi)_i}{\left\{1-(v_{x\phi})_i(v_{\phi x})_i\right\}}$$

$$(Q_{12})_i=\frac{(v_{\phi x})_i(E_x)_i}{\left\{1-(v_{x\phi})_i(v_{\phi x})_i\right\}}$$

$$(Q_{21})_i=\frac{(v_{\phi x})_i(E_\phi)_i}{\left\{1-(v_{x\phi})_i(v_{\phi x})_i\right\}},$$

$$(v_{\phi x})_i(E_x)=(v_{x\phi})_i(E_\phi)_i \text{ making } (Q_{12})_i=(Q_{21})_i$$

$$(Q_{66})_i=(G_{x\phi})_i;\ (C_{44})_i=(G_{\phi z})_i;\ (C_{55})_i=(G_{xz})_i$$

where $(E_x)_i$, $(E_\phi)_i$; $(v_{x\phi})_i$, $(v_{\phi x})_i$; and $(G_{x\phi})_i$, $(G_{xz})_i$, $(G_{\phi x})_i$, are respectively the Young's moduli, the Poisson's ratios and the shear moduli of the material of the ith layer.

The strain energy $\overline{U}$ of the shell is given by

$$\overline{U} = \frac{1}{2}\sum_{i=1}^{N}\int_x\int_\phi\int_z\Big[(\rho_{xx})_i(\epsilon_{xx})_i + (\sigma_{\phi\phi})_i(\varepsilon_{\phi\phi})_i + (\tau_{\phi z})_i(\gamma_{\phi z})_i + (\tau_{zx})_i(\gamma_{zx})_i + (\tau_{x\phi})_i(\gamma_{x\phi})_i\Big]\zeta_i^{-1}dzd\phi dx \quad (5)$$

where N is the number of layers and L is the slant length of the shell. Considering transverse, meridional translatory and rotatory inertias, kinetic energy $\overline{T}$ of the multilayered conical shell is given by

$$\overline{T} = \frac{1}{2}\sum_{i=1}^{N}\int_x\int_\phi\int_z\Big[\rho_i\{\dot{w}^2 + \dot{u}_{Ij}^2 + \dot{v}_{zi}^2\}\Big]\zeta_i^{-1}dzd\phi dx \quad (6)$$

The work done $\overline{W}$ by the external excitation forces $f(x)$ sin ωt is given by

$$\overline{W} = \int_x\int_\phi f(x)\sin\omega t\; w\; d\phi dx \quad (7)$$

Performing the variation term by term and making use of Hamilton's energy equation

$$\int_{t_1}^{t_2}[\delta\overline{U} - \delta\overline{T} + \delta\overline{W}]\,dt = 0,$$

and considering the fact that for axisymmetric vibrations of multilayered conical shell, the deformations $u_1, u_2, u_3, \ldots\ldots\ldots u_{N+1}, v_1, v_2, v_3, \ldots\ldots v_{N+1}$ and ω are independent of angular coordinate ϕ, the governing equations of motion and boundary conditions for axisymmetric vibrations of the shell are obtained {Appendix: Eqns. (12) -(14)}.

3. SOLUTION FOR AXISYMMETRIC VIBRATIONS OF CONICAL SHELL

Although the free-free condition is most easily achieved experimentally, it is perhaps the most difficult condition from an analytical viewpoint, particularly in the case of a multilayered shell, because of the difficulty in finding functions that satisfy the boundary conditions.

From the fact that the displacements and the rotations at the free edges will always be unrestrained, the series of consine functions having non-zero values at the free edges are taken as approximate solution.

$$U_i = \sum_{m=1}^{\infty} U_{i,m}\cos\frac{m\pi x}{L}\sin\omega t$$

$$V_i = \sum_{m=1}^{\infty} V_{i,m}\cos\frac{m\pi x}{L}\sin\omega t$$

$$W = \sum_{m=1}^{\infty} W_{,m}\cos\frac{m\pi x}{L}\sin\omega t$$

$(m = 1, 2, 3, 4, \ldots\ldots\ldots\ldots\ldots n)$

The excitation may be expanded as

$$F = \sum_{m=1}^{\infty} F_m \sin\frac{m\pi x}{L}\sin\omega t \quad (9)$$

Substituting the assumed solution functions {Eqn. (8)} in the governing differential equations [Eqns. (12)-(14)], one set of m $(N+2)$ algebraic equations in terms of the meridional displacements $\{U_{m1}, U_{m2}, U_{m3}, \ldots U_{m(N+1)}\}$ and the transverse displacement (W_m) and another set of m $(N+1)$ algebraic equations in terms of the circumferential displacements $\{V_{m1}, V_{m2}, V_{m3}, \ldots V_{m(N+1)}\}$ are obtained. Replacing the real moduli of the layer materials by their complex moduli according to the correspondence principle of linear viscoelasticity, the first set of equations form a complex eigenvalue problem of the type.

$$\left[A - \omega_1^2 B\right]\{X\} = 0$$

where the column matrix $\{X\} = \{U_{m1}, U_{m2}, U_{m3}, \ldots, U_{m(N+1)}, W_m\}^T$, A and B are square matrices of order $m(N+2)$ and the eigenvalues ω_1^2 give resonance frequencies and associated system loss factors for coupled meridional and radial modes of vibrations.

Similarly, the second set of equations form a complex eigenvalue problem as:

$$\left[C - \omega_2^2 D\right]\{Y\} = 0 \quad (11)$$

where the column vector $\{Y\} = \{V_{m1}, V_{m2}, V_{m3}, \ldots, V_{m(N+1)}\}^T$, C and D are square matrices of order $m(N+1)$ and the eigenvalues ω_2^2 give resonance frequencies and associated system loss factors for torsional and other circumferential shear modes.

The elements of the matriqes A, B, C, and D are the functions of the geometric and the material properties of the shell. The real part ω of the complex eigenvalue is the resonance frequency and the ratio of the imaginary part to the real part is the associated system loss factor η_s (Rao and Nakra[8]). It can be shown that η_s is the ratio of the imaginary to the real part of the generalised complex stiffness and also the ratio of energy dissipated per cycle to the maximum strain energy during a cycle (Ungar and Kerwin[9]).

The above procedure has been programmed to compute the resonance frequencies and the associated system loss factors for all the modes of the families of modes of axisymmetric vibrations of a general multilayered conical shell with free edges. In the present analysis, five-term solution (m = 0,1,2,3,4) is taken. Vibrating modes consist of one family of 5 (N+2) coupled modes having meridional and radial deformations, and the other family of 5(N+1) modes having torsional and circumferential shear deformations. Though the modes of the first family are coupled, deformations occur predominantly along meridional or radial direction and they are named accordingly.

4. COMPARISON WITH REPORTED RESULTS

The results of the natural frequencies with the use of Rayleigh-Ritz solution, given by Siu and Bert[7] for a free-free homogeneous conical shell are computed with the present analysis for the data:

$$\alpha = 14.20^\circ \quad L = 17.3 \text{ in.}, \; t = 0.005 \text{ in.}, \quad R_0 = 2.72 \text{ in}$$

$$E_x = E_\phi = 0.295\times 10^8 \text{ lb/in.}^2$$

$$\rho = 0.773\times 10^{-3} \text{ lb sec}^2\text{/in}$$

$$G_{x\phi} = G_{\phi z} = G_{xz} = 0.113\times 10^8 \text{ lb/in.}^2$$

The minimum natural frequency is found to be 17.86315 Hz, whereas the reported one is around 20 Hz.

The natural frequencies of a sandwich conical shell with free edges consisting of elastic layers have been determined with the present analysis and have been found to be in good agreement (Table 1) with the results reported by Wilkins *et al*[6] for the data :

$$\alpha = 5.07^\circ, \qquad L = 72.5 \text{ in.}$$

$$R_{01} = 22.290 \text{ in.}, \quad R_{02} = 22.450 \text{ in.}, \quad R_{03} = 22.609 \text{ in}$$

$$t_1 = 0.021 \text{ in.}, \quad t_2 = 0.3 \text{ in.}, \quad t_3 = 0.021 \text{ in.}$$

$$E_{x1} = E_{x3} = E_{\phi 1} = E_{\phi 3} = 3.64\times 10^6 \text{ lb/in}^2$$

$$G_{zx2} = 3.2\times 10^4 \text{ lb/in.}^2,$$

$$G_{\phi z2} = 1.83\times 10^4 \text{ lb/in.}^2,$$

$$G_{zx1} = G_{zx3} = G_{\phi z1} = G_{\phi z3} = G_{x\phi 1} = G_{x\phi 3} = 1.0\times 10^6 \text{ lb/in.}^2$$

$$\nu_{x\phi 1} = \nu_{x\phi 3} = \nu_{\phi x1} = \nu_{\phi x3} = 0.2$$

$$\rho_1 = \rho_3 = 0.265\times 10^{-3} \text{ lb-sec}^2\text{/in.}^4,$$

$$\rho_2 = 0.3368\times 10^{-5} \text{ lb.sec}^2\text{/in.}^4$$

where suffix 1 is for inner face layer, 2 for core and 3 for outer face layer of the sandwich conical shell.

The resonance frequencies and the associated system loss factors for three- and five-layered cylindrical shells (two elastic face layers sandwiching a viscoelastic core) have been computed with the present analysis by taking zero apex angle and these have been found to be in close agreement with results reported by Alam and Asnani[10].

Table 1. Comparison with analytical frequencies (Hz) for free-free sandwich conical shells reported by Wilkins, *et al.*[6]

Lowest	Second Lowest
0.207	736.8
(0.26)	(724.7)
There are torsion modes	
31.96	407.8
(32.4)	(408)

Values in (...) are from table reported in Ref. 6

5. MULTILAYERED CONICAL SHELL

The multilayered conical shell considered in the present analysis consists of alternate elastic and viscoelastic layers such that the face layers are always elastic. All of the elastic layers are assumed to be of the same thickness and of specially orthotropic material, as are all of the viscoelastic layers. The ratio of thickness of the viscoelastic layer to that of the elastic layer is denoted by thickness ratio parameter V. Total thickness parameter (T/R_1) denotes the ratio of total thickness of the shell to the radius of the first layer of the shell. The ratio of mass density (ρ) of viscoelastic to that of elastic material is taken to be 0.5. The loss factor η of the viscoelastic core in shear as well as in extension is taken to be 0.5. Length parameter (R_1/L) is defined as the ratio of inner radius of the first layer at the small end of the shell to the slant length of the shell and is taken to be

0.1. Poisson's ratio (ν) of the elastic material is taken to be 0.3 and the ratio of Poisson's ratio of the viscoelastic material to that of the elastic material is taken to be 1.33. Cone apex angle, α, is taken to be 5.07°. The shear parameter, δ, defined as the ratio of the inphase component of the shear modulus ($G_{x\phi}$) of the viscoelastic cores to the Young's modulus (E) of the elastic layers is taken to be 10^{-4}.

6. RESULTS & DISCUSSION

Variation of the resonance frequencies ω and associated system loss factors η_s with thickness ratio parameter V and total thickness parameter (T/R_1) have been discussed for axisymmetric vibrations of three-, five- and seven-layered conical shells with alternate elastic and viscoelastic layers.

Results have been presented for the shell with data for the face elastic layer as follows:

Young's modulus : $E_x = E_\phi = 3.64 \times 10^6$ lb/in^2 = 0.252874×10^{11} N/m^2

Shear modulus : $G_{x\phi} = G_{\phi z} = G_{xz} = 1.399994 \times 10^6$ lb/in^2 = 0.972591×10^{10} N/m^2

Density: $\rho = 0.265 \times 10^{-4}$ lb-sec^2/in^4 = $0.28535327 \times 10^{-4}$ Ns^2/m^4

Thickness : t = 0.03 in. = 0.762×10^{-5}m

Radius : R_1 = 7.2 in. = 0.18288 m

In the analysis, the designation m_1 denotes the lowest resonance frequency and its corresponding system loss factor; m_2 denotes the second lowest frequency, etc. For a three-layered sandwich shell, families of modes have been shown for m_1 and m_2, whereas for the sake of clarity in figures for multilayered shells the curves have been drawn for the first lowest frequencies only and their corresponding system loss factors for families of modes.

Variation of resonance frequency ω and associated system loss factor η_s with V for axisymmetric vibrations of three-, five- and seven-layered conical shells with free edges has been shown in Figs. 2-4 for (T/R_1) = 0.5.

For multilayered shell ω for radial mode increases for lower values of V, reaches a maximum (900 Hz) at V = 5 and then decreases with further increasing values of V as the stiffness of the shell reduces with further increase in V. For this mode η_s increases with V. Similar trend is observed for resonance frequency ω and associated system loss factor η_s for meridional and torsional modes for multilayered shells. There is only a marginal increase in η_s for these modes with more layers. Thus for getting high values of η_s for radial,

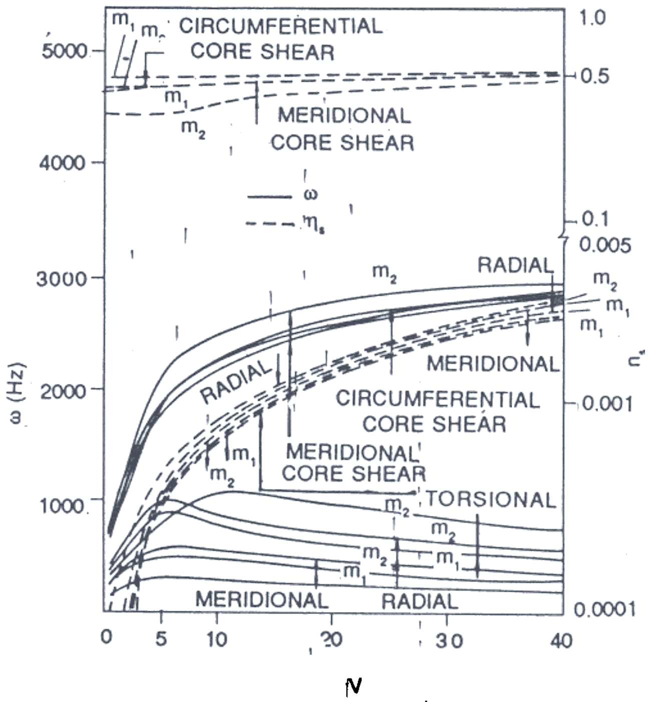

Figure 2. Variation of ω and η_s with V for axisymmetric vibrations of a free-free three-layered conical shell.

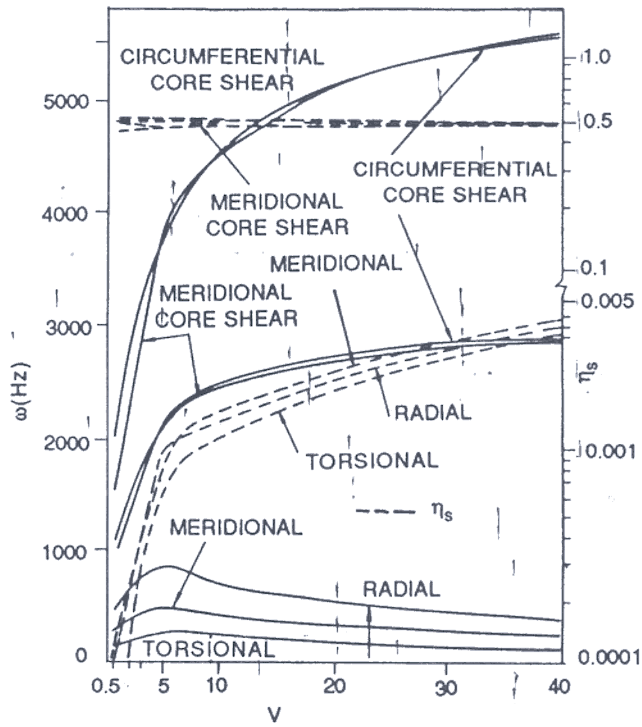

Figure 3. Variation of ω and η_s with V for axisymmetric vibrations of a free-free five-layered conical shell.

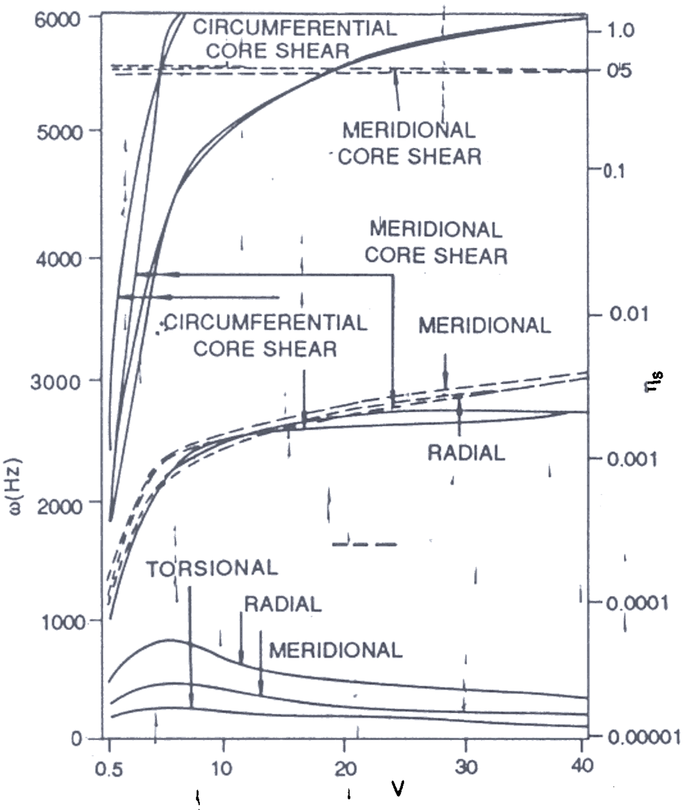

Figure 4. Variation of ω and η_s with V for axisymmetric vibrations of a free-free seven-layered conical shell.

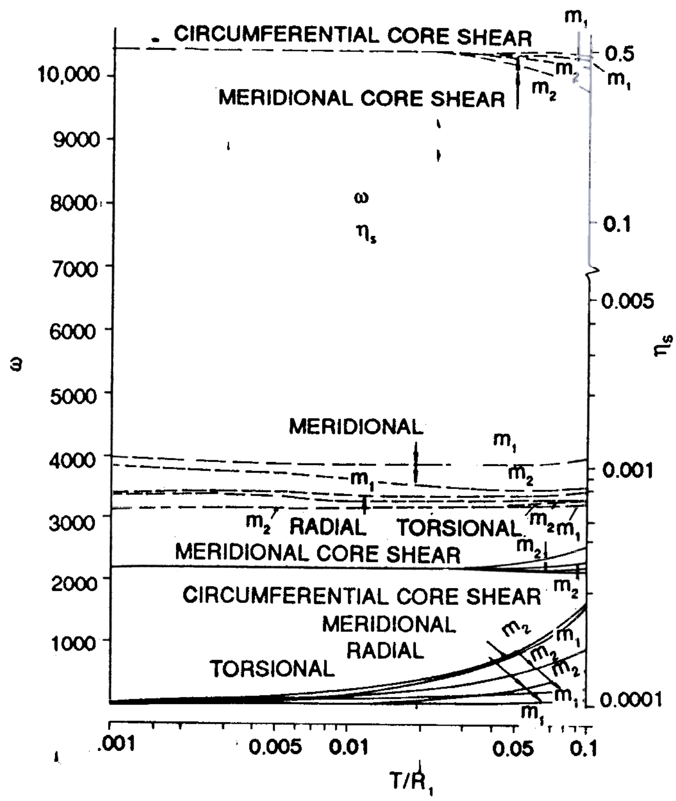

Figure 5. Variation of ω and η_s with (T/R_1) for axisymmetric vibrations of a free-free three-layered conical shell.

meridional and torsional modes, one should go for higher of V.

For meridional and circumferential core shear modes ω increases with V. For these modes η_s increases marginally with V and reaches a maximum, equal to the material loss factor of the core. η_s is more for these thickness core shear modes with more number of layers in the multilayered shell for a particular value of V. Thus it is observed that uniformly high values of η_s for all families of modes of vibrations are obtained if $V > 5$.

Figures 5-7 show the variation of resonance frequency ω and associated system loss factor η_s with total thickness parameter T/R_1 for axisymmetric vibrations of three-, five- and seven-layered conical shells with free edges for $V = 10$.

For three-layered sandwich shell ω for radial mode increases with T/R_1. A considerable increase in ω for this mode is observed in the high range of T/R_1. Similar trend is noticed for ω for this mode for five-and seven-layered conical shells with T/R_1. The η_s for radial

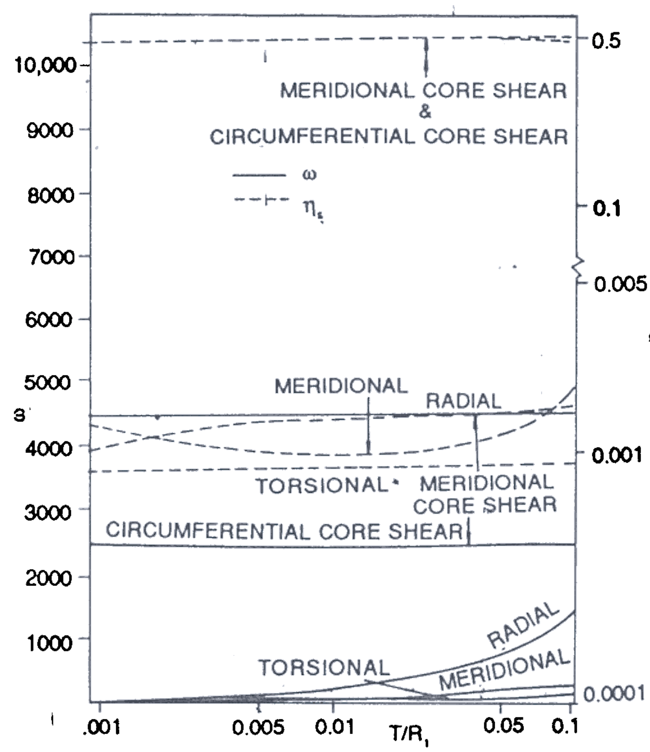

Figure 6. Variation of ω and η_s with (T/R_1) for axisymmetric vibrations of a free-free five-layered conical shell.

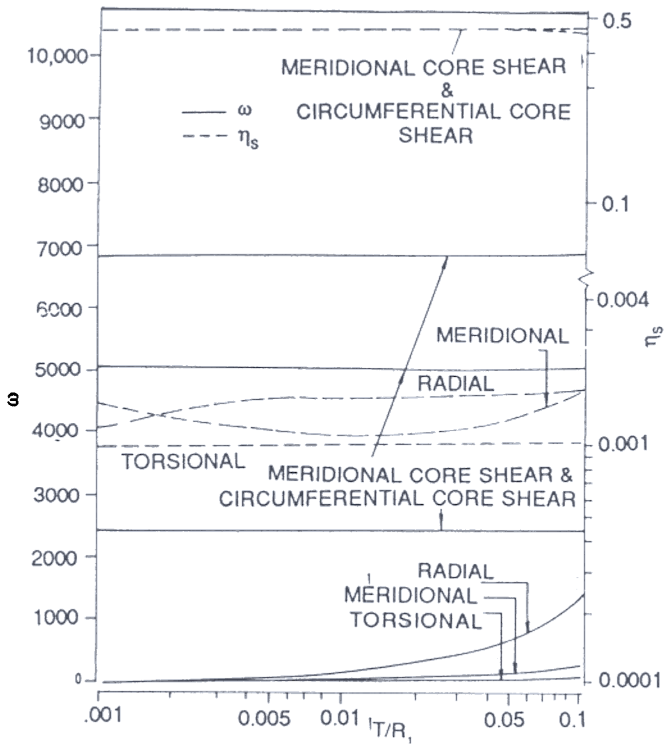

Figure 7. Variation of ω and η_s with (T/R_1) for axi-symmetric vibrations of a free-free seven-layered conical shell.

mode for three-layered conical shell remains nearly constant for the lower values of T/R_1, that is, for thin shells and decreases marginally in the higher range of T/R_1 whereas η_s for this mode for five and seven-layered shells first increases up to $T/R_1 = 0.005$, then remains almost constant upto $T/R_1 = 0.01$, and again increases marginally for the further increase of T/R_1. Also η_s increases with the number of layers in the shell.

For meridional and torsional modes for multilayered shells ω increases with the increase of (T/R_1). There is no appreciable change in η_s for meridional mode for three-layered sandwich conical shell up to $T/R_1 = 0.01$ an increase in η_s for this mode for three-layered shell is observed with the higher values of (T/R_1). For this mode for five-layered shell η_s decreases at the lower values of T/R_1, reaches a minimum at $(T/R_1) = 0.007$ and then increases with further increasing values of (T/R_1). For meridional mode for seven-layered shell η_s follows the same trend as that for five-layered shell but the rate of decrease and increase in the lower and higher ranges of (T/R_1) respectively is comparatively reduced. For meridional mode η_s is small for a three-layered shell which increases substantially for a five-layered shell. There is a decrease in η_s for this mode in the higher ranges of (T/R_1) when the number of layers in the shell is increased from five to seven. There is no substantial change in η_s for torsional mode with (T/R_1) for free-free multilayered conical shell; η_s for this mode increases with the number of layers.

Resonance frequencies for meridional and circumferential core shear modes for multilayered shell remain almost constant in the chosen range of (T/R_1), that is, ω for these modes is same for thin and thick shells. There is no change in associated system loss factor η_s for core shear modes for multilayered conical shells and it is observed to be equal to the material loss factor of the viscoelastic core layer of the shell.

The vibration and damping analysis of general multilayered conical shells of constant thickness with free edges presented here shows that increase in number of layers increases maximum obtainable system loss factor for most of the modes of vibration with proper selection of geometric parameters—thickness ratio parameter and total thickness parameter. It is observed that there is a considerable increase in system loss factor when the number of layers in the shell is increased from three to five but the increase is only mariginal when the number of layers is further increased to seven. It is noticed that uniformly high values of system loss factor for all families of modes of vibration are obtained for high values of thickness ratio parameter. For a thick shell, more layers are advisable for getting relatively high values of system loss factor for all families of modes.

7. CONCLUSION

The vibration and damping analysis of general multilayered conical shells of constant thickness with free edges presented here shows that increase in number of layers increases maximum obtainable system loss factor for most of the modes of vibration with proper selection of geometric parameters—thickness ratio parameter and total thickness parameter. It is observed that there is a considerable increase in system loss factor when the number of layers in the shell is increased from three-to-five but the increase is only mariginal when the number of layers is further increased to seven. It is noticed that uniformly high values of system loss factor for all families of modes of vibration are obtained for high values of thickness ratio parameter. For a thick shell, more layers are advisable for getting relatively high values of system loss factor for all families of modes.

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APPENDIX

Governing Equations of Motion and Boundary Conditions for Axisymmetric Vibrations of Multilayered Conical Shell

$$\left\{\frac{1}{3}\frac{t_i}{(R_{oi}+x\sin\alpha)}+\frac{1}{12}\frac{t_i^2\cos\alpha}{(R_{oi}+x\sin\alpha)^2}+\frac{1}{48}\frac{t_i^3\cos^2\alpha}{(R_{oi}+x\sin\alpha)^3}\right\}(Q_{22})_i u_i\sin^2\alpha+\left\{\frac{1}{6}\frac{t_i}{(R_{oi}+x\sin\alpha)^3}+\frac{1}{48}\frac{t_i^3\cos^2\alpha}{(R_{oi}+x\sin\alpha)^3}\right\}$$

$$\times(Q_{22})_i u_{i+1}\sin^2\alpha+\left\{\frac{1}{2}\frac{t_i}{(R_{oi}+x\sin\alpha)}+\frac{1}{12}\frac{t_i^2\cos\alpha}{(R_{oi}+x\sin\alpha)^2}+\frac{1}{24}\frac{t_i^3\cos^2\alpha}{(R_{oi}+x\sin\alpha)^3}\right\}(Q_{22})_i\omega\sin\alpha\cos\alpha$$

$$+(C_{55})_i(R_{oi}+x\sin\alpha)\left\{\frac{1}{t_i}u_i-\frac{1}{t_i}u_{i+1}-w_{,x}\right\}-\frac{1}{3}t_i(Q_{11})_i\{u_{i,x}\sin\alpha+(R_{oi}+x\sin\alpha)u_{i+1,xx}\}-\frac{1}{2}t_i(Q_{12})_i w_{,x}\cos\alpha$$

$$+\left\{\frac{1}{3}\frac{t_{i-1}}{(R_{oi-1}+x\sin\alpha)}-\frac{1}{12}\frac{t_{i-1}^2\cos\alpha}{(R_{oi-1}+x\sin\alpha)^2}+\frac{1}{48}\frac{t_{i-1}^3\cos^2\alpha}{(R_{oi-1}+x\sin\alpha)^3}\right\}+\left\{\frac{1}{6}\frac{t_{i-1}}{(R_{oi-1}+x\sin\alpha)}+\frac{1}{48}\frac{t_{i-1}^3\cos^2\alpha}{(R_{oi-1}+x\sin\alpha)^3}\right\}(Q_{22})_{i-1}u_{i-1}\sin^2\alpha$$

$$\left\{\frac{1}{2}\frac{t_{i-1}}{(R_{oi-1}+x\sin\alpha)}-\frac{1}{12}\frac{t_{i-1}^2\cos\alpha}{(R_{oi-1}+x\sin\alpha)^2}+\frac{1}{24}\frac{t_{i-1}^3\cos^2\alpha}{(R_{oi-1}+x\sin\alpha)^3}\right\}(Q_{22})_{i-1}w\sin\alpha\cos\alpha$$

$$+(C_{55})_{i-1}(R_{oi-1}+x\sin\alpha)\left\{\frac{\cdot}{t_{i-1}}u_i-\frac{\cdot}{t_{i-1}}u_{i-1}+w_{,}\right\}-\frac{1}{3}t_{i-1}(Q_{11})_{i-1}$$

$$\times\left\{u_{i,x}\sin\alpha+\left(R_{oi-1}+x\sin\alpha+\frac{1}{4}t_{i-1}\cos\alpha\right)u_{i,xx}\right\}-\frac{1}{6}t_{i-1}(Q_{11})_{i-1}$$

$$\times\{u_{i-1,x}\sin\alpha+(R_{oi-1}+x\sin\alpha)u_{i-1,xx}\}-\frac{1}{2}t_{i-1}(Q_{12})_{i-1}w_{,x}\cos\alpha+\frac{1}{3}\rho_i t_i(R_{oi}+x\sin\alpha)\left\{\ddot{u}_i+\frac{1}{2}\ddot{u}_{i+1}\right\}$$

$$+\frac{1}{3}\rho_{i-1}t_{i-1}(R_{oi-1}+x\sin\alpha)\left\{\ddot{u}_i+\frac{1}{2}\ddot{u}_{i-1}\right\}-\frac{1}{12}\{\rho_i t_i^2-\rho_{i-1}t_{i-1}^2\}\ddot{u}_i=0$$

{For $i=1,2,3,\ldots\ldots\ldots\ldots(N+1)$, these are $(N+1)$ equations.} (12)

$$\left\{\frac{1}{3}\frac{t_i}{(R_{oi}+x\sin\alpha)}+\frac{1}{12}\frac{t_i^2\cos\alpha}{(R_{oi}+x\sin\alpha)^2}+\frac{1}{48}\frac{t_i^3\cos^2\alpha}{(R_{oi}+x\sin\alpha)^3}\right\}v_i\{(Q_{66})_i\sin^2\alpha-(C_{44})_i\cos^2\alpha\}$$

$$+\left\{\frac{1}{6}\frac{t_i}{(R_{oi}+x\sin\alpha)}+\frac{1}{48}\frac{t_i^3\cos^2\alpha}{(R_{oi}+x\sin\alpha)^3}\right\}v_{i+1}\{(Q_{66})_i\sin^2\alpha+(C_{44})_i\cos^2\alpha\}$$

$$-\frac{1}{3}t_i(Q_{66})_i\left(R_{oi}+x\sin\alpha-\frac{1}{4}t_i\cos\alpha\right)v_{i,xx}-\frac{1}{6}t_i(Q_{66})_i(R_{oi}+x\sin\alpha)\;v_{i+1,xx}$$

$$+\left\{\frac{1}{3}\frac{t_{i-1}}{(R_{oi-1}+x\sin\alpha)}-\frac{1}{12}\frac{t_{i-1}^2\cos\alpha}{(R_{oi-1}+x\sin\alpha)^2}+\frac{1}{48}\frac{t_{i-1}^3\cos^2\alpha}{(R_{oi-1}+x\sin\alpha)^3}\right\}$$

$$\times v_i\{(Q_{66})_{i-1}\sin^2\alpha+(C_{44})_{i-1}\cos^2\alpha\}+\left\{\frac{1}{6}\frac{t_{i-1}}{(R_{oi-1}+x\sin\alpha)}+\frac{1}{48}\frac{t_{i-1}^3\cos^2\alpha}{(R_{oi-1}+x\sin\alpha)^3}\right\}v_{i-1}$$

$$\times\{(Q_{66})_{i-1}\sin^2\alpha+(C_{44})_{i-1}\cos^2\alpha\}+\frac{1}{t_{i-1}}(C_{44})_{i-1}(R_{oi-1}+x\sin\alpha)(v_i-v_{i-1})$$

$$-(C_{44})_{i-1}v_i\cos\alpha-\frac{1}{3}t_{i-1}(Q_{66})_{i-1}\left\{R_{oi-1}+x\sin\alpha+\frac{1}{4}t_{i-1}\cos\alpha\right\}v_{i,xx}$$

$$-\frac{1}{6}t_{i-1}(Q_{66})_{i-1}\{R_{0i-1}+x\sin\alpha\}v_{i-1,xx}-\frac{1}{3}t_{i-1}(Q_{66})_{i-1}\sin\alpha\left(v_{i,x}+\frac{1}{2}v_{i-1,x}\right)$$

$$+\frac{1}{3}\rho_i t_i(R_{0i}+x\sin\alpha)\left(\ddot{v}_i+\frac{1}{2}\ddot{v}_{i+1}\right)+\frac{1}{3}\rho_{i-1}t_{i-1}(R_{0i-1}+x\sin\alpha)\left(\ddot{v}_i+\frac{1}{2}\ddot{v}_{i-1}\right)-\frac{1}{12}\left(\rho_i t_i^2-\rho_{i-1}t_{i-1}^2\right)\ddot{v}_i\cos\alpha=0$$

$\{$For $i=1,2,3,$ $(N+1)$, these are $(N+1)$ equations$\}$

and

$$\sum_{i=1}^{N}\left[\frac{1}{2}t_i(Q_{12})_i\cos\alpha\{u_{i,x}+u_{i+1,x}\}+\left\{\frac{t_i}{(R_{0i}+x\sin\alpha)}+\frac{1}{12}\frac{t_i^3\cos^2\alpha}{(R_{0i}+x\sin\alpha)^3}\right\}(Q_{22})_i w\cos^2\alpha\right.$$

$$+\left\{\frac{1}{2}\frac{t_i}{(R_{0i}+x\sin\alpha)}-\frac{1}{12}\frac{t_i^2\cos\alpha}{(R_{0i}+x\sin\alpha)^2}+\frac{1}{24}\frac{t_i^3\cos^2\alpha}{(R_{0i}+x\sin\alpha)^3}\right\}(Q_{22})_i u_{i+1}\sin\alpha\cos\alpha$$

$$+\left\{\frac{1}{2}\frac{t_i}{(R_{0i}+x\sin\alpha)}+\frac{1}{12}\frac{t_i^2\cos\alpha}{(R_{0i}+x\sin\alpha)^2}+\frac{1}{24}\frac{t_i^3\cos^2\alpha}{(R_{0i}+x\sin\alpha)^3}\right\}(Q_{22})_i u_i\sin\alpha\cos\alpha$$

$$-(C_{55})_i t_i\{w_{,x}\sin\alpha+(R_{0i}+x\sin\alpha)w_{,xx}\}-(C_{55})_i$$

$$\times\{u_{i+1}\sin\alpha+(R_{0i}+x\sin\alpha)u_{i+1,x}\}+(C_{55})_i\{u_i\sin\alpha+(R_{0i}+x\sin\alpha)u_{i,x}\}$$

$$+\rho_i t_i(R_{0i}+x\sin\alpha)\ddot{w}+f(x,\phi)g(t)=0$$

The boundary conditions obtained at $x=0$ and $x=L$ are
either $u_i=0$ or

$$\frac{1}{3}t_i(Q_{11})_i\left\{R_{0i}+x\sin\alpha-\frac{1}{4}t_i\cos\alpha\right\}u_{i,x}+\frac{1}{6}t_i(Q_{11})_i(R_{0i}+x\sin\alpha)u_{i+1,x}+\frac{1}{3}t_i(Q_{12})_i$$

$$\times\left\{u_i\sin\alpha+\frac{1}{2}u_{i+1}\sin\alpha+\frac{3}{2}w\cos\alpha\right\}+\frac{1}{3}t_{i-1}(Q_{11})_{i-1}\left\{R_{0i-1}+x\sin\alpha+\frac{1}{4}t_{i-1}\cos\alpha\right\}u_{i,x}+\frac{1}{6}t_{i-1}$$

$$\times(Q_{11})_{i-1}(R_{0i-1}+x\sin\alpha)u_{i-1,x}+\frac{1}{3}t_{i-1}(Q_{12})_{i-1}\left\{u_i\sin\alpha+\frac{1}{2}u_{i+1}\sin\alpha+\frac{3}{2}w\cos\alpha\right\}=0$$

$\{$For $i=1,2,3,\ldots\ldots\ldots\ldots\ldots,(N+1)$, these are $(N+1)$ equations.$\}$
either $v_i=0$
or

$$\frac{1}{3}t_i(Q_{66})_i\left\{R_{oi}+x\sin\alpha-\frac{1}{4}t_i\cos\alpha\right\}v_{i,x}+\frac{1}{6}t_i(Q_{66})_i\times(R_{oi}+x\sin\alpha)_i v_{i+1,x}-\frac{1}{3}t_i(Q_{66})\left\{v_i\sin\alpha+\frac{1}{2}v_{i+1}\sin\alpha\right\}$$

$$+\frac{1}{3}t_{i-1}(Q_{66})_{i-1}\left\{R_{oi-1}+x\sin\alpha+\frac{1}{4}t_{i-1}\cos\alpha\right\}v_{i,x}+\frac{1}{6}t_{i-1}(Q_{66})_{i-1}$$

$$\times(R_{oi-1}+x\sin\alpha)v_{i-1,x}-\frac{1}{3}t_{i-1}(Q_{66})_{i-1}\left\{v_i\sin\alpha+\frac{1}{2}v_{i-1}\sin\alpha\right\}=0$$

$\{$For $i=1,2,3,\ldots.$ $.,(N+1)$, these are $(N+1)$ equations.$\}$

and either $w=0$ or

$$\sum_{i=1}^{N}\left[(C_{55})_i(R_{oi}+x\sin\alpha)\{t_i w_{,x}+u_{i+1}-u_i\}\right]=0$$