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Modulation of Rectangular & Triangular Bar Targets by Optical Systems using Hamming Filters

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ABSTRACT

The modulation in incoherent imaging systems for rectangular and triangular bar targets has been investigated using Hamming apodisation filters. The results of contrast and cutoff frequency responses have been reported with the help of certain profiles. Different apodisation parameters with different values have been used for the calculation of image modulation.

1. INTRODUCTION

Investigations on the use of rectangular and triangular bar targets for the frequency response studies of optical systems under various imaging conditions have been continually made during the last three decades[1-4]. These studies have paved the way for proper selection of test targets for the experimental measurements of the optical transfer function of various optical systems. A proper understanding of the performance of optical systems with different imaging conditions can be had from these investigations. This paper discusses the modulation in incoherent images of rectangular and triangular bar targets by optical systems using Hamming filters.

Studies on the transfer function and the equivalent pass-band of various apodising filters have indicated the possibility of having an optical system better than Airy or perfect lens approximation regarding white signal power transmission. Obviously, the next step in this direction is to study the modulation of periodic targets of various shapes; the results of certain targets with optical systems apodised with Kaiser filters[5] have already been reported.

2. THEORETICAL FORMULATION

A periodic rectangular target waveform can be represented by Fourier series as[6]

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$$A(u) = a_1 - a_2 + 2\alpha a_2 + \frac{4a_2}{\pi} \sum_{n=1}^{\infty} \frac{\sin n\alpha\pi}{n} \cos n\omega u \quad (1)$$

where

a_1 is the average irradiance and a_2 the modulation height about the average irradiance; α is the ratio of width of bright portion to the width of dark portion of the intensity pattern; ω is the reduced angular spatial frequency in the direction u. The Fourier transform of Eqn (1) represents the object intensity spectrum at the entrance pupil of the optical system. Thus

$$a(x,y) = \iint_{-\infty}^{\infty} [(a_1 - a_2) + 2\alpha a_2 + \frac{4a_2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha\pi)}{n} \cos(n\omega u)] e^{-i(ux+vy)} dxdy \quad (2)$$

which, on further simplification becomes

$$a(x,y) = \delta(x,y) [(a_1 - a_2) + 2\alpha a_2] + \frac{4a_2}{2\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha\pi)}{n} \delta(x - n\omega,y) + \frac{4a_2}{2\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha\pi)}{n} \delta(x + n\omega,y) \quad (3)$$

where

$\delta(x)$ is the Dirac delta function.

The modified spectrum on the exit pupil is given by

$$a^1(x,y) = a(x,y)\, T(x,y) \quad (4)$$

where $T(x,y)$ is the incoherent transfer function of the system.

Then

$$a^1(x,y) = \delta(x,y) \quad (a_1 - a_2) + 2\alpha a_2] \; T(x,y) + \frac{4a_2}{2\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha\pi)}{n} \delta(x - n\omega,y)\, T(x,y) + \frac{4a_2}{2\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha\pi)}{n} \delta(x + n\omega,y)\, T(x,y) \quad (5)$$

The inverse Fourier transform of $a^1(x,y)$ gives rise to the image intensity distribution. It may be written as

$$B^1(u^1,v^1) = \iint_{-\infty}^{\infty} a^1(x,y) \exp\{i(u^1x + v^1y)\}\, dxdy \quad (6)$$

$$B^1(u^1,v^1) = \iint_{-\infty}^{\infty} [\delta(x,y) \{(a_1 - a_2) + 2\alpha a_2\}\, T(x,y) + \frac{4a_2}{2\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha\pi)}{n} \delta(x - n\omega,y)\, T(x,y) + \frac{4a_2}{2\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha\pi)}{n} \delta(x - n\omega,y)\, T(x,y) \exp\{i(u^1x + v^1y)\}\, dx\, dy \quad (7)$$

Using the properties of Dirac delta function, the transfer function $T(n\omega) = 0$ for $n\omega \geq 2$, the maximum value of n is n^1 such that $n^1 \omega \leq 2$. Then Eqn (7) after simplification becomes

$$B^1(u^1,v^1) = \{(a_1 - a_2) + 2\alpha a_2\}\, T(o) + \frac{4a_2}{2\pi} \sum_{n=1}^{n'} \frac{\sin(n\alpha\pi)}{n} T(n\omega) \cos n\omega u \quad (8)$$

If the transfer function $T(n\omega)$ is normalised such that $T(o) = 1$, then Eqn (8) can be written as

$$B^1(u^1,v^1) = (a_1 - a_2) + 2\alpha a_2 + \frac{4a_2}{2\pi} \sum_{n=1}^{n'} \frac{\sin(n\alpha\pi)}{n} T(n\omega) \cos n\omega u^1 \quad (9)$$

The Hamming apodisation filters represented by the pupil transmission function[7] are

$$f(r) = 0.54 + 0.46 \cos \beta \pi r \qquad (10)$$

where

r is the normalised distance of a point on the circular aperture and β is the apodisation parameter which determines the nature of the nonuniformity of transmission through the pupil.

Obviously, the auto-correlation of the aperture function is, by definition, the optical transfer function of the pupil and can be expressed as

$$T(\omega)_N = \int_{\omega/2}^{1} \int_{0}^{\sqrt{1-x^2}} [0.54 + 0.46 \cos \beta\pi (x^2+y^2)^{1/2}] \{0.54 + 0.46 \cos \beta\pi\, [(x-\omega)^2 + y^2]^{1/2}\}\, dxdy \qquad (11)$$

where

$T(\omega)_N$ is the normalised optical transfer function for the spatial frequency ω, with the condition that $T(0) = 1$. The term x and y in Eqn (11) are the Cartesian coordinates corresponding to the polar coordinates, r.

The values of $T(n\omega)$ can be obtained by the numerical integration of Eqn (11).

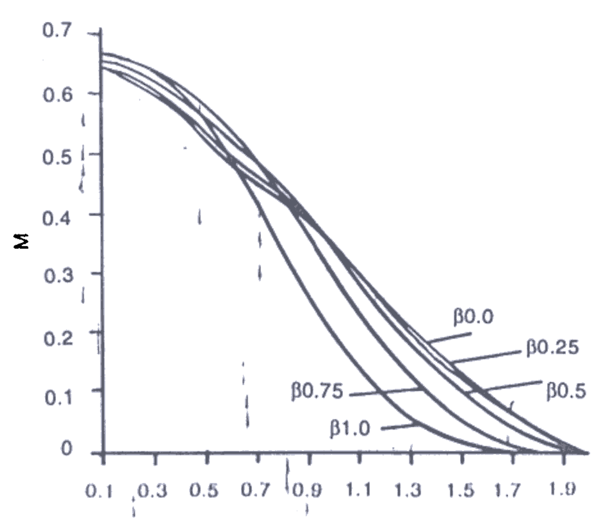

Figure 1. Modulation of rectangular targets for $\alpha = 0.25$

3. RESULTS & DISCUSSION

The intensity distributions in the images of periodic rectangular and triangular targets were obtained by evaluating Eqn (9). The modulation in the images of the system to these targets have been computed by the formula.

$$B = \frac{B^1(u^1)_{max} - B^1(u^1)_{min}}{B^1(u^1)_{max} + B^1(u^1)_{min}} \qquad (12)$$

where

$B^1(u^1)_{max}$ and $B^1(u^1)_{min}$ are the maximum and minimum intensities of the image, respectively.

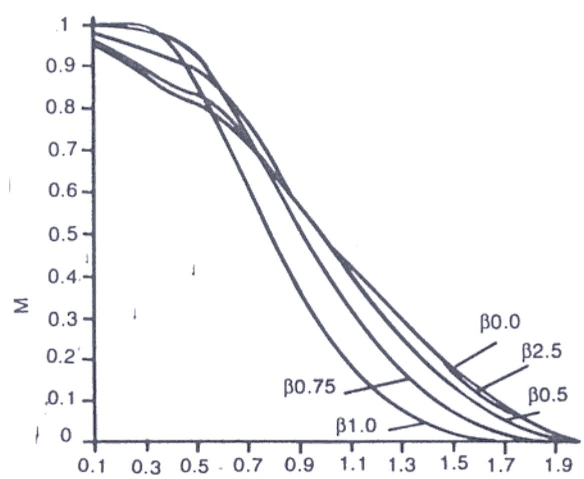

Figure 2. Modulation of rectangular targets for $\alpha = 0.50$

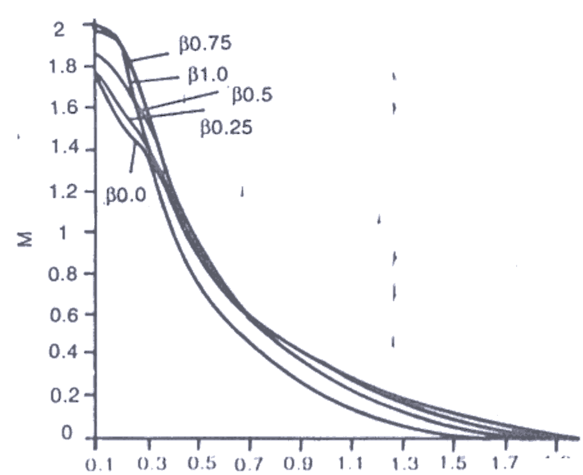

Figure 3. Modulation of rectangular targets for $\alpha = 0.75$

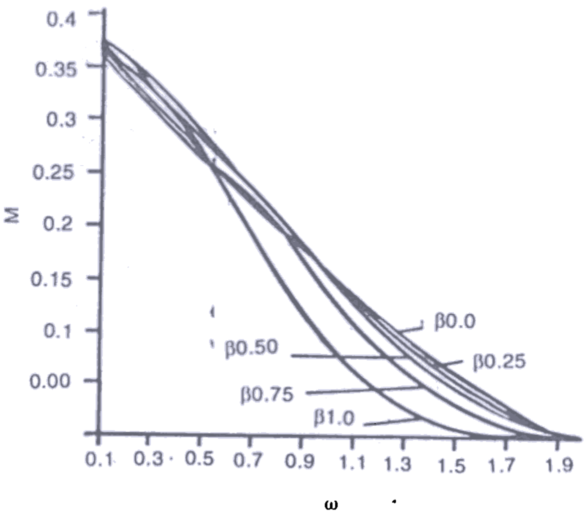

Figure 4. Modulation of triangular targets for α = 0.25

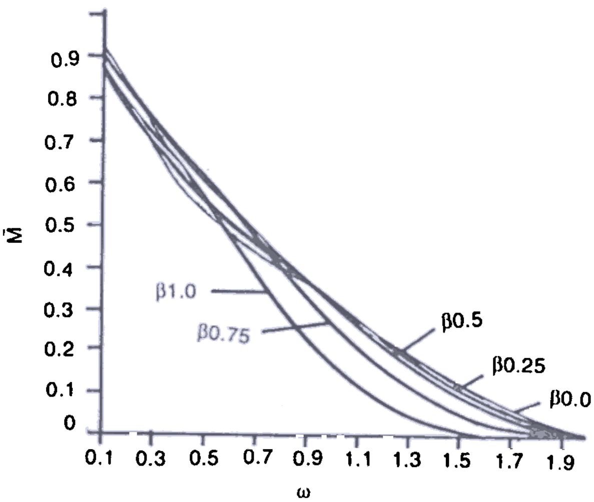

Figure 5. Modulation of triangular targets for α = 0.50

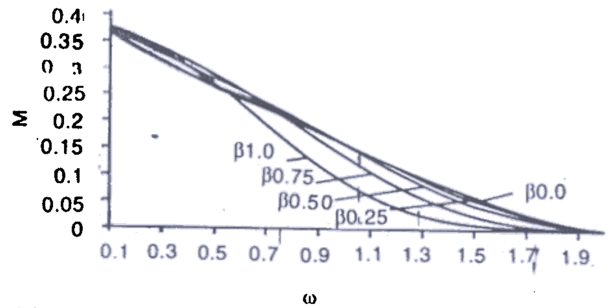

Figure 6. Modulation of triangular targets for α = 0.75

Figures 1-3 represent the profiles of M versus ω for β = 0, 0.25, 0.50, 0.75 and 1 values and α = 0.25, 0.50 and 0.75 for rectangular targets and Figs 4-6 represent the corresponding profiles for triangular targets for the same values of β and α.

It has been found that for higher values of β the cutoff frequency decreases, for both rectangular and triangular targets. In other words, with increasing apodisation parameter, β, the ability of the pupil to cutoff certain frequency ranges increases. The nonlinear profiles of rectangular and triangular wave responses show a continuously varying slope which, in turn, indicates that the modulation of intensity in the image is also nonlinear. In rectangular targets, the contrast of the images increases with increasing values of β, i.e., the lesser the cutoff frequency, the more is the contrast. While studying the profiles of triangular targets, it has been observed that for α = 0.5, the image modulation or contrast is relatively more when compared with those for α = 0.25 and α = 0.75. Hence to attain more image contrast, a moderate value of α = 0.5 can be utilised. It has also been observed that for increasing α and increasing β the image contrast increases. Table 1 shows the numerical values of modulation profiles for different values of α and β.

Based on the results, a quantitative as well as qualitative behaviour of Hamming filter could be well understood. By increasing the values of apodisation parameter, β, the user can have relative control on the image contrast when the optical system is designed to record images of feeble objects, like stars. As the present investigation has been carried out with incoherent light, the wide range of frequencies is a hurdle on the image quality. Hence, an approximation has to be made and the same has been carried out using Hamming apodisers, i.e. certain frequency range has been cutoff. By increasing β, image contrast increases at the cost of decreasing cut-off frequency. This has been observed in both rectangular and triangular

Table 1. Numerical values of modulation profiles of rectangular and triangular targets

	Modulation of rectangular bar targets					Modulation of triangular bar targets				
					$\alpha = 0.25$					
ω	β= 0.0	β= 0.25	β= 0.50	β= 0.75	β= 1.0	β= 0.0	β= 0.25	β= 0.50	β= 0.75	β= 1.0
0.0	0.9002	0.9002	0.9002	0.9002	0.9002	0.4052	0.4052	0.4052	0.4052	0.4052
0.2	0.6223	0.6294	0.6453	0.6606	0.6621	0.3367	0.3407	0.3506	0.3583	0.3545
0.4	0.5737	0.5816	0.6008	0.6126	0.5973	0.2863	0.2917	0.3052	0.3138	0.3018
0.6	0.4849	0.4960	0.5238	0.5400	0.4977	0.2406	0.2462	0.2591	0.2621	0.2335
0.8	0.4261	0.4354	0.4535	0.4365	0.3424	0.2003	0.2041	0.2106	0.1997	0.1549
1.0	0.3520	0.3541	0.3488	0.2984	0.1938	0.1584	0.1594	0.1570	0.1343	0.0872
1.2	0.2563	0.2532	0.2345	0.1758	0.0916	0.1154	0.1140	0.1056	0.0792	0.0412
1.4	0.1694	0.1632	0.1391	0.0878	0.0348	0.0762	0.0735	0.0626	0.0395	0.0157
1.6	0.0937	0.0876	0.0672	0.0340	0.0098	0.0422	0.0394	0.0303	0.0153	0.0044
1.8	0.0337	0.0303	0.0205	0.0079	0.0017	0.0152	0.0136	0.0092	0.0035	0.0008
					$\alpha = 0.5$					
0.0	1.2727	1.2727	1.2727	1.2727	1.2727	0.8099	0.8099	0.8099	0.8099	0.8099
0.2	0.9172	0.9284	0.9580	0.9886	0.9984	0.7793	0.7907	0.8192	0.8398	0.8238
0.4	0.8303	0.8528	0.9141	0.9745	0.9643	0.6307	0.6438	0.6752	0.6902	0.6501
0.6	0.7783	0.7983	0.8442	0.8511	0.7476	0.5086	0.5200	0.5453	0.5446	0.4764
0.8	0.6423	0.6537	0.6716	0.6320	0.4874	0.4087	0.4160	0.4274	0.4022	0.3102
1.0	0.4976	0.5006	0.4932	0.4219	0.2740	0.3167	0.3185	0.3138	0.2685	0.1744
1.2	0.3624	0.3580	0.3315	0.2486	0.1295	0.2306	0.2278	0.2110	0.1582	0.0824
1.4	0.2394	0.2308	0.1967	0.1241	0.0493	0.1524	0.1468	0.1252	0.0790	0.0314
1.6	0.1325	0.1238	0.0950	0.0481	0.0139	0.0843	0.0788	0.0605	0.0306	0.0088
1.8	0.0476	0.0428	0.0289	0.0111	0.0024	0.0303	0.0273	0.0184	0.0071	0.0015
					$\alpha = 0.75$					
0.0	0.8991	0.8991	0.8991	0.8991	0.8991	0.4042	0.4042	0.4042	0.4042	0.4042
0.2	1.5086	1.5658	1.7324	1.9171	1.8952	0.3361	0.3400	0.3500	0.3576	0.3538
0.4	1.1158	1.1458	1.2075	1.1777	0.9813	0.2857	0.2911	0.3045	0.3131	0.3011
0.6	0.6987	0.7114	0.7310	0.6924	0.5659	0.2400	0.2456	0.2585	0.2615	0.2330
0.8	0.4859	0.4923	0.4981	0.4575	0.3467	0.1998	0.2036	0.2101	0.1992	0.1545
1.0	0.3515	0.3536	0.3484	0.2981	0.1936	0.1580	0.1590	0.1566	0.1340	0.0870
1.2	0.2560	0.2529	0.2342	0.1756	0.0915	0.1151	0.1137	0.1053	0.0790	0.0411
1.4	0.1691	0.1630	0.1390	0.0877	0.0348	0.0760	0.0733	0.0625	0.0394	0.0156
1.6	0.0936	0.0874	0.0671	0.0340	0.0098	0.0421	0.0393	0.0302	0.0153	0.0044
1.8	0.0336	0.0303	0.0204	0.0078	0.0017	0.0151	0.0136	0.0092	0.0035	0.0008

targets. This kind of apodisation of optical systems is a better alternative for night vision devices used in defence armament where feeble objects have to be targetted.

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