Defence Science Journal, Vol 48, No 2, April 1998, pp. 133-140

Predictive Control-Based Optimal Nonlinear Reentry Guidance Law

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ABSTRACT

This paper discusses a new nominal riding reentry guidance law design based on a nonlinear optimal predictive control approach. In this method, the error between the actual trajectory and the nominal trajectory is predicted, and a quadratic cost function of these predicted errors is minimised, resulting in an optimal feedback guidance law design. The guidance law thus obtained does not require linearisation of the equations of motion. A nominal trajectory is selected which satisfies the vehicle constraints and mission objectives. This nominal data is used with actual data to evaluate the guidance law. Numerical simulations have been carried out for a planar trajectory for a variety of initial condition errors and also for off nominal conditions and the results are presented.

NOMENCLATURE

A	Reference area
C_L, C_D	Lift and drag force coefficients
D	Drag force
e	Error vector
g	Gravitational acceleration
L	Lift force
L_i	Lower limit on control
m	Mass
Q_1, Q_2	Weighting matrices on state errors
R	Weighting matrix on control
t	Time
u	Control vectors
U_i	Upper limit on control
V	Velocity
x	State vector
X	Downrange
Z	Altitude
γ	Flight path angle
ρ	Atmospheric air density
δ	Small time increment

1. INTRODUCTION

Extensive research has been carried out in the area of reentry guidance and control, and a variety of schemes have appeared in literature[1]. The objective of the reentry guidance is to guide a

Received 29 August 1996, revised 17 July 1997

reentry vehicle from its point of reentry into the atmosphere to its destination on ground, without violating the physical constraints of the vehicle. The different guidance schemes have been mainly classified into three categories as:

(a) Explicit guidance schemes

(b) Guidance using closed-form solutions

(c) Implicit guidance schemes.

Implicit guidance schemes use the nominal concept, i.e. the guidance law is designed to control a reentry vehicle on the pre-selected nominal or reference trajectory, which usually is generated through an optimisation process. The guidance law should guide the vehicle back to the nominal trajectory whenever the vehicle deviates from it. Linear optimal control techniques, such as linear quadratic regulator (LQR)[2], have been used to design such optimal guidance laws, in which a control command is obtained by minimising a quadratic performance index. Such linear techniques necessitate linearisation of nonlinear equations of motion and thus affect the performance of guidance law whenever the assumptions of linearisation are violated.

In this paper, a predictive control-based optimal, nonlinear reentry guidance law is proposed. In this approach, the guidance law minimises the predicted difference between the actual trajectory and the nominal trajectory. The performance index is quadratic and is based on the predicted errors. Since the equations of motion are not linearised, the drawbacks of linear control techniques overcome.

Initially, a reference trajectory, which satisfies the mission objectives and vehicle constraints is chosen. The nominal data is stored as a function of X variable instead of t, since it has been shown that choice of t as an independent variable for formulation of the guidance law offers poor performance[3]. The nonlinear point mass equations of motion of the vehicle are then simulated using the predictive control-based guidance law. The simulations were carried out for various perturbed initial entry conditions for planar motion. Simulations were also carried out for off-nominal conditio[...] results are presented.

2. PREDICTIVE CONTROL

Predictive control theory resu[...] feedback law which computes a control [...] by minimising the predicted errors be[...] actual and the nominal trajectory. A bri[...] of this approach[4] is presented here for th[...] completeness.

Consider a nonlinear system descr[...]

$$\dot{x}_1 = f_1\,[x\,(t)]$$

$$\dot{x}_2 = f_2\,[x\,(t)] + g_2\,[(x\,(t),\,u\,(t)]$$

where the state vector $x = [x_1^T x_2^T]^T \in R^n$,[...] and $x_2 \in R^{n2}$. The functions $f_1 : R^n \rightarrow R^{n1}$, [...] R^{n2} and $g_2 : R^n \times R^m \rightarrow R^{n2}$ are contin[...] differentiable nonlinear functions and the [...] vector $u \in U = \{u \in R^m \mid L_i \le u_i \le U_i\}$ w[...] lower and upper L_i and U_i specified.

Suppose the desired response of the [...] is given by a reference trajectory $x^*(t)$ fo[...] reference control $u^*(t)$ and initial cond[...] From Eqns (1) and (2), it can be seen [...] $\dot{x}_1(t)$ and $\dot{x}_2(t)$ depend on $u\,(t)$ explicitly. [...] the choice of $u\,(t)$ at any arbitrary t[...] influences the response of the system after [...] time $(t+\delta)$, where δ is a small time increme[...] the state vectors $x_1\,(t+\delta)$ and $x_2\,(t+\delta$[...] expanded in second-and first-order Tay[...] series, respectively, one obtains:

$$x_1(t+\delta) \approx x_1(t) + \delta f_1[x(t)] + \left(\frac{\delta^2}{2}\right) \{F_{11}[x(t)]\, f_1[x(t)] + F_{12}\,[x(t)]\, f_2[x(t)] + F_{12}[x(t)]\;\, g_2[x(t), u\,(t)]\}$$

$$x_2(t+\delta) \approx x_2(t) + \delta\,\{f_2[x(t)] + g_2[x(t), u(t)]\}$$

where

$$F_{11}(x) = \frac{\partial f_1}{\partial x_1} \text{ and } F_{12}(x) = \frac{\partial f_1}{\partial x_2}$$

Similarly partitioning the reference trajectory $x^*(t)$ as $x^*(t) = [x_1^{*T}(t)\, x_2^{*T}(t)]$ and expanding $x_1^*(t+\delta)$ and $x_2^*(t+\delta)$ in second - and first-order Taylor's series respectively, one gets:

$$x_1^*(t+\delta) \approx x_1^*(t) + \delta \dot{x}_1^*(t) + \frac{\delta^2}{2}\ddot{x}_1^*(t) \tag{6}$$

$$x_2^*(t+\delta) \approx x_2^*(t) + \delta\, \dot{x}_2^*(t) \tag{7}$$

The tracking errors at time $(t+\delta)$ are

$$e_1(t+\delta) = x_1(t+\delta) - x_1^*(t+\delta) \tag{8}$$

$$e_2(t+\delta) = x_2(t+\delta) - x_2^*(t+\delta) \tag{9}$$

The performance index to be minimised is defined as:

$$\min_{u \in U} J = \min_{u \in U} \frac{1}{2}\Big[e_1^T(t+\delta)\, Q_1 e_1(t+\delta) + e_2^T(t+\delta) Q_2\, e_2(t+\delta) + [u(t) - u^*(t)]^T R\, [u(t) - u^*(t)]\Big] \tag{10}$$

where

Q_1, Q_2 are weighting matrices on the predicted tracking errors $e_1(t+\delta)$, and $e_2(t+\delta)$, while R is the weighting on the deviation of the present control input from its nominal value. The weightings Q_1 and Q_2 are to be positive semi-definite, while R is positive definite matrix. All weightings are of appropriate dimensions. However, matrix R can be set to zero, if $g_2(x, u)$ is linear in u, i.e. when Eqn (2) is of the form:

$$\dot{x}_2 = f_2(x) + B_2(x)\, u \tag{11}$$

Minimising the performance index by setting $\partial J / \partial u = 0$ or the case when system equations are linear in control u and assuming no-control saturation, the control command can be obtained in closed form as:

$$u(t) = -P(x)\left\{\frac{1}{2\delta^2}(F_{12}B_2)^T Q_1 [e_1 + \delta \dot{e}_1 + \frac{\delta^2}{2}(F_{11}f_1 + F_{12}f_2 - \ddot{x}_1^*)] + \frac{1}{\delta^3} B_2^T Q_2 [e_2 + \delta(f_2 - \dot{x}_2^*)]\right\} \tag{12}$$

where

$$P(x) = \left[\frac{1}{4}(F_{12}B_2)^T Q_1 F_{12} B_2 + \frac{1}{\delta^2} B_2^T Q_2 B_2\right]^{-1} \tag{13}$$

and

$$e_i(t) = x_i(t) - x_i^*(t); \quad i=1,2$$
$$\dot{e}_i(t) = \dot{x}_i(t) - \dot{x}_i^*(t); \quad i=1,2 \tag{14}$$

The control law given by Eqn (12) is a function of the nominal and actual state variables and is evaluated on the actual trajectory. To avoid possible control saturation, limits can be placed on the control command, such as

$$L_i \le u_i(t) \le U_i \tag{15}$$

where

L_i and U_i are the lower and upper bounds on the control.

3. GUIDANCE LAW

The coordinate frame employed for the equations of motion is centered at a point where the nominal downrange is zero with the X-axis pointing X and Z-axis along the local vertical and positive upward. The wind-axis frame is used for derivation of the equations of motion for the point mass vehicle moving in X-Z plane. The equations thus obtained are as follows:

$$\frac{dX}{dt} = V\cos\gamma$$
$$\frac{dZ}{dt} = V\sin\gamma$$
$$\frac{dV}{dt} = -\frac{D}{m} - g\sin\gamma$$
$$\frac{d\gamma}{dt} = \frac{L}{mV} - \frac{g\cos\gamma}{V} \tag{16}$$

where

$$D=\frac{1}{2}\rho V^2 C_D A$$

$$L=\frac{1}{2}\rho V^2 C_L A$$

The choice of an independent variable to formulate the guidance law is important. Time, which appears to be as a natural choice for an independent variable, offers poor performance. The other variables which can be used as independent variables [1-3] are the X, Z and the V or components of the V. Which variable should be chosen as an independent variable depends upon the nature of the nominal trajectory. If the nominal path consists of vertical descent, then Z may be a better choice as compared to X. In this work X has been chosen[3] as an independent variable as the reference trajectory chosen does not have perfect vertical descent. However, it is obvious that the choice of any particular variable as an independent variable, for formulation of the guidance law does not change the basic dynamics of the system.

Since X is used as an independent variable for guidance command, it is necessary to transform the equations of motion Eqn (16) with respect to the independent variable. The resulting equations with X as the independent variable are:

$$\frac{dZ}{dX}=\tan\gamma$$

$$\frac{dV}{dX}=-\frac{D}{mV\cos\gamma}-\frac{g}{V}\tan\gamma$$

$$\frac{d\gamma}{dX}=-\frac{g}{V^2}+\frac{L}{mV^2\cos\gamma} \tag{17}$$

Taking lateral acceleration (L/m) as the control input, Eqn (17) can be expressed as:

$$\frac{dZ}{dX}=\tan\gamma$$

$$\frac{dV}{dX}=-\frac{D}{mV\cos\gamma}-\frac{g}{V}\tan\gamma$$

$$\frac{d\gamma}{dX}=-\frac{g}{V^2}+\frac{1}{V^2\cos\gamma}u$$

where

u is the control input i.e. lateral acceleratio (18) is now in the form to which control la (12) can be applied. Consider the state vec $[Z\ V\ \gamma]^T$. Partitioning the state vector $=[x_1^T x_2^T]^T$, where $x_1=[Z\ V]^T$ and $x_2=[\gamma$ comparing Eqn (18) with Eqns (1) and (11), w

$$f_1=\begin{bmatrix}\tan\gamma\\ -\dfrac{D}{mV\cos\gamma}-\dfrac{g}{V}\tan\gamma\end{bmatrix}$$

$$f_2=\left[-\frac{g}{V^2}\right];\quad B_2=\left[\frac{1}{V^2\cos\gamma}\right]$$

$$e_i(t)=x_i(t)-x_i^*(t);\quad i=1,2$$

$$\dot{e}_i(t)=\dot{x}_i(t)-\dot{x}_i^*(t);\quad i=1,2$$

where the 'dot' represents differentiation w.r.t independent variable X. The matrices F_{11} and can be easily computed from Eqn (5). quantity $\dot{x}_1^*$ in the control law is obtained differentiating $f_1(x)$ w.r.t. X and is computed w generating the nominal trajectory. All quanti required in the control law are known, except weighting matrices Q_1 and Q_2 and the paramete which are to be selected by the designer. T weighting matrix R is set to zero as control appe linearly in the system equations and hence ne not be considered. The parameter δ is now meters as the equations have been transformed fro t as an independent variable to X as the independe variable. The control is evaluated at discrete poin in the actual trajectory by using the stor nominal data and the actual data, and is ke constant till a new value of the control is compute at the next point. To account for possible contro uration, upper and lower limits are fixed on the ntrol. It is important to note that the control mmand does not depend upon the nominal ntrol history.

SIMULATIONS & RESULTS

To assess the performance of the predictive ntrol-based guidance law, it is necessary to nerate a reference trajectory for the reentry hicle. The reference trajectory is usually btained by employing certain optimisation chniques incorporating the physical constraints f the vehicle, terminal conditions constraints to e satisfied, while meeting the objectives of the ission. However, to assess the performance of the uidance law, it is not a must that the trajectory be ptimal as long as it is a feasible and realistic olution that satisfies the equations of motion.

In this paper, a reference trajectory was generated on similar lines. The vehicle data needed for this purpose is taken from Ref. (5). The rajectory was generated by giving an open loop control and simulating the equations of motion, Eqn (16), by a fourth-order Runge-Kutta integration routine. The variables required for evaluating the guidance law were stored as a function of X. Since this work deals with a planer trajectory, only lateral acceleration in X-Z plane was needed as input for the equations of motion. International Standard Atmosphere was used for obtaining the density as a function of altitude.

After generating the nominal trajectory, it was necessary to choose the parameter δ and weighting matrices Q_1 and Q_2. It can be noticed that the term $1/\delta$ in control law acts as a controller gain and it has been shown that δ need not to a constant[6]. In other words, δ can be tuned suitably to achieve better performance. These quantities are usually obtained by carring out extensive simulations due to the lack of any standard methodology for their selection. In this work, certain bounds on the reentry condition errors are assumed and these quantities are obtained through simulations, such that the terminal errors for the assumed bound on the entry conditions are within specified limits.

By following this approach, δ was tuned and reduced from 5000 m at start to 100 m at the end with a linear decrement. The weightings in the control law were chosen as:

$$Q_1 = diag\ [1,\ 1];\ Q_2 = [\delta^2] \quad (20)$$

After selecting the controller parameter δ and weightings, the guidance law was simulated for various perturbed initial conditions. The initial conditions, Z (0) and V (0) and γ (0), were perturbed from their nominal values, and equations of motion were simulated by incorporating the guidance law. The results for various perturbed

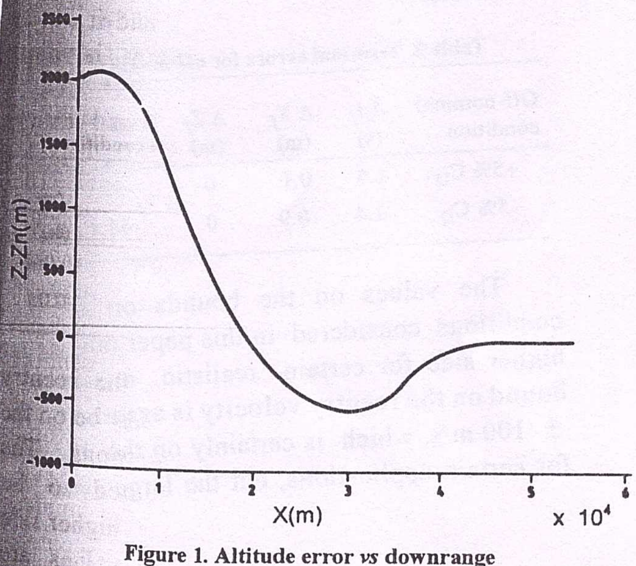

Figure 1. Altitude error *vs* downrange

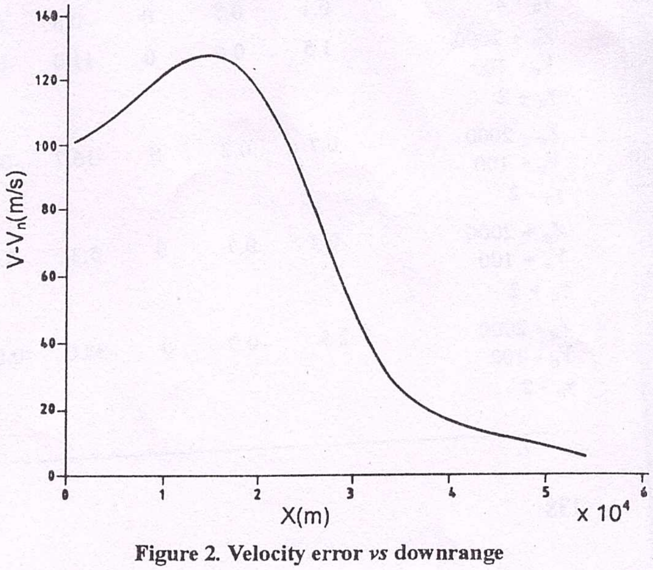

Figure 2. Velocity error *vs* downrange

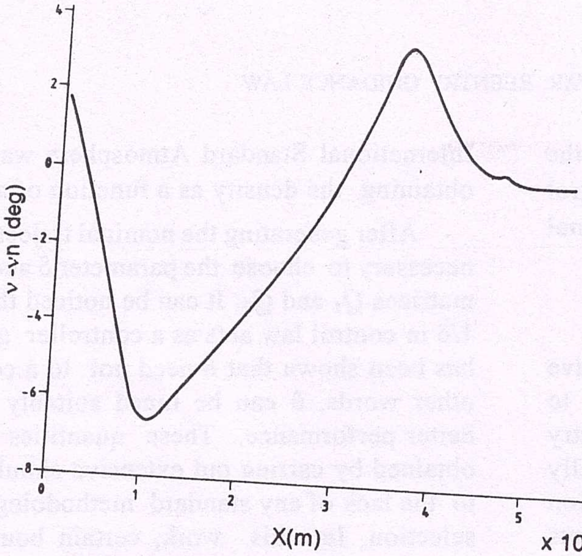

Figure 3. Flight path angle error *vs* downrange

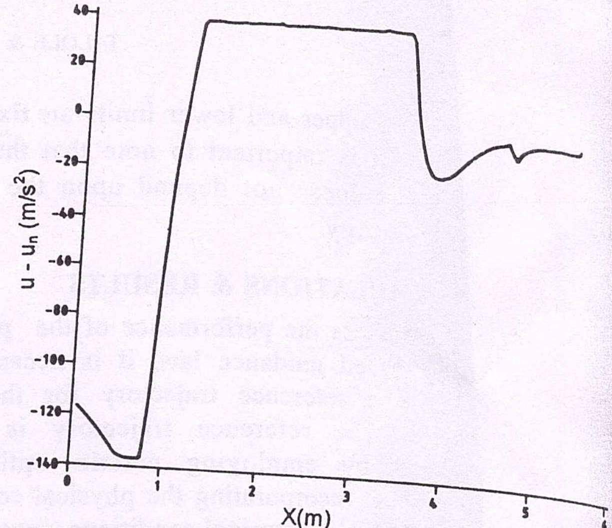

Figure 4. Deviation of actual control from nominal *vs* downrange.

entry conditions are given in Table 1. It can be noticed that for various perturbed initial conditions, the terminal errors are very small and satisfactory. In Table 1, readings (1) to (6) are for perturbations in individual entry variable while readings (7) to (10) are for simultaneous perturbations in all the entry variables. Figs 1-3 show the state errors (i.e. $Z - Z_n$, $V - V_n$ and $\gamma - \gamma_n$) as a function of X for reading (9) of Table 1 and it can be observed that the state errors reduce smoothly almost to zero as the destination is reached. The deviation of the actual control from its nominal value ($u - u_n$) is shown in Fig. 4. In the simulations, lower and upper limits were placed on the control to avoid control saturations.

Table 1. Terminal errors for initial condition perturbations

Initial condition error	Δt_f (s)	ΔX_f (m)	ΔZ_f (m)	ΔV_f (m/s)	$\Delta \gamma_f$ (deg)
$Z_n + 2000$	0.45	-0.2	0	-1.0	-0.04
$Z_n - 2000$	0.6	-0.1	0	11.0	-0.04
$V_n + 100$	-1.0	0.0	0	13.0	-0.04
$V_n - 100$	1.2	0.2	0	-12.0	-0.04
$\gamma_n + 2$	0.0	-0.5	0	0.9	-0.02
$\gamma_n - 2$	0.1	0.7	0	-0.8	-0.03
$Z_n + 2000$, $V_n - 100$, $\gamma_n + 2$	1.5	0.6	0	11.0	-0.05
$Z_n - 2000$, $V_n + 100$, $\gamma_n - 2$	0.7	0.2	0	-16.7	-0.04
$Z_n + 2000$, $V_n + 100$, $\gamma_n + 2$	-0.3	0.7	0	5.3	-0.05
$Z_n - 2000$, $V_n - 100$, $\gamma_n - 2$	2.5	-0.9	0	-33.0	-0.06

Simulations were also carried out for off-nominal conditions for the same set of weighting matrices and tuning parameter δ. The results for off-nominal conditions in drag coefficient are shown in Table 2, and it can be noticed that the terminal accuracy is quite satisfactory.

Table 2. Terminal errors for off-nominal conditions

Off-nominal condition	Δt_f (s)	ΔX_f (m)	ΔZ_f (m)	ΔV_f (m/s)	$\Delta \gamma_f$ (deg)
+5% C_D	1.6	0.1	0	-39.0	-0.062
-5% C_D	-1.4	-0.9	0	44.4	-0.03

The values on the bounds on the reentry conditions considered in this paper may be on the higher side for certain realistic missions. The bound on the reentry velocity is assumed to be ± 100 m/s, which is certainly on the higher side for certain applications, but the large values are chosen to show that the predictive control-based guidance algorithm can handle comparatively larger initial errors while delivering satisfactory performance. The performance of the guidance law will be certainly satisfactory if the initial errors are smaller than what have been considered.

Simulation results also show that the choice of weighting matrices Q_1 and Q_2 and the controller parameter δ has a direct effect on the terminal accuracy, and so the choice of these vital design parameters is important for satisfactory performance. Thus, a designer can obtain a better performance from the guidance law by properly choosing these parameters. If the reentry conditions have larger perturbation than what has been considered here, these parameters may be modified appropriately to obtain the terminal accuracy within specified limits.

CONCLUSION

This paper considers design and simulation of an optimal, nonlinear guidance law for reentry vehicles. The design of the guidance law, based on a nonlinear predictive control approach does not require linearisation of the equations of motion of the vehicle. Based on a nominal following concept, the physical constraints of the vehicle are incorporated by choosing an appropriate reference trajectory. A suitable choice was made for the controller parameter δ and weighting matrices necessary for evaluation of the guidance law and control constraints were incorporated to avoid control saturation. Simulations were carried out to assess the performance of the predictive control-based reentry guidance law for various perturbed entry conditions for a planar motion. The results show that the state errors reduce smoothly to acceptable values, thus guiding the reentry vehicle on to its nominal trajectory efficiently. Simulations were also carried out for off-nominal conditions and the results are found to be quite satisfactory.

ACKNOWLEDGEMENTS

The authors wish to thank Prof U.C. Durgapal and Prof S.B. Phadke for their invaluable suggestions and discussions during the course of this work.

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