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Thermal Stress Analysis of Laminated Composite Plates using Shear Flexible Element

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ABSTRACT

Using C° shear flexible QUAD-9 plate element, stresses and deflections in composite laminated plates due to thermal loads are analysed. A formulation based on first order shear deformation theory has been employed for the analysis. The effects of various parameters, such as ply-angle, number of layers, thickness and aspect ratios on stresses and deflections are brought out. The present formulation is being extended for studying composite shell structures.

1. INTRODUCTION

Composite materials are being looked upon as the key answer to man's eternal search for better materials to meet the exacting requirements of various critical applications. By virtue of their high strength-to-weight and stiffness-to-weight ratios and because of their mechanical properties in various directions, they can be tailored as per the requirements. Further, they combine a number of unique properties, including corrosion resistance, high damping, temperature resistance and low thermal coefficient of expansion. These unique properties have resulted in the expanded use of the advance composite materials in structures subjected to severe thermal environment. These structures are usually referred to as high temperature structures. Examples are provided by the structures used in high speed aircraft, spacecraft and re-entry vehicles, etc. However, the inhomogeneity and anisotropy of these materials pose a great challenge for the designer-analyst and analysis of heated composite structures remains an area of active research.

A large variety of computational/analytical models have been developed for studying the laminated composite plates under mechanical loadings. However, the literature dealing with thermal loads on composite structures is meagre. The thermal stress analysis of laminated composite plates is carried out using analytical approaches[1-6]. For problems involving complex boundary conditions and different types of loads, analytical methods, are not easily applicable and numerical methods, like finite element method (FEM), are preferred. Investigations based on analysis by FEM are reported in the literature[7-11].

The effect of shear deformation, depending on geometrical and material properties, plays a significant role in determining the global characteristics of the structure. Hence, for the finite element analysis of laminated structures, it is preferable to use a shear flexible theory.

In this paper a simple, C° continuous, nine-noded quadrilateral plate element based on the field consistency approach[12] is employed to study the stress

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analysis of laminated anisotropic plates subjected to thermal loads. The present model includes the effects of shear deformation. The influence of various parameters, such as number of layers, ply-angle, aspect and thickness ratios and boundary conditions, on the thermal stress behaviour of plates is brought out.

2. FORMULATION

A laminated composite plate is considered with the coordinates x,y along the in-plane directions and z along the thickness direction. Using Mindlin formulation, the displacements u, v, w at a point (x,y,z) from the median surface are expressed as functions of mid-plane displacements u_0, v_0 and independent rotations θ_x and θ_y of the normal in xz and yz planes, respectively as

$$\begin{aligned} u(x,y,z,t) &= u_o(x,y,t) + z\,\theta_x(x,y,t) \\ v(x,y,z,t) &= v_o(x,y,t) + z\,\theta_y(x,y,t) \\ w(x,y,z,t) &= w(x,y,t) \end{aligned} \tag{1}$$

Strains based on shear flexible theory, are

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_p^o \\ 0 \end{Bmatrix} + \begin{Bmatrix} -z\varepsilon_b \\ \varepsilon_s \end{Bmatrix} \tag{2a}$$

The mid-plane strains $\{\epsilon_p^o\}$, and bending strains $\{\epsilon_b\}$ and shear strains $\{\epsilon_s\}$ in Eqn (2a) are written as

$$\{\varepsilon_p^o\} = \begin{cases} u_{o,x} \\ v_{o,y} \\ u_{o,y} + v_{o,x} \end{cases} \tag{2b}$$

$$\{\varepsilon_b\} = \begin{cases} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{cases} \tag{2c}$$

$$\{\varepsilon_s\} = \begin{Bmatrix} \theta_x - w_{,x} \\ \theta_y - w_{,y} \end{Bmatrix} \tag{2d}$$

where subscript comma denotes the partial derivative with respect to the spatial coordinate succeeding it.

If (N) represents the membrane stress resultants (N_{xx}, N_{yy}, N_{xy}) and (M) the bending stress resultants (M_{xx}, M_{yy}, M_{xy}), one can relate these to membrane strain (ϵ_p^o) and bending strains (ϵ_b) through the constitutive relations as

$$\begin{aligned} \{N\} &= [A_{ij}]\{\varepsilon_p^o\} + [B_{ij}]\{\varepsilon_b\} - \{N^T\} \\ \{M\} &= [B_{ij}]\{\varepsilon_p^o\} + [D_{ij}]\{\varepsilon_b\} - \{M^T\} \end{aligned} \tag{3b}$$

where (A_{ij}), $(B_{ij}$ and (D_{ij}), $(i,j=1,2,6)$ are extensional, bending-extensional coupling and bending stiffness coefficients, respectively of the composite laminate. (N^T) and (M^T) are thermal stresses and moment resultants. Similarly, the transverse shear force (Q) representing the quantities (Q_{xz}, Q_{yz}) are related to the transverse shear strains $(\epsilon_s\}$ through the constitutive relations as

$$\{Q\} = [E_{ij}]\{\varepsilon_s\} \tag{4}$$

where (E_{ij}) $(i,j=4,5)$ are the transverse shear stiffness coefficients of the laminate.

For a composite laminate of thickness h, consisting of N layers with stacking angles ϕ_i $(i=1,N)$ and layer thickness h_i $(i=1,N)$, the necessary expressions to compute the stiffness coefficients, available in the literature[13], are used here. The potential energy functional U is given by

$$\begin{aligned} U(\delta) = \left(\frac{1}{2}\right)\iint_A \Big[&\{\varepsilon_p^o\}^T[A_{ij}]\{\varepsilon_p^o\} + \{\varepsilon_p^o\}^T[B_{ij}]\{\varepsilon_b\} + \\ &\{\varepsilon_b\}^T[B_{ij}]\{\varepsilon_p^o\} + \{\varepsilon_b\}^T[D_{ij}]\{\varepsilon_b\} + \{\varepsilon_s\}^T \\ &[E_{ij}]\{\varepsilon_s\} + \{\varepsilon_p^o\}^T\{N^T\} + \{\varepsilon_b\}^T\{M^T\}\Big]\, dA \end{aligned} \tag{5}$$

where δ is the vector of degrees of freedom.

The minimisation of the potential energy given by Eqn (5) with respect to the generalised displacement vector δ gives the following set of equations:

$$[K]\{\delta\} = \{F_T\} \tag{6}$$

where $[K]$ and $\{F_T\}$ are stiffness matrix and load vector, respectively.

3. DESCRIPTION OF THE ELEMENT

The laminated plate element employed in this investigation is a C^o continuous shear flexible element and needs five nodal degrees of freedom u, v, w, θ_x, and θ_y at nine nodes in QUAD-9 element, as shown in Fig. 1.

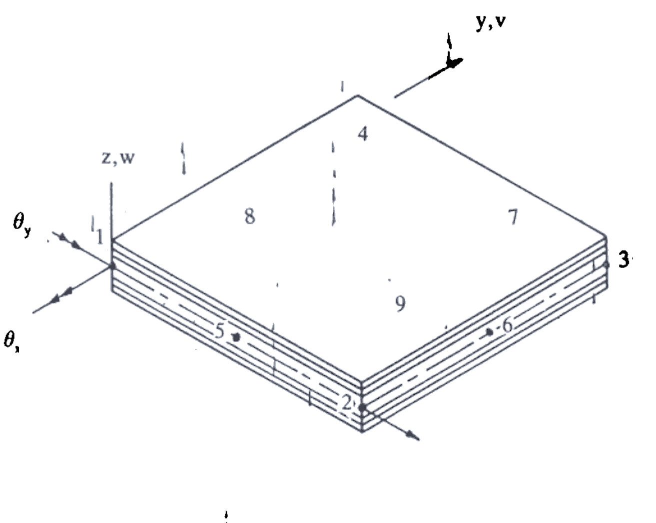

Figure 1. Geometry and coordinate systems of laminated plate element.

If the interpolation functions for QUAD-9 are used directly to interpolate the five variables u to θ_y in deriving the shear strains and membrane strains, the element will lock and show oscillations in the shear and membrane stresses. Field consistency requires that the transverse shear strains and membrane strain must be interpolated in a consistent manner. Thus u, v, θ_x and θ_y terms in the expressions for $\{\epsilon_s\}$ given in Eqn (2d) have to be consistent with field functions $w_{,x}$ and $w_{,y}$ as shown in the literature[12]. This is achieved by using field-redistributed, substitute shape-functions to interpolate those specific terms which must be consistent[12]

4. RESULTS AND DISCUSSION

Structural response of laminated plates due to temperature variation is considered here. For analysis, unless otherwise specified, one quarter of the plate with 2 × 2 mesh is assumed. The shear correction factor is taken as 5/6. The boundary conditions considered for the analysis are:

Simply supported:

$$v = w = \theta_y = 0 \qquad \text{on } x = 0, a$$

$$u = w = \theta_x = 0 \qquad \text{on } y = 0, b$$

Clamped supported:

$$u = v = w = \theta_x = \theta_y = 0 \qquad \text{on } x = 0, \text{ and } y = 0, b$$

The material properties used for the parametric study are: $E_L/E_T = 40$, $G_{LT}/E_T = 0.5$, $G_{TT}/E_T = 0.2$, $\nu_{LT} = 0.25$, $\alpha_L = \alpha_T = 1.0 \times 10^{-6}$ where L and T are the longitudinal and transverse directions, respectively with respect to the fibres. All the layers are of equal thickness and the ply-angle is measured with respect to x-axis (longitudinal axis).

The temperature distribution, without any external load, is assumed as

$$T(x,y,z) = (C_0 + C_1 z) T_s(x,y)$$

where

$$T_s(x,y) = T_0 + 2T_1 y/b \qquad 0 \le y \le b/2$$
$$= T_0 - 2T_1(y/b - 1) \qquad b/2 \le y \le b$$

and C_0, C_1, T_0, T_1 are constants.

To demonstrate the correctness of the present formulation in solving the problems of thermal stress, two cases for which closed form/exact solutions are available, have been solved using present formulation and results are compared with exact solutions.

Table 1. Non-dimensional central deflection $\bar{w}$ (= w/h) of isotropic plate subjected to linear temperature distribution varying through the thickness for simply supported boundary condition ($C_1 = T_0 = 1$, $C_0 = T_1 = 0$, $\nu = 0.3$, $\alpha = 1 \times 10^{-6}$, $E = 1 \times 10^7$, $a/b = 1$, $a/h = 100$)

	Ref. 1	Present work
Central – deflection (ω)	0.19079×10^{-4}	0.19139×10^{-4}

In Table 1, non-dimensional displacement $\bar{w}$ (= w/h) at the centre of the plate ($a/b = 1$, $a/h = 100$) with simply supported boundary conditions and subjected to linear temperature distribution, is compared with those of closed form/exact solution[1]. The parameters describing the temperature distribution are $C_1 = T_0 = 1$, $C_0 = T_1 = 0$, $\nu = 0.3$, $\alpha = 1 \times 10^{-6}$, $E = 1 \times 10^7$. Next, a laminated plate with sinusoidal temperature distribution [$T(x, y, z) = T_1 \sin(\pi x/a) \sin(\pi y/b)$] is considered for the analysis. The results are compared with those reported in the literature[3] in Table 2. One can observe from Tables 1 and 2 that the results based on the present formulation agree well with the available solutions.

Detailed parametric study is carried out for non uniform temperature distribution ($C_0 = 1$, $C_1 = T_0 = 0$,

Table 2. Central deflection of (0°/90°/0°....) plate subjected to sinusoidal temperature distributed load for simply supported boundary condition ($C_1 = T_n = 0$, $C_o = T_1 = 1$, $E_1 = 25E_2$, $G_{12} = G_{13} = 0.5\ E_2$, $G_{23} = 0.2\ E_2$, $\nu_{12} = 0.25$, $\alpha_1/\alpha_2 = 3$, $a/b = 1$, $a/h = 10$, $R/a = \infty$)

Central deflection ($W=10w/(\alpha_1 T_1 b^2)$)	Ref. 4	Present work
Two layers	1.1504	1.15051
Ten layers	1.0331	1.03338

T_1 =150). The results are presented along the line x/a=0.5. Figure 2 shows the variation of in-plane displacement $v = v/h$ in the y direction for different aspect and thickness ratios for simply-supported (45°/–45°) laminate. It is observed from Fig. 2 that the displacement increases with Y ($=y/b$) and is antisymmetric about the centre line of the plate for $a/b \geq 1$ whereas it is opposite for $a/b<1$. Similar plot is given in Fig. 3 for clamped six-layered angle-ply (45°/–45°/45°/–45°/45°/–45°) as well as cross-ply (0°/90°/0°/90°/0°/90°) laminates. It is noticed from Fig. 3 that the maximum displacement for angle-ply is higher than that for the cross-ply plates. The stress and moment resultants distributions are indicated in Figs

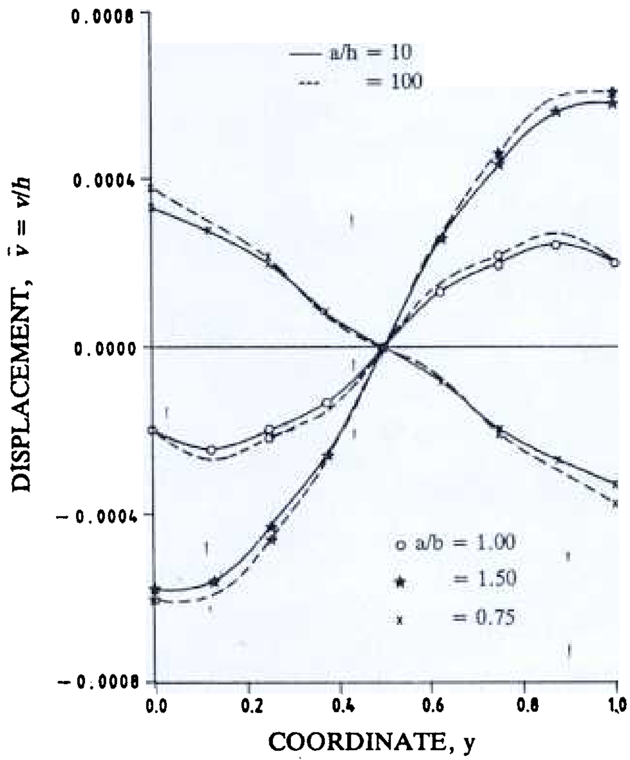

Figure 2. Distribution of displacement $\bar{v}=(v/h)$ ([45°/–45°], simply supported).

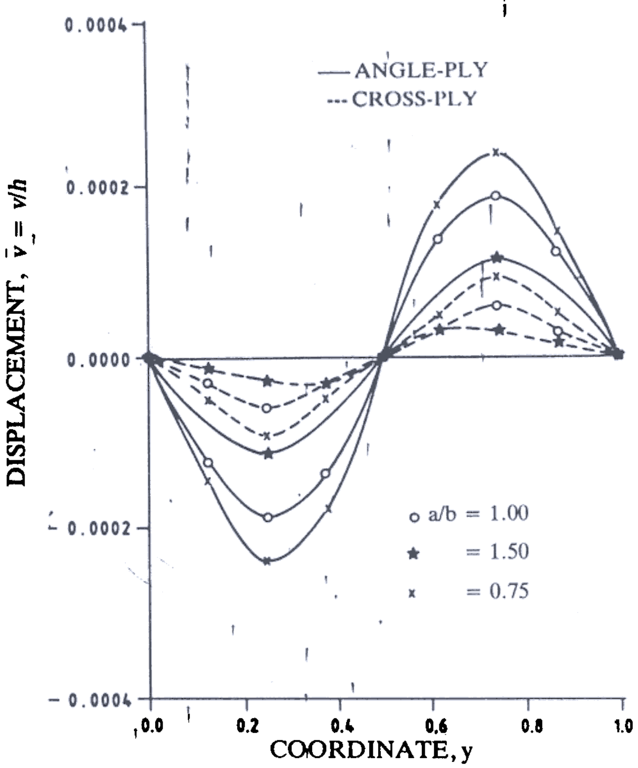

Figure 3. Distribution of displacement $\bar{v}=(v/h)$ (a/h= 100, 6 layers, clamped).

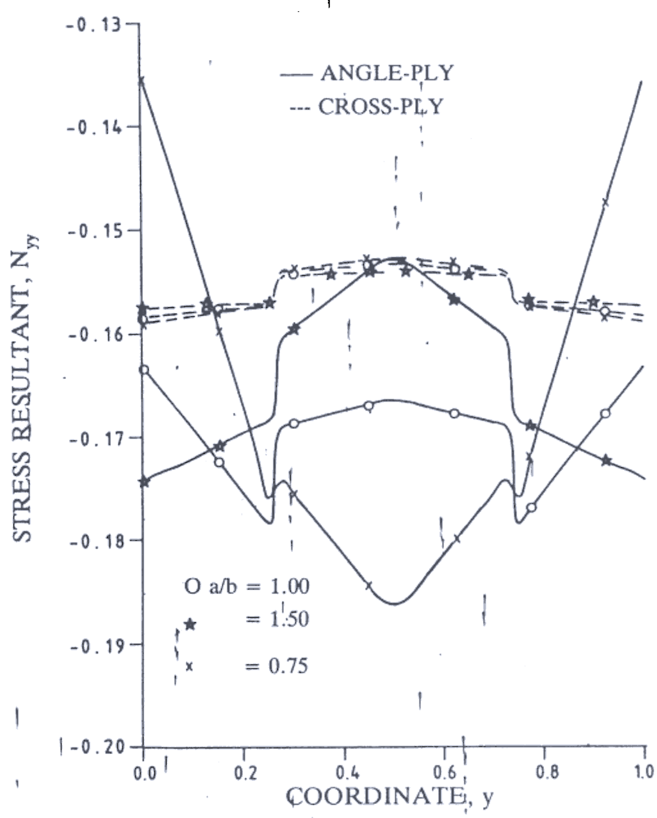

Figure 4. Distribution of force resultant N_{yy} (a/h = 100, 6 layers, clamped).

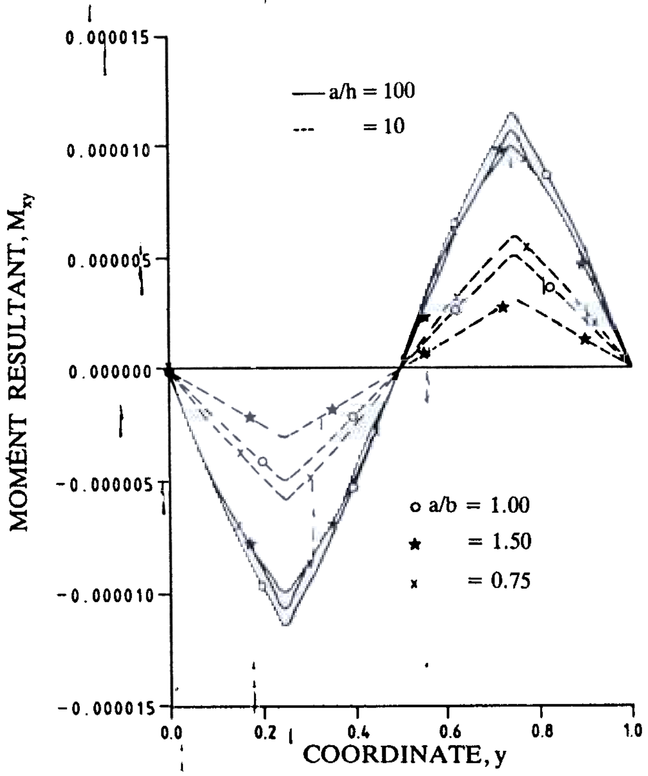

Figure 5. Distribution of moment resultant M_{xy}(6 layers cross-ply, clamped).

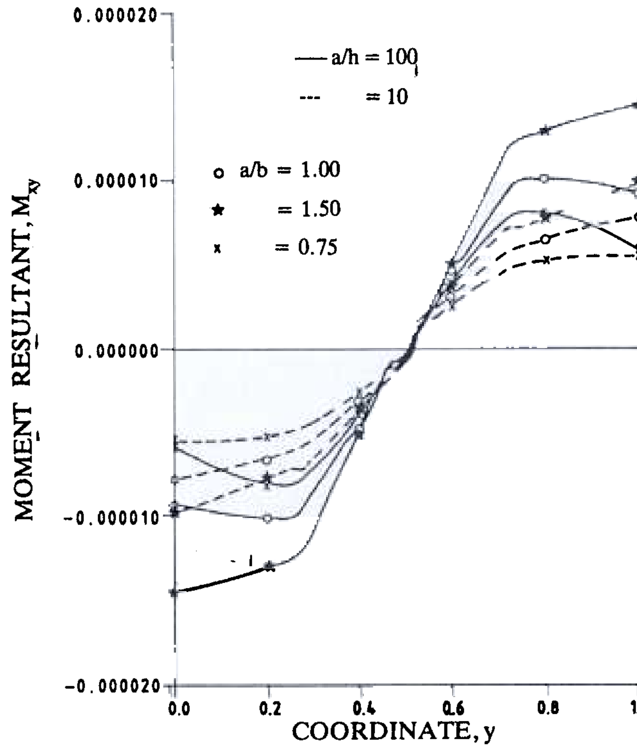

Figure 7. Distribution of moment resultant M_{xy} (6 layers, cross-ply, simply supported).

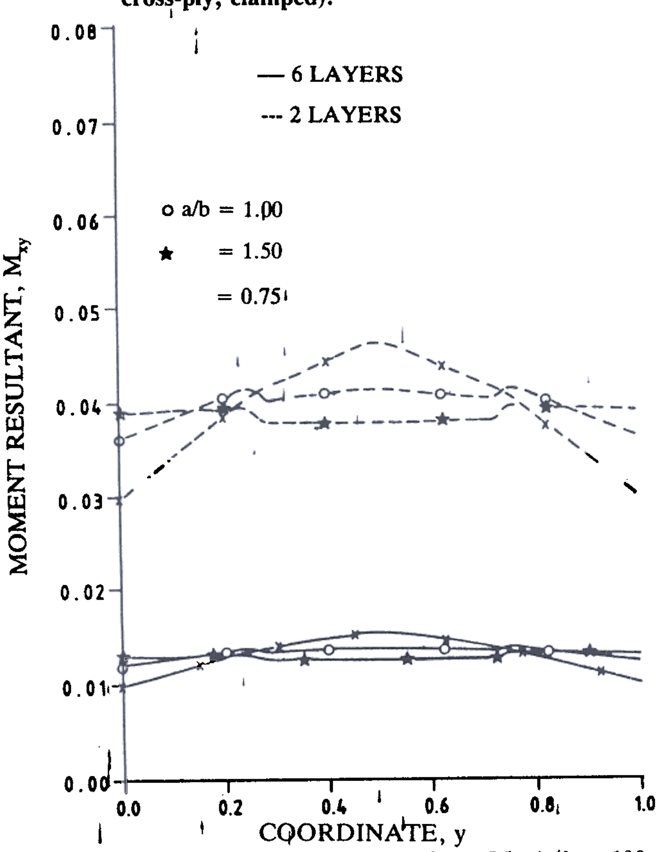

Figure 6. Distribution of moment resultant M_{xy} (a/h = 100, angle-ply, clamped, 2 layers, 6 layers).

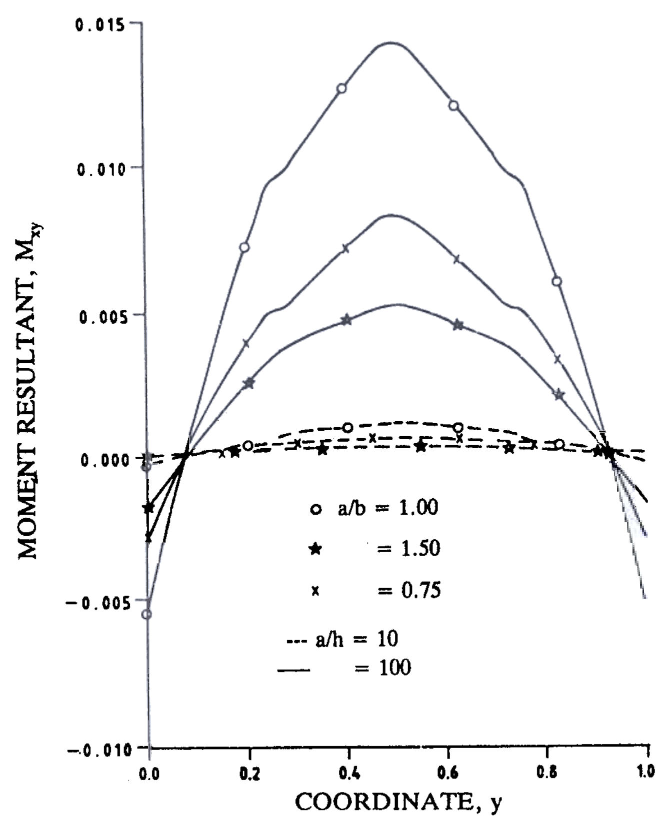

Figure 8. Distribution of moment resultant M_{xy} (6 layers, angle-ply, simply supported).

4-6 for clamped plate. It is concluded from these figures that the level of stress/moment resultant is more for angle-ply laminate in comparison with cross-ply case. Also it is observed that the effect of bending-extensional coupling in two-layered laminate is to increase the resultants. Similar plots for moment distributions are presented in Figs 7 and 8 for simply-supported laminated plates. In all the cases studied in the present analysis, the aspect and thickness ratios play a significant role in determining the displacement/stress levels.

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