Defence Science Journal, Vol. 61, No. 5, September 2011, pp. 485-490, DOI: 10.14429/dsj.61.403

Air Object Height Estimation with 2-D Radars using Fuzzy Logic

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ABSTRACT

Multi-sensor tracking is a widely used technique in aerospace applications to precisely estimate the target kinematics. Particularly naval-based tracking systems utilise different types of radars (2-D, 3-D) in multi-sensor tracking scenarios for robust estimation. As the supplied information from 2-D radar contains only range and azimuth values, it is difficult to estimate the height of an air object using 2-D radar. To overcome the limitation, a geometric method is considered to combine the information obtained from two 2-D radars located as two different locations. As the solution of the geometric method depends upon certain geometric features, it is not possible to get good results with one pair of sensors. However, to obtain better results, the proposed method combines geometric as well as fuzzy logic-based validation and experimented with more than two 2-D radars. The issues related to 2-D radar tracking and the method comprising triangulation geometry and a fuzzy logic-based selection method to improve the height estimation accuracy in real time have been discussed.

Keywords: Fuzzy decision making, fuzzy logic, target tracking, 2-D radars, air object height estimation multi-sensor tracking, naval-based tracking system

1. INTRODUCTION

The development of radar technology started in the early decades of the 20th century during the World War II. All the forces used radars to exert control over skies and seas. The radar technology developed in that era is still used to track air and land objects. Target tracking is an important area of research that embraces large domain of defence as well as commercial applications[1]. Understanding air situation is an essential task for air space control. Multi radar tracking (MRT) is an extensively used technique for naval-based target tracking applications. In multi-sensor data fusion systems, the information obtained from multiple radars located at different places are fused to get robust estimates[7,8]. Different types of radars comprising 2-D, 3-D are utilised in the MRT process to estimate the target positions in 3-D. Particularly 2-D radars provide range and azimuth information of a target that is used for ship-based tracking. As the plot data supplied by the 2-D radar contains only range and azimuth information, it is not possible to estimate target height with a single sensor due to observability problem, so that there is a need to combine the information (range and azimuth) obtained from two 2-D radars located on two different ships as shown in Fig. 1.

As radar-target geometric values are affecting the accuracy of the solution, it is not possible to get good results always. Gai-Ming-jiu[2], *et al.* and Hakl[10], *et al.* provided a solution for tracking 3-D targets with a single 2-D radar, but this solution works in some specialised conditions that include: (i) air object speed is known, (ii) air object flies at a constant altitude. However, it is observed that the accuracy of the geometric solution is a function of three parameters, i.e., angle making target with the radar stand points and slant ranges from the radar stand points. In order to obtain optimal geometric solution, it is proposed that more than two 2-D radars located at different locations be deployed and a pair of radars be selected, which produces good results computed using fuzzy logic.

2. GEOMETRIC SOLUTION

Consider a 3-D object (air target) located at *E* tracked by two 2-D radars located at *A* and *B*. For each iteration, radars at *A* and *B* (Radar 1, Radar 2) are supplying range and azimuth information of the target E (R1, Az1 and R2, Az2). Using the geometric solution described in Fig. 2, air object height is estimated. Performance of triangulation geometry is highly dependent on the accuracies of the observed measurements and geometric values.

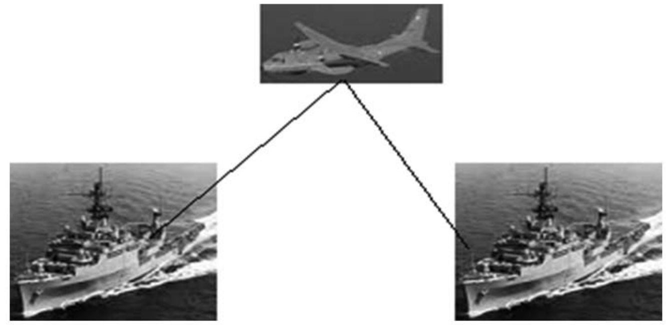

Figure 1. Tracking 3-D object with 2-D radars.

The following diagram describes the geometry of air object tracking from two 2-D radars (Radar 1, Radar 2) located

Received 12 March 2010, published online 2 September 2011

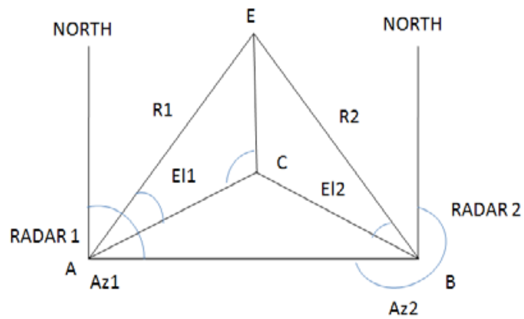

Figure 2. Geometry with two 2-D radars.

on two ships separated geographically.

By considering the radar positions in geodetic coordinate frame, located at A and B, and the obtained ranges ($R1$, $R2$) and azimuths ($Az1$, $Az2$) in each iteration, elevation angles wrt to each radar located at A and B are computed. C is a point of intersection of two azimuths ($Az1$, $Az2$) observed of A and B and $\angle ACE$ and $\angle BCE$ are right angles. The elevation angle obtained with the second radar, that is angle $\angle CBE$ is converted wrt the Radar1 position. In turn, this value is combined with angle $\angle CAE$ (obtained with Radar 1) to compute the mean elevation. The algorithm explains the process in detail for a single iteration as follows:

Step 1: Compute the distance between radar stand points A and B using the following equation.

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

where $A = (x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$.

Step 2: Compute the angles $\angle BAC$ and $\angle ABC$ using the observed azimuth values $Az1$ and $Az2$.

Step 3: Compute the angle $\angle ACB = \pi - \angle ABC - \angle BAC$.

Step 4: Compute the side lengths BC and AC using the following trigonometric equations and the computed values in steps as discussed.

$$BC = (sin(\angle BAC) * AB) / sin(\angle ACB)$$

$$AC = (sin(\angle ABC) * AB) / sin(\angle ACB)$$

Step 5: Using BC and AC, compute the target elevations $El1$ and $El2$ wrt Radar 1 and Radar 2 using the following trigonometric equations.

$$E/1 = \cos^{-1}(AC/R1)$$

$$E/2 = \cos^{-1}(BC/R2)$$

Step 6: Converting the two polar coordinates (range, azimuth, elevation) wrt a point of interest either (Radar 1 or Radar 2 stand point) and average the elevation value.

Using the elevation value obtained from the Step 6 and existing range and azimuth values, the air object height is estimated in ENV (East, North, Vertical) frame.

3. FUZZY LOGIC

In 2-D radar tracking, multiple 2-D radars track the 3-D targets from the ships and it is observed that certain target geometric values are affecting the accuracy of the solution. A fuzzy selection[4] method is considered to obtain the pair of 2-D radars, which gives optimal accuracy.

Fuzzy logic is an extension of Boolean logic supports in between values between true and false. The concept of fuzzy logic was introduced by Zadeah[5]. The goal was to develop a model that could be close to natural language process.

3.1 Fuzzification

Fuzzification is the first step in a fuzzy system. In this process, all crisp inputs are converted into fuzzy variables. The fuzzification process partitions the universe of discourse spanned by each variable into a number of fuzzy sets, assigning a linguistic variable[6]. Fuzzy sets are represented by fuzzy membership functions (MF). Different types of membership functions can be used for fuzzification process.

3.1.1 Membership Function

In fuzzy systems, fuzzy sets are represented by fuzzy membership functions (MF). Triangular, trapezoidal and Gaussian, are some of the membership functions[5], used as per the convenience and nature of the problem. Figure 3 describes the considered membership functions for the fuzzification process of input 'angle' for the present problem.

For every crisp value, membership functions represent a degree, which indicates the strength of its concern to the fuzzy set. The degree of membership is always in the range [0,1]. For example, any crisp value has a membership degree (or degree of support) 0, indicates that the value is not in the fuzzy set. If the degree is 1, it means, the value is completely within the fuzzy set. The following equations explain how to calculate a membership value ($m(x)$) for a crisp value x, using a triangular membership function.

$$\mu(x) = \begin{cases} 0 & if \;\; x < a_1 \\ (x - a_1)/(a_2 - a_1) & if \; a_1 \le x \le a_2 \\ (a_3 - x)/(a_2 - a_3) & if \; a_2 \le x \le a_3 \\ 0 & if \;\; x > a_3 \end{cases}$$

where a_1, a_2, and a_3 are the x coordinates of the start, middle, and end points of a triangular fuzzy membership function.

3.2 Fuzzy Inference

Fuzzy inference is a process of formulating a mapping from inputs to output using fuzzy logic. In fuzzy inference, knowledge is expressed in terms of fuzzy if-then rules. The computation of fuzzy inference consists of two components: aggregation (computation of if-part) and composition (computation of then-part). The most common inference

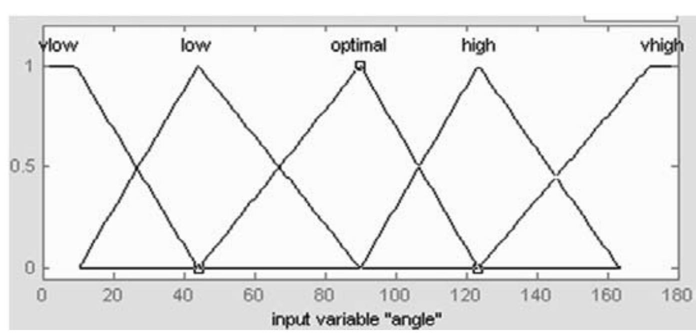

Figure 3. Membership functions for the input angle.

methods AND, OR, and NOT are used in majority of fuzzy logic applications.

In this case, a numerical output is required from the inference process, a defuzzification step must follow the fuzzy inference. Mamdani type fuzzy inference is considered. A weight factor is used to tune the fuzzy logic system for optimal performance, the weight factor represents the strength of the rule belongs to the interval [0,1]. It is multiplied with the aggregation result in the composition step. Table 1 some of the fuzzy rules considered for the present problem are described.

3.3 Defuzzification

Defuzzification process translates fuzzy values to crisp value. Most of the fuzzy systems required this process to visualise the result in a numerical form rather than fuzzy. Defuzzification step also requires membership functions as shown in Fig. 4. The following equation represents computation of crisp value U using proposed centre of area defuzzification method for the present problem. Defuzzification method computes a crisp value U, which is in range of [0, 100].

$$U = \sum_{i=1}^{n} (A_i * L_i) / \sum_{i=1}^{n} A_i$$

where A_i represents the area covered under each fuzzy set and L_i represents mid-point of the fuzzy set on x-axis and n = 5 (number of fuzzy sets in the defuzzification process).

4. COMBINATION OF THE METHODS

To obtain optimal results, it is considered that more than two 2-D radars are deployed on different ships, which are separated geographically. Each 2-D radar deployed on the ship tracks the air object, if it is within the coverage area. So that for a particular air object, tracked radar supplies range and azimuth information. For estimating air object height, there is a need to process the received information by considering the geometric solution mentioned in Section 2. During the experimental phase, it is observed that angles and the slant ranges play an important role in the height estimation process and identifying the best pair based on the angles and slant ranges is difficult part with conventional approach. It is well known that fuzzy systems perform well in nonlinear dynamic applications[9], where mathematical models are complex. To get optimal accuracy, fuzzy logic based selection is considered for the optimal pair from the available pairs. For the present study, three 2-D radars are considered for tracking the target, so that three data sets, i.e., T1, R1, Az1 (time, range, azimuth) from Radar 1 T2, R2, Az2, from Radar 2 and T3, R3, Az3 from Radar 3 are obtained for the process in each process. The flow chart as shown in Fig. 5 describes the process for each iteration.

Table 1. Fuzzy inference (rules) considered for the present problem

Rule	Angle	Avg Side	Selection	Weight
1	Optimal	Less	Best	1
2	Optimal	Medium	Best	0.7
3	Optimal	More	Better	1
4	Low	Less	Best	1
5	High	Less	Best	0.9
6	V High	Less	Bad	1
7	Low	Medium	Good	1
8	High	Medium	Better	1
9	Low	More	Good	0.8
10	V Low	More	Bad	1

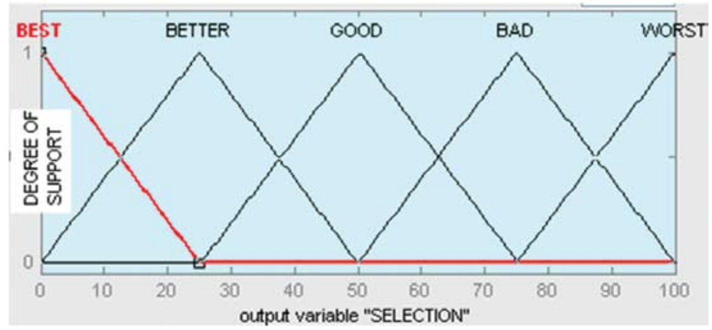

Figure 4. Membership functions for selection output.

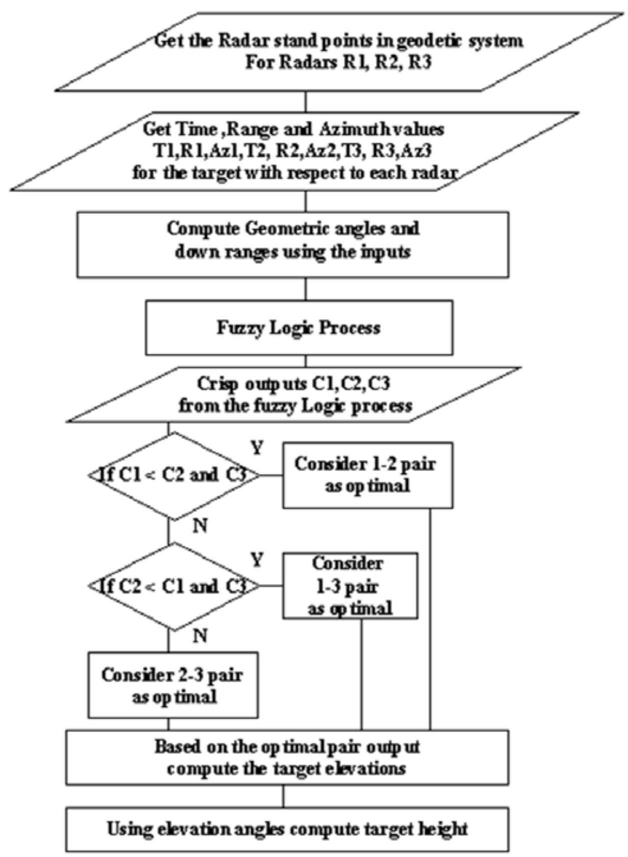

Figure 5. Proposed method to estimate air object height with three 2-D radars using fuzzy logic.

5. SIMULATION AND RESULTS

Using this method, a software has been developed using MATLAB 7.0. To test the software, aircraft trajectories were simulated by considering target dynamics. Software was tested with different data sets obtained from the simulation models and the results are shown in MATLAB graphs. To compare the performance of the proposed method, mean error (ME) was computed with the following equation.

$$ME = \frac{1}{n}\sum_{i=1}^{n}\left|H_R - H_E\right|$$

where H_R represents real height of air object and H_E is the height computed by one pair of radars or optimal pair, obtained by fuzzy selection method. Considering different dynamics of aerodynamic targets, the software has been tested and mean errors of all the experiments are given in Table 5.

In Experiment 1, a simulated maneuvering target was considered . In this simulation, the sampling time t =1(s) . The initial position of the target is given by $[x(0), y(0), z(0)]$ = [2,500 (m),14,000 (m),8,000 (m)] with an initial speed of $[v_x, v_y, v_z]$ = [150(ms^{-1}), -60(ms^{-1}), 4(ms^{-1})]. The target moved with a constant acceleration of $[u_x(0), u_y(0), u_z(0)]$ = [–0.5(ms^{-2}), –0.5(ms^{-2}), 0(ms^{-2})] until t = 100(s), Then it started to maneuver with acceleration $[u_x(101), u_y(101), u_z(101)]$ = [2(ms^{-2}), 10(ms^{-2}), –3(ms^{-2})]. This acceleration continued to t = 200s. Then, the target started another maneuver with acceleration $[u_x(201), u_y(201), u_z(201)]$ = [–10 (ms^{-2}), –30(ms^{-2}), 5(ms^{-2})]. This acceleration continued to t = 300 (s) and at t = 301 (s) target took one more maneuver with acceleration $[u_x(301), u_y(301), u_z(301)]$ =[–10 (ms^{-2}), 20 (ms^{-2}), –6 (ms^{-2})] and it continues up to t = 600 (s). Ships/Radars locations are represented in ENV frame. Ships/radars positions considered for the experiment are given in Table 2. Figures 6, 7 and 8 describe the estimation errors of available pairs with fuzzy selection method.

In Experiment 2, the trajectory generated for Experiment 1 was considered and the ship locations were changed as mentioned in Table 3. Figures 9, 10 and 11 describe the estimation errors of available pairs with fuzzy selection method.In Experiment 3, a simulated medium-maneuvering target, which was far from all the ships, was considered. In this simulation, the sampling time t = 1 (s) . The initial position of the target is given by $[x(0), y(0), z(0)]$ = [50,500(m), 34,000(m), 10,000(m)] with an initial speed of $[v_x, v_y, v_z]$ = [150 (ms^{-1}), -60 (ms^{-1}), 4(ms^{-1})]. The target moved with a constant acceleration of $[u_x(0), u_y(0), u_z(0)]$ = [-0.5(ms^{-2}), -0.5(ms^{-2}), 0(ms^{-2})] until t = 100(s), Then it started to maneuver with acceleration $[u_x(101), u_y(101), u_z(101)]$ = [2(ms^{-2}), 3 (ms^{-2}), -3(ms^{-2})]. This acceleration continues to t = 200s. Then, the target starts another maneuver with acceleration $[u_x(201), u_y(201), u_z(201)]$ = [5 (ms^{-2}), -3 (ms^{-2}), 5 (ms^{-2})]. This acceleration continues to t = 300(s) and at t = 301(s) target took one more manoeuver with acceleration $[u_x(301), u_y(301), u_z(301)]$ =[6 (ms^{-2}), -5(ms^{-2}), -5(ms^{-2})] and it continued up to t = 600(s). Ships/radars locations are

Table 2. Ships/radars locations in ENV frame for experiment 1.

Ship	X-position (m)	Y-position (m)	Z-position (m)
1	3778	1592	0
2	7845	2180	0
3	889	4902	0

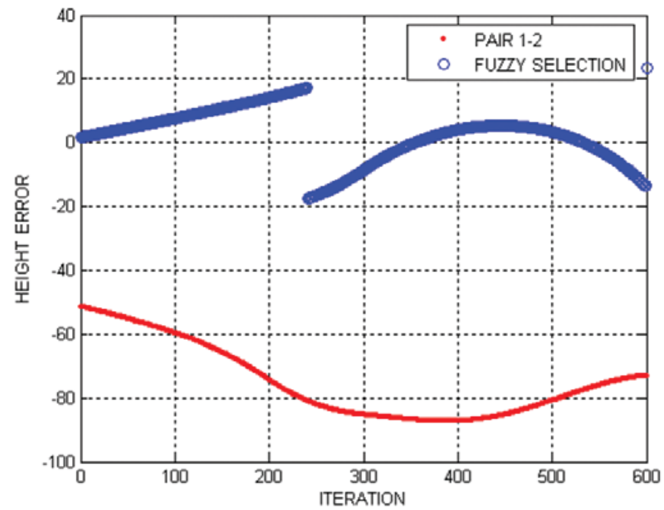

Figure 6. Air object height estimation errors of PAIR 1-2 and fuzzy selection (in m).

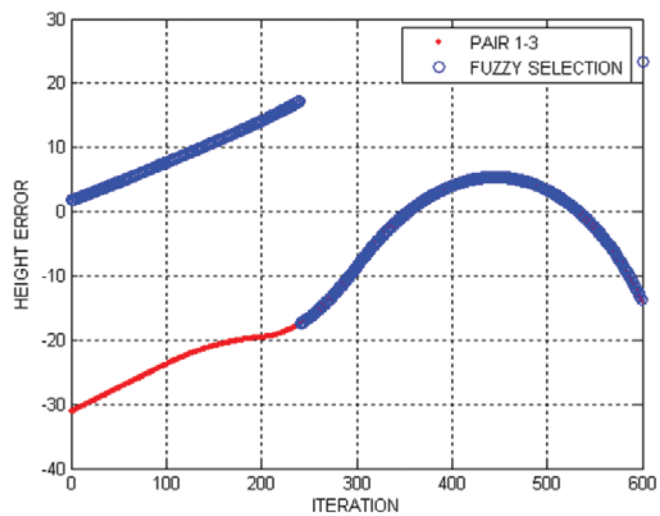

Figure 7. Air object height estimation errors of PAIR 1-3 and fuzzy selection (in m).

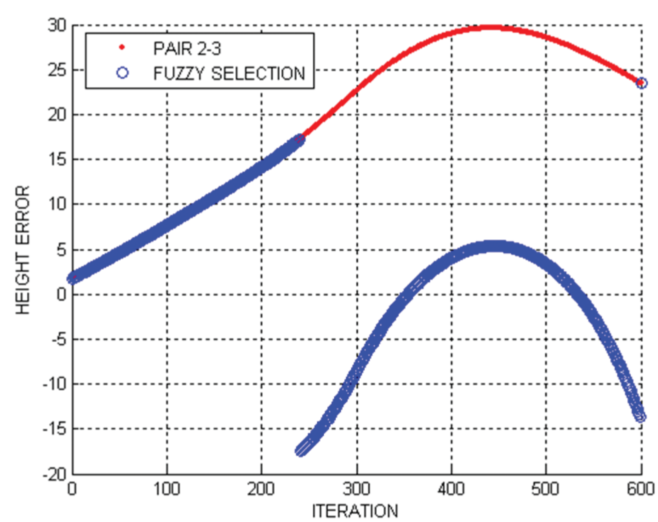

Figure 8. Air object height estimation errors of PAIR 2-3 and fuzzy selection (in m).

Table 3. Ships/radars locations in ENV frame for experiment 2.

Ship	X-position (m)	Y-position (m)	Z-position (m)
1	3111	4353	0
2	6556	18824	0
3	19111	6863	0

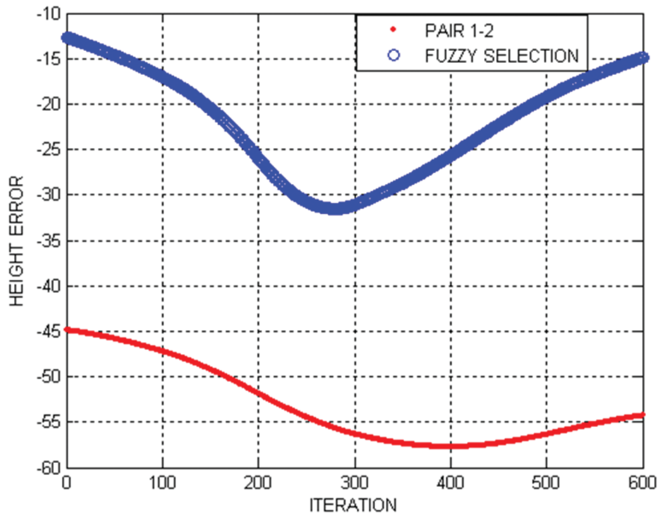

Figure 9. Air object height estimation errors of PAIR 1-2 and fuzzy selection (in m).

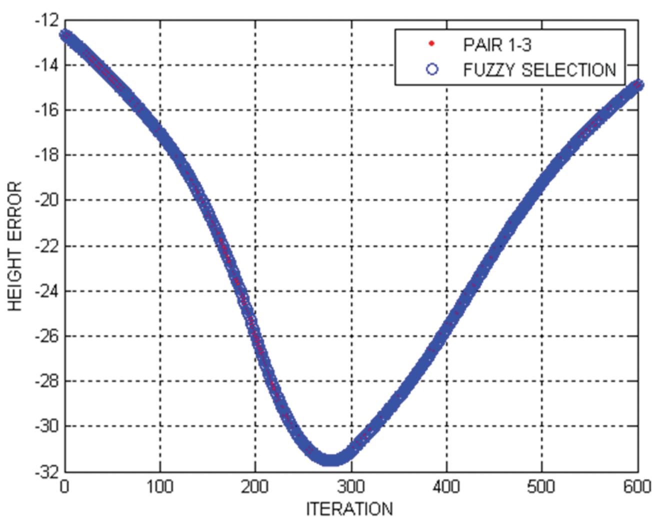

Figure 10. Air object height estimation errors of PAIR 1-3 and fuzzy selection (in m).

represented in ENV frame are given in Table 4. Figures 12, 13 and 14 describe the estimation errors of available pairs with fuzzy selection method.

6. CONCLUSIONS AND FUTURE RESEARCH

In this paper, a new algorithm has been presented that combines a geometric solution of 2-D radars with fuzzy logic-based selection technique for improving the accuracy of air target height estimation. Based on the proposed algorithm, software was designed and developed to estimate the 3-D target height using 2-D radar data. Software was tested with

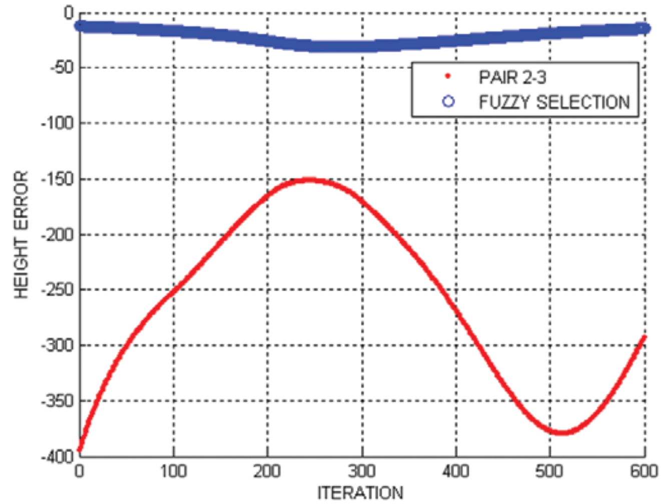

Figure 11. Air object height estimation errors of PAIR 2-3 and fuzzy selection (in m).

Table 4. Ships/radars locations in ENV frame for experiment 3

Ship	*X*-position (m)	*Y*-position (m)	*Z*-position (m)
1	7067	4128	0
2	8400	8130	0
3	5067	10569	0

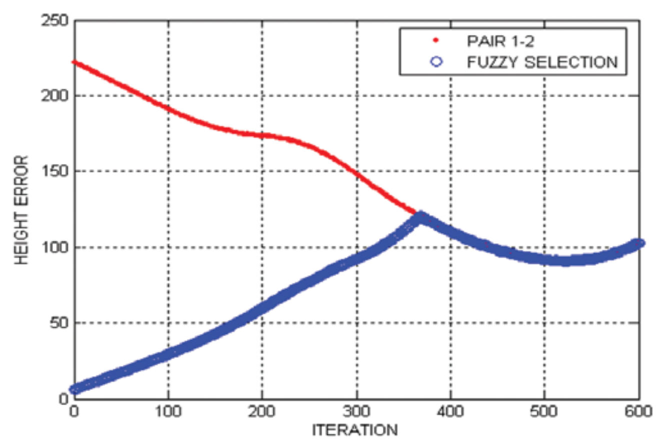

Figure 12. Air object height estimation errors of PAIR 1-2 and fuzzy selection (in m).

different sets of aircraft trajectories. In this experimental study, Mamdani type fuzzy logic technique was utilised to select a pair of sensors to get optimal values, among all available sensor pairs. It was observed that the proposed algorithm has considerable benefits over normal 2-D tracking methods. As all the fuzzy rules were generated and tuned manually, software may not provide optimal pair in case the solutions are relatively close. To enhance the accuracy, it is required to use genetic, neural techniques for optimal inference rules by considering various target geometries. All the fuzzy inference rules generated manually and using trial and error method, fuzzy rules are modified by tuning the weight values. Random errors caused by sensors are not considered. It is observed that height estimation error is increasing, when the distance

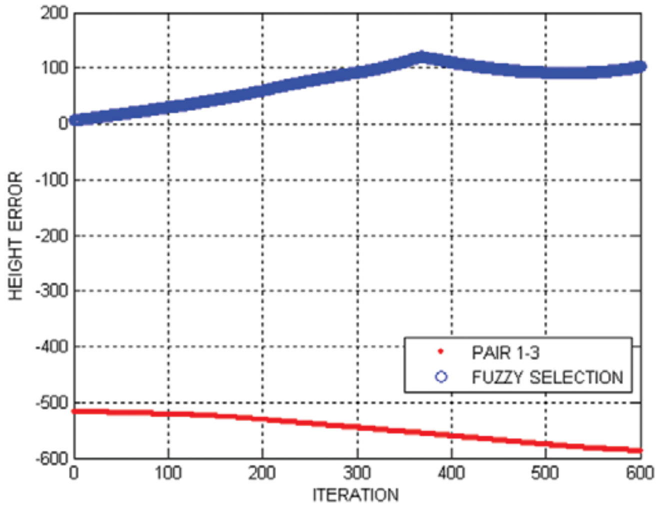

Figure 13. Air object height estimation errors of PAIR 1-3 and fuzzy selection (in m).

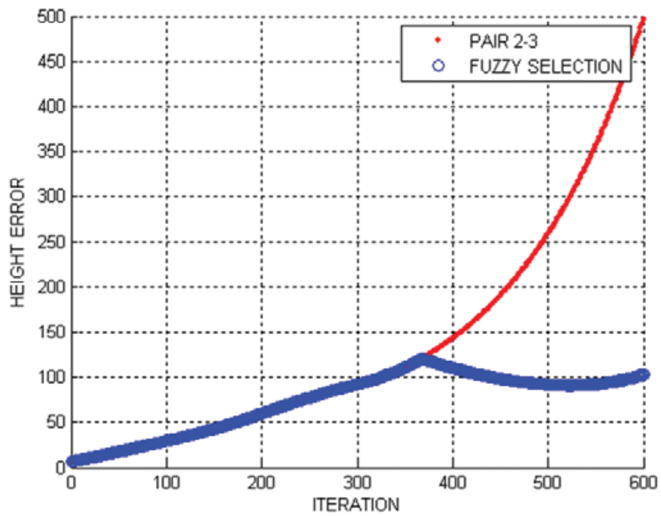

Figure 14. Air object height estimation errors of PAIR 2-3 and fuzzy selection (in m).

Table 5. Mean errors (in m) of all radar pairs with fuzzy selection.

	Mean error (m)			
Experi-ment	Pair 1-2	Pair 1-3	Pair 2-3	Fuzzy selection
1	75.06	12.8	19.3	7.1
2	53.2	22.23	262.1	22.23
3	145.3	547.8	134.9	73.3

between ships/radars is more (due to earth curvature). To control random disturbances in measurements, statistical estimation techniques[3] (e.g., Kalman filter) are utilised frequently, which require target dynamic models and sensor uncertainties. For future enhancement, wavelets-based denoising techniques, either in combination of statistical approaches or alone are considered, as multi-resolution techniques do not require any pre-assumptions.

ACKNOWLEDGEMENTS

The authors express their deep sense of gratitude to University Grants

commission for sanctioning DRS-III/SAP-I to the department of mathematics, Osmania University, Hyderabad whose infrastructural facilities were used during the course of our research. They are thankful to Shri K Srinivas, Director DCCT for giving necessary support.

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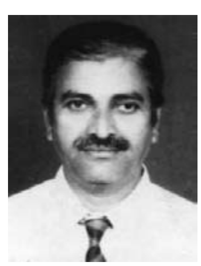

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