Defence Science Journal, Vol 47, No 1, January 1997, pp. 107-117

REVIEW PAPER

Estimation of Target Damage due to Submunition-Type Missile Warheads using Simulation Model/Technique

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ABSTRACT

The damage analysis of two targets due to submunition-type missile warheads has been studied. The paper discusses (a) damage to a battlefield (comprising army personnel, tanks, armoured personnel carriers and soft-skinned vehicles) due to bomblet-type warhead and (b) the denial of an airfield comprising runway tracks inclined to each other due to blast-cum-earth shock (BCES) type of warheads. Simulation technique has been used in both the cases. In addition, a mathematical model has been discussed in the second case to compare the results of the simulation model. For comparison, a particular methodology for checking the denial criterion called conventional methodology has been used. Later, a new methodology has been incorporated for checking the denial criterion in the simulation model. A mathematical formulation of the new methodology has also been given.

1. INTRODUCTION

Missiles are capable of carrying different types of warheads, viz., submunition, biological, concussion, incendiary, etc. The submunition type of warhead is mainly used against area targets, like troop concentrations, airfields, etc. The extent of damage to the targets depends on the type of the warhead and its lethal capabilities.

During a war, one of the prime objectives of the friendly- forces is to deny the enemy airfields and also to destroy his aircraft on ground. Heavy casualties to enemy troops and armoured vehicles is also an important factor in winning a war.

This paper is aimed to estimate the damage to specified ground targets due to submunition-type missile warhead, using a simulation technique. Only two types of warheads, viz., bomblet-type and blast-cum-earth shock (BCES) type are discussed in this paper.

In the first part of this paper, damage to a battlefield comprising army personnel, tank, armoured personnel carriers (APCs) and soft-skinned vehicles (SSVs) due to bomblet-type warhead[1] has been discussed. Second part discusses the denial of an airfield comprising runway tracks inclined to each other using BCES type of warheads[2,3]

2. Damage Assessment of a Typical Battlefield using Bomblet-Type Missile Warhead

The trajectories of individual launch tubes and bomblets have been computed after their respective ejection timings from the warhead. The impact points of the bomblets on ground have been considered for the determination of distribution pattern of the bomblets on ground. This has been repeated for various missile warhead velocities. It is observed that the lethal radius of the warhead and the distribution pattern of the bomblets vary with the velocities of warhead and the heights of release.

Received 05 February 1996

2.1 Computation of Trajectories

The following assumptions have been made for the computation of the trajectories of launch tubes and bomblets:

(a) The warhead is falling freely.

(b) The aerodynamic force acting on the modules and bomblets is the drag force (which includes various forces due to parachutes) acting opposite to the direction of the velocity vector.

(c) Parachute is a mass-less extension of the main body.

(d) Indian standard atmosphere, sea level condition, exist.

The origin of reference frame for the computation of the trajectories is considered to be positioned at the point of ejection of the first module. Its Y-axis is vertically downwards and the X and Z axes together form a right-handed coordinate system.

A three-dimensional point-mass trajectory model has been used for the computation of flight paths of the modules and bomblets, and the equations used for this purpose are

$$m\frac{d^2x}{dt^2} = -\frac{1}{2}\rho S C_D V^2 \cos(\theta)\cos(\varphi) \quad (2.1.1)$$

$$m\frac{d^2y}{dt^2} = -\frac{1}{2}\rho S C_D V^2 \sin(\theta) + mg \quad (2.1.2)$$

$$m\frac{d^2z}{dt^2} = -\frac{1}{2}\rho S C_D V^2 \cos(\theta)\sin(\varphi) \quad (2.1.3)$$

$$\theta = \tan^{-1}\left(\frac{dy/dt}{\sqrt{((dx/dt)^2 + (dz/dt)^2)}}\right) \quad (2.1.4)$$

$$\varphi = \tan^{-1}\left(\frac{dz/dt}{dx/dt}\right) \quad (2.1.5)$$

where

θ = Angle of elevation

φ = Angle of azimuth

C_D = Drag coefficient

ρ = Density of air

g = Acceleration due to gravity

m = Mass of the body

For the computation of the trajectories of bomblets, skin friction coefficient of the ribbon has also been taken care of.

2.2 Denial Criteria

For the estimation of the damage to the target, the following denial criteria have been considered. In this model, a person falling within the lethal radius of the warhead as well as the bomblet, has been considered as killed. In the case of tanks, APCs and SSVs, a bomblet hit on it is taken as the condition for the denial. Mathematically, the same can be described as

If (X_w, Y_w), (X_b, Y_b) and (X_p, Y_p) are the coordinates of the warhead, a typical bomblet and personnel respectively, the person is considered to be killed if the following conditions are satisfied:

$$(X_w - X_p)^2 + (Y_w - Y_p)^2 \le lrwh^2$$
$$(X_b - X_p)^2 + (Y_b - Y_p)^2 \le lrb^2 \quad (2.2.1)$$

where

lrwh = Lethal radials of the warhead

lrb = Lethal radius of the bomblet

Similarly, if (X_T, Y_T) are the coordinates of a typical tank/APC/SSV, then it is considered to be killed, if

$$\frac{(X_b - X_T)^2}{(l_T/2)^2} + \frac{(Y_b - Y_T)^2}{(b_T/2)^2} \le 1 \quad (2.2.2)$$

where l_T and b_T are respectively, the length and breadth of the tank and it is assumed that the tank is an ellipsoid.

2.3 Model

A rectangular target of $L \times B\ m^2$ on which N_p, number of personnel, N_T, tanks, N_{APC}, armoured personnel carriers and N_S, soft-skinned vehicles uniformly distributed, has been taken as the scenario. The N_p pairs of uniform random numbers within the target area have been generated to locate the personnel positions. Same method has been followed to generate the tank's, APC's and SSV's positions. These points are stored as a structured array in the computer.

The aim points to drop the missile warheads are pre-decided. Warhead makes an impact at a point which is normally distributed around the aim point. Taking these points as the warhead mean impact points, the (x_b, y_b) coordinates of bomblet positions have been generated.

To compute the damage to the target, the (x,y) coordinates of personnel, tanks, APCs and SSVs have been checked to ascertain whether the target is falling within the lethal radius of the warhead. If a person is falling within the lethal radius of the warhead as well as the bomblet, he is assumed to be killed and the counter is incremented by one and the (x,y) coordinates of that person are removed from the array. In the case of tanks, APCs and SSVs, the condition is checked for the possibility of a bomblet falling on it. If it is so, the counters are incremented by one and their (x,y) positions are removed from the array. This process is repeated for all persons, tanks, APCs and SSVs lying within the lethal radius of the warhead.

The trial is repeated n times, and the ratios (number of targets killed/total number of targets) are computed for personnel, tanks, APCs and SSVs. These ratios give the probability of denial of personnel, tanks, etc.

3. DENIAL OF RUNWAY TRACKS USING BCES-TYPE WARHEADS

BCES-type warhead, generally used against runway tracks, is capable of inflicting craters to the tracks, making them unserviceable. An airfield consisting of three tracks inclined at arbitrary angles (a main runway denoted 'RW', a carway denoted 'CW' and another runway denoted 'ARW') is considered for attack. Here, a simple layout of the airfield tracks where 'CW' makes an angle of '0°' and 'ARW' makes an angle of '90°' with the RW has been considered. The denial criterion of the airfield is that a strip of dimension $L_d \times W_d$ sufficient for an aircraft to take off in an emergency is not available on the track. A particular methodology for checking the denial criterion called conventional methodology has been used here for the comparison of simulation and mathematical model results. Later, the simulation results are modified by incorporating a new methodology for checking the denial criterion.

3.1 Conventional Methodology for Checking Denial Criterion

Consider the case of a runway of length L and width W. Certain number of areas (called DMAI's) are cut on the runway and are divided into parallel strips so that, if one bomblet falls in a strip, it is assumed to be denied[4]. Thus, if all the strips of all DMAI's of the runway are denied, the whole runway is considered to be denied. This methodology is termed as conventional methodology for checking the denial criterion. In the following sections, this methodology is first used to estimate the number of missiles required to deny the runway. Later, it is modified.

DMAI's and strips are chosen in such a way that, if each strip has one bomblet, nowhere a strip of dimension $L_d \times W_d$ will be available. Number of strips N_s of effective width W_s in a DMAI is given by[2]

$$N_s = \begin{cases} 1, & \text{if } W_d = W \\ \left(\left(\dfrac{2W}{W_d + 2r_b}\right)\right) + 1, & \text{otherwise} \end{cases} \quad (3.1)$$

where

W, W_d are the width and denial width of RW respectively, and r_b, the lethal radius of the bomblet.

3.2 Simulation Model for Missile Attack

In this section, Monte Carlo technique of simulation is used to find the number of missiles required to be dropped on the runway tracks to ascertain a specified level of damage.

Aim points are taken as the centre of DMAI's. Let (x_d, y_d) be the coordinates of one of the aim points. To find the impact point, two normal random numbers x and y are generated as[5,6]

$$x = \sqrt{-2\log(u_1)} \quad \sin(2\pi u_2)$$

and

$$y = \sqrt{-2\log(u_1)} \quad \cos(2\pi u_2)$$

where u_1 and u_2 are independent uniform random numbers in the interval (0,1). Then the coordinates of the impact point are given by

$$x_I = x_d + x\sigma_x,$$

$$y_I = y_d + y\sigma_y, \quad (3.2.1)$$

where σ_x and σ_y are the standard deviations of impact point in x and y directions respectively.

Thus, due to its circular error probability (CEP), the warhead aimed at point (x_d, y_d) has fallen on point (x_I, y_I). Assume that the warhead contains n_b number of bomblets, each of lethal radius r_b which, after detonation, are distributed uniformly within a circle centred at (x_I, y_I), and of radius R_{wh}, which is called the lethal radius of the warhead. To generate the (x_i, y_i) coordinates of the i^{th} bomblet, take a pair of independent uniform random numbers (v_1, v_2) from different streams of random numbers between 0 and 1 and put

$$x_i = (x_I - R_{wh}) + (2R_{wh})v \quad \&$$

$$y_i = (y_I - R_{wh}) + (2R_{wh})v_2$$

The condition for the bomblet to lie within the lethal circle is given as

$$\sqrt{(x_i - x_I)^2 + (y_i - y_I)^2} \le (R_{wh} - r_b) \quad (3.2.2)$$

If this condition is not satisfied, go on generating different pairs of (x_i, y_i) till the condition is satisfied.

Knowing the position of all the bomblets, it is ascertained that each strip of width W_s has at least one bomblet. If all DMAI's are denied, the experiment is a success, otherwise it is a failure. Trial is repeated say 1000 times and the probability of denial is calculated as the ratio of the number of successes to the number of trials. To ascertain the correct probability of denial, probability has been calculated n times (say 15 times) and the actual probability of denial has been obtained as the average of these n probabilities.

In the next section, a mathematical model is presented for the comparison of simulation model.

3.3 Mathematical Model

In this section, a mathematical model has been presented for comparison with the simulation model proposed in Section 3.2. In the mathematical model, the old methodology for denial criterion has been used. It is assumed that, if the results of mathematical and simulation models agree for the old methodology, it will hold good for the new methodology (Section 3.6) too.

At first, consider the case of a single DMAI. Let this DMAI (say i^{th}) be divided into N_s^i number of strips. Then

L_i = Length of i^{th} DMPI

W_i = Width of the i^{th} DMPI

L_i^k = Length of the k^{th} strip of i^{th} DMPI

W_i^k = Width of the k^{th} strip of i^{th} DMPI

According to the old methodology described in Section 3.1, a DMAI is considered as denied if each of its strips simultaneously has at least one bomblet. If $E_i^{s,k}$ is defined as the event that k^{th} strip of the i^{th} DMAI is denied, then the probability of denial of whole DMAI is the probability that all the strips of the DMAI are denied, i.e.

$$P_i^d = P\left(\bigcap_{k=1}^{N_s^i} E_i^{s,k}\right) \quad (3.3.1)$$

Using the additive law of probabilities, one gets

$$P(E_i^{s,k} \cap E_i^{s,l}) = P(E_i^{s,k}) + P(E_i^{s,l}) - P(E_i^{s,k} \cup E_i^{s,l}) \quad (3.3.2)$$

Equation (3.3.2) is substituted in Eqn. (3.3.1) for all the strip combinations and a generalised equation involving probabilities of events and their unions is obtained. To find these probabilities, the expected number of bomblets falling on the combination of strips taken one, two, ... N_s^i at a time is to be evaluated. Knowing the expected number of bomblets on a typical area, the probability of at least one bomblet falling on it can be evaluated by Poisson's distribution law.

3.4 Expected Number of Bomblets over Specific Area

First of all, the coverage of individual strips of any DMAI when one warhead is aimed at any other DMAI, has to be evaluated. Define $C^k_{i,j}$ as the coverage of k^{th} strip of i^{th} DMAI, when a missile warhead is dropped at the centre of j^{th} DMAI. The expression for $C^k_{i,j}$ is given by[7]

$$C^k_{i,j} = \frac{1}{A^k_i} \int A^k_i \int P(Rwh, tj)\,dx\,dy$$

where the integral is taken over A^k_i which is the area of k^{th} strip of i^{th} DMAI and $P(R_{wh}, t_j)$ is the circular coverage function given by

$$P(R_{wh}, t_j) = \frac{1}{\pi}\, e^{-\left(\frac{1}{2}(x-\xi)^2 + (y-\eta)^2\right)}$$

$$\int_0^{R_{wh}} \int_0^{\pi} e^{-\frac{1}{2}\left(r^2 + 2r\sqrt{(x-\xi)^2 + (y-\eta)^2}\cos\theta\right)} r\,dr\,d\theta$$

R_{wh} = lethal radius of the warhead.

$t_j = \sqrt{(x-\xi)^2 + (y-\eta)^2}$, is the distance of an arbitrary point (x,y) of the target from the aim point $P_j(\xi,\eta)$ where P_j is the centre of the j^{th} DMAI.

Average area covered by one bomblet of the missile is

$$A_{av} = \frac{\pi (R_{wh})^2}{n_b}$$

where

n_b = Number of bomblets in one warhead, distributed uniformly within its lethal radius.

Thus the expected number of bomblets falling on k^{th} strip of i^{th} DMAI when one warhead is dropped on j^{th} DMPI is given by

$$n^k_{ij} = \frac{L^k_i \times W^k_i}{A_{av}} \times C^k_{ij} \qquad (3.4.1)$$

As a corollary of the above relation, the number of bomblets falling on k^{th} strip of i^{th} DMAI due to all DMAI's, when n_j warheads are dropped at j^{th} DMAI is

$$n^k_i = \sum_{j=i}^{N_d} \left(n_j \times n^k_{i,j}\right) \qquad (3.4.2)$$

where N_d is the total number of DMPI's.

Similarly, it is shown in the succeeding sub-sections that the expected number of bomblets falling on the union of strips is the sum of the expected bomblets falling on the individual strips.

3.5 Probability of Denial of Complete Airfield

Let the airfield tracks have N_d (N_d = 8 in this case) number of DMAI's, each DMAI divided into N^i_s number of strips. Thus there are in all $N_s = \sum_i^{N_d} N^i_s$ strips, irrespective of the DMAI to which they belong. Following the concept of addition of expected number of bomblets, $n^{k,l}_{i,j}$, $n^{k,l,m}_{i,j}$ and $n^{k,l,m}_{i,j}$..... are defined as the average number of bomblets falling on the union of (k & l)th, (k & l & m)th.... strips of i^{th} DMPI when one warhead is dropped at j^{th} DMPI. Then

$$n^{k,l}_{i;j} = n^k_{i;j} + n^l_{i;j}$$

$$n^{k,l,m}_{i;j} = n^k_{i;j} + n^l_{i;j} + n^m_{i;j} \qquad (3.5.1)$$

Using Eqn (3.4.2), the average number of bomblets falling on the union of (k & l)th, (k & l & m)th ...... strips of i^{th} DMPI when n_j warheads are dropped at j^{th} DMPI can be calculated.

Similarly, the number of bombs falling on the combination of any number of strips is nothing but the sum of the bomblets falling on the individual strips.

Let $E^{s;klm....}_{pqr....j}$ be the event that at least one of the strips out of k^{th} strip of p^{th}, l^{th} strip of q^{th} and so on DMPI, due to a warhead dropped at j^{th} DMPI is occupied. The probability of occurrence of this event is defined by

$$P^{s;klm}_{pqr..j} = P\left(E^{s;klm}_{pqr..j}\right) \qquad (3.5.2)$$

Thus the probability that at least one of the strips out of k^{th} strip of p^{th} DMPI, l^{th} strip of q^{th} DMPI and so on is occupied due to a bomb dropped at j^{th} DMPI is given by

$$P_{pqr..;j}^{s;klm} = 1 - e^{-n_{pqr...j}^{klm...}} \qquad (3.5.3)$$

where

$$n_{pqr..;j}^{klm} = n_{p;j}^{k} + n_{r;j}^{m} + ..$$

Similarly, the probability that union of (k, l, m, ...) strips of p^{th} DMPI has at least one bomblet, when warhead is dropped at j^{th} DMPI is

$$P_{p;j}^{klm..} = 1 - e^{-n_{p;j}^{klm...}} \qquad (3.5.4)$$

Thus, if P_i^d is the probability that all the strips of i^{th} DMPI are occupied due to n_1, n_2 .... n_8 warheads dropped respectively at P_1, P_2 .... P_8 DMPI's, then

$$P_d^i = \sum_{k} P(E_i^{s;k}) - \sum_{k,l} P(E_i^{s;k} \cup E_i^{s;l}) + \sum_{k,l,m} P(E_i^{s;k} \cup E_i^{s;l} \cup E_i^{s;m}) - \sum_{k,l,m,n} P(E_i^{s;k} \cup E_i^{s;l} \cup E_i^{s;m} \cup E_i^{s;n}) \qquad (3.5.5)$$

where k,l,m,n stand for strip numbers of i^{th} DMPI.

Probability, P of total runway denial is the probability that all DMAIs are denied. Thus, if P is the probability of occurrence of event E_i^d, i.e., denial of i^{th} DMPI due to n_j warheads dropped at j^{th} DMPI, then the total probability that all the DMPI's are simultaneously denied is

$$P = P\left(\bigcap_{i=1}^{8} E_i^d\right) \qquad (3.5.6)$$

When intersection is converted to union, one

$$P = \sum_{i=1}^{8} P(E_i^d) - \sum_{i,j} P(E_i^d \cup E_j^d) + \sum_{i,j,k} P(E_i^d \cup E_j^d \cup E_k^d) \qquad (3.5.7)$$

Probability, P is the level of assurance with which runway can be denied be n_j warheads dropped at j^{th} DMPI, where $j \pm 1$ to 8. If this level of assurance is less than the stipulated level of assurance, n_j can be increased, on a DMPI, on which probability of denial is low.

Equation (3.5.7) can be written in a simplified way for computation. The condition of denial of whole runway is that all the DMPI's should be denied, which in turn, means that all the strips should have at least one bomblet due to n_1, n_2 ..n_8 warheads simultaneously dropped at P_1, P_2..... P_8 DMPI's. Thus Eqn (3.5.7) can be written in the form

$$P = \sum_{k=1}^{Ns} P(E^{s;k}) - \sum_{\substack{k,l=1 \\ k \neq 2}}^{N_s} P(E^{s;k} \cup E^{s;l}) + \sum_{\substack{k,l,m=1 \\ k \neq l, k \neq m}}^{N_s} P(E^{s;k} \cup E^{s;l} \cup E^{s;m}) \qquad (3.5.8)$$

Here the identification of strip by DMPI number has been dropped.

3.6 Modified Methodology for Checking Denial Criterion

It is observed that the old methodology of checking the denial condition is sufficient but not always necessary. It can be seen that, in some of the cases, even if a strip does not have a bomblet, the distance between two bomblets in neighbouring strips is less than W_d. Keeping this in mind conventional methodology is modified. In the following sections, determination of the aim points has been explained and then a mathematical formulation of the methodology has been presented[3].

3.7 Determination of Aim Points (DMPI's)

Considering the case of runway, let the runway be divided into N_p number of sections given by

$$N_p = \begin{cases} Int\,(L/L_d) + 1, \text{if } remainder \neq 0 \\ L/L_d, \; otherwise \end{cases} \quad (3.7.1)$$

where L and L_d are the length of runway and the denial length, respectively. Thus the length L_p of each section is given by

$$Lp = \frac{L}{N} \quad (3.7.2)$$

and

$$N_{dmpi} = N_p - 1$$

where N_{dmpi} is the number of aim points which are in the middle of corresponding two sections. These points are the aim points for the missile warheads. Due to errors in landing, let these warheads fall at two extreme ends at a distance 3σ from the aim point. Thus if L_f is the free-length available in a particular strip, then

$$L_f = L_p - 2R_{wh} + 6\sigma \quad (3.7.3)$$

Since the criterion for the runway denial of each runway is that nowhere an area of dimensions L_d x W_d should be available for the runway to be denied,

$$6\sigma < L_f < L_d \quad (3.7.4)$$

If this condition is not true, then the number of sections is increased by one.

Equation (3.1.1) gives the number of strips in which full runway is divided. Similarly, the aim points and strips on other tracks also can be determined. Runway, carway and auxiliary runway are attacked by dropping a desired number of missile warheads on each of these DMPI's.

After attacking the airfield with missile warheads the position of each bomblet is simulated. Then each bomblet is checked whether it falls on runway, carway or auxilliary runway. After finding the simulated position of each bomblet, it is found that on which strip of the tracks bomblets fall. The strips are numbered from top to bottom and on each strip the bomblets are arranged in the increasing order of their x-coordinates. The methodology for checking the denial criterion is described here.

3.8 Mathematical Formulation of the Methodology

Consider the case of a runway. After a desired number of warheads are dropped on the runway, the position of each bomblet on the runway is found and that the RW is denied or not is ascertained. Let (rwax, rway) and (rwcx, rwcy) be the respective left-top end and right-bottom end of the runway.

Let the runway be divided into n number of strips. Strips are numbered from top to bottom. X-Y coordinate system is chosen; such that positive Y-axis is down towards the bottom of RW and RW is taken in the first quadrant. For all the strips j, let mj be the total number of bomblets falling on j^{th} strip (Fig.1). Let (x_i^j, y_i^j) be the position of i^{th} bomblet in j^{th} strip for i = 1, 2, ...mj.

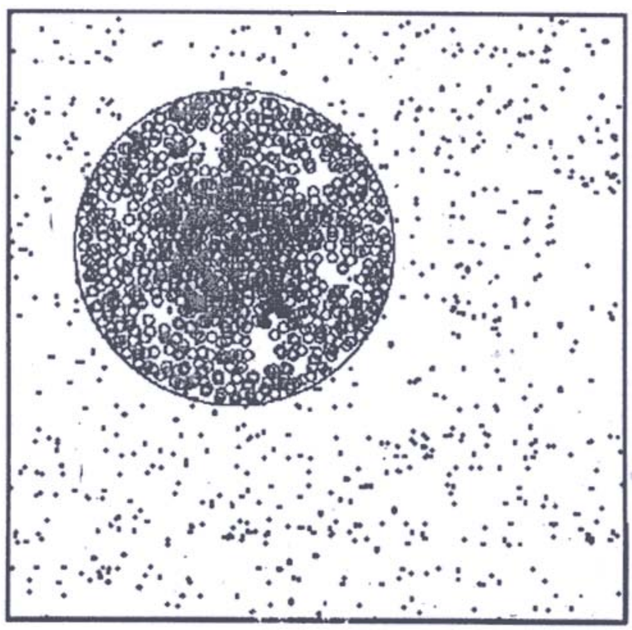

Figure 1. Bomblet-type warhead dropped on an area target

Put x_o^j = rwax and x_{mj+1}^j = rwcx, for all j.

For all j define the set B_j = {1, 2, 3.... mj, mj+1}

Now for all $i \in B_j$, j = 1, 2, ... n, define the pair (x_l^j, x_r^j) as follows

If $m_j \neq 0$, define

$$x_i^j = \begin{cases} x_{i-1}^j, \text{ if } i = 1 \\ x_{i-1}^j + r_b, \text{ if } 1 < i \leq mj + 1 \end{cases}$$

$$x_r^j = \begin{cases} x_i^j, \text{ if } i = mj + 1 \\ x_i^j - r_b, \; i < mj + 1 \end{cases}$$

if mj = 0, define

x_i^j = rwax and x_r^j = rwcx

Let j ∈ J, where J = {1,2, ..., n}

for i = 1, check the inequality

$$x_r^j - x_i^j \geq L_d \qquad (3.8.1)$$

If Eqn (3.8.1) does not hold, we say the trial is a success for i^{th} bomblet on j^{th} strip, and replace x_i^j and x_r^j in (3.8.1) for the next i and continue the process. Otherwise,

if n = 1, trial is a failure on the runway

if j = 1, put yt = rwax and xyt = rwax

if j = n, put yb = rwcy and xyb = rwax

if $j \neq 1$, define the set

$$U = y_i^{j-1}, x_i^j < x_i^{j-1} < x_i^j + L_d$$

if $j \neq n$, define the set

$$L = \{ y_i^{j+1}, x_i^j < x_i^{j+1} < x_i^j + L_d$$

If U or L is empty, trial is a failure on the runway (or runway is not denied). Otherwise,

Let y_{i0}^{j-1} = maximum of the set U and

y_{i1}^{j+1} = minimum of the set L

Put yt = $y_{i0}^{j-1} + r_b$ and xyt = $x_{i0}^{j-1} + r_b$

yb = $y_{i1}^{j+1} - r_b$ and xyb = $x_{i1}^{j+1} + r_b$

Now check for yb - yt ≥ W_d (3.8.2)

If Eqn (3.8.2) holds, the trial is a failure on the runway.

Otherwise, replace x_i^j in (3.8.1) by

$$x_i^j = \begin{cases} max(xyt, xyb), \text{ if } j=1 \text{ or } j=n \\ min(xyt, xyb), \text{ otherwise} \end{cases}$$

and continue the process.

If the trial is a success for all i = 1,2, ... mj, mj + 1, on j, we say trial is a success on the strip j. If the trial is a success on all the strips, it is a success on the runway, i.e. the runway is denied. Similarly, the denial of other tracks can be determined. (Fig.2).

4. DATA USED

4.1 Bomblet-Type Warhead

Length of the target	: 1000 m
Breadth of the target	: 1000 m
Number of bomblets per warhead	: 1150
Number of persons per km^2	: 720
Number of tanks per km^2	: 52 m
Number of armoured personnel carriers per km^2	: 42
Number of soft-skinned vehicles per km^2	: 62
CEP of warhead	: 100 m

4.2 BCES-Type Warhead

Runway dimensions	: Length	= 3100 m,
	Breadth	= 50 m
Carway dimensions	: Length	= 3100 m,
	Breadth	= 25 m
Auxiliary runway dimensions	: Length	= 2100 m,
	breadth	= 50 m
Denial parameters	: Denial length	= 1000 m,
	Denial width	= 25 m
CEP of the warhead		: 150 m
Lethal radius of the warhead		: 250 m
Number of bomblets n_b		: 32
Lethal radius of the bomblet		: 3.2 m

5. RESULTS & CONCLUSION

Table 1 gives the kill probabilities of a typical battlefield comprising personnel, tanks, APCs and SSVs, due to bomblet-type warheads. The mathematical model of Section 3.3 is quite generalised and takes into account any number of DMPI's. In

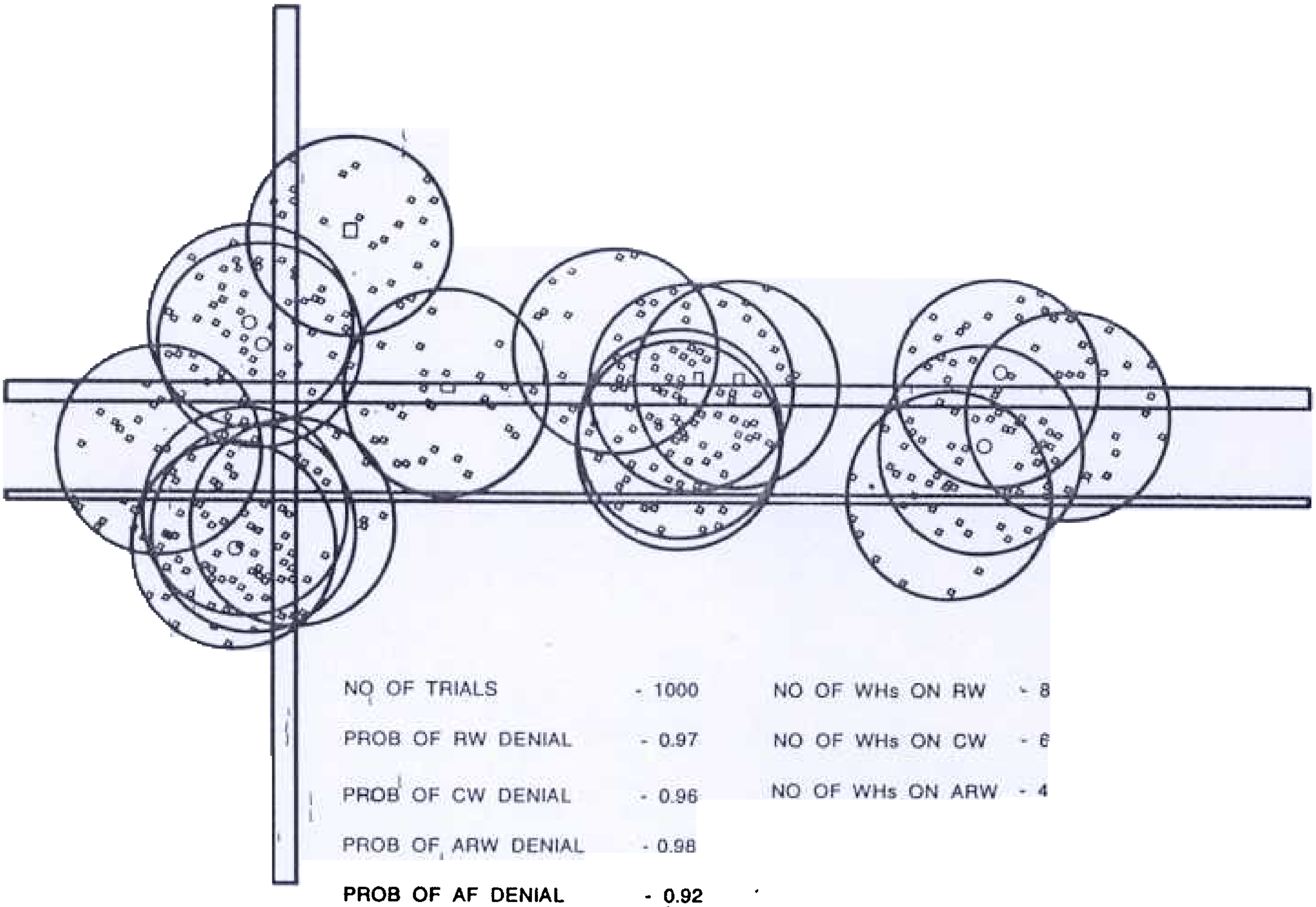

Figure 2. Simulation of a typical trial

the case of old methodology for checking the denial criteria, the results of simulation when different number of missile warheads are dropped on different DMPI's have been compared with those obtained by mathematical model. A good agreement by both the methods has been observed.

With the data given in Section 4 and using the old methodology for denial criteria, the simulation model has shown that 48 warheads are required to be dropped on the airfield to achieve a denial probability of 90 per cent (Table 2). The mathematical model, when similar number of missiles are dropped, also gives 93 per cent probability of denial. By taking into consideration the modified methodology for checking the denial criteria, the number of missiles required is much less, viz., 18. If mid-bombing method is used (dropping warheads on DMPI's located in between RW and CW and at the crossings of the tracks), the number of warheads required for 90 per cent denial probability is still less, viz., 17 (Table 3). Figures

Table 1. Kill probabilities due to bomblet-type warhead

	One warhead dropped at the centre of four sectors of the target	One warhead dropped at the centre of the target
Personnel	0.560	0.160
Tank	0.070	0.018
APC	0.040	0.012
SSV	0.060	0.013

1 and 2 give computer outputs of simulation results using bomblet-type warheads and BCES-type warheads, respectively.

Table 2. Comparison of probabilities of denial of airfield (old methodology) by simulation and mathematical models

No. of warheads on three DMPI's on RW	No. of warheads on three DMPI's on CW	No. of warheads on two DMPI's on ARW	Probabilities of denial	
			Simulation model	Mathematical model
1,1,1	1,1,1	1,1	0.0000	0.0000
2,2,2	2,2,2	2,2	0.0145	0.0160
3,3,3	3,3,3	3,3	0.1604	0.1874
4,4,4	4,4,4	4,4	0.4195	0.4786
5,5,5	5,5	5,5	0.6612	0.7119
6,6,6	6,6,6	6,6	0.8211	0.8502
10,13,13	1,0,0	6,5	0.9045	0.9339

Table 3. Probabilities of denial of airfield using simulation (new methodology)

No. of warheads on RW DMPI's	No. of warheads on CW DMPI's	No. of warheads on ARW DMPI's	No. of warheads on MID DMPI's	Probabilities of denial
1,1,1	1,1,1	1,1	0,0,0	0.19
2,2,2	2,2,2	2,2	0,0,0	0.82
3,3,3	3,3,3	3,3	0,0,0	0.98
2,3,3	2,2,2	2,2	0,0,0	0.90
0,0,0	0,0,0	4,4	0,5,5	0.94
0,0,0	0,0,0	4,4	0,4,5	0.91

ACKNOWLEDGEMENT

The authors are thankful to Dr N.K. Srinivasan, Director, CASSA, Bangalore, for giving permission to publish this paper.

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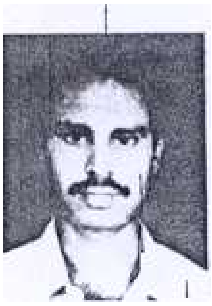

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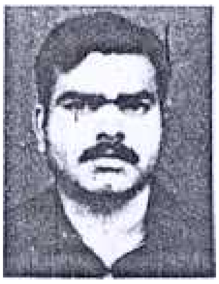

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