

## New Method For Calculating The Input Impedance of Rectangular Patch Antenna

R. K. Mishra, G. K. Patra, A. Patnaik and S. K. Dash  
 Berhampur University, Berhampur - 760 007.

### ABSTRACT

The cavity model has been modified to account for the impedance boundary condition at the edges of a rectangular microstrip antenna. Results have been compared with those from the old cavity model and experimental data.

### 1. INTRODUCTION

Cavity model being used to find the input impedance of a rectangular patch antenna, treats the antenna as a cavity with ideal magnetic walls at the boundary<sup>1,2</sup>. However, the boundary is not ideal and possesses finite impedance. In practice, power is lost from the cavity formed by microstrip patch due to radiation from the fringing fields at the periphery. Effects of fringing fields are accounted for by extending the boundary. In the present work, the impedance boundary condition,  $\vec{H} = Y_w (\hat{n} \times \vec{E})$ , suggested by Carver<sup>2</sup> is used to account for radiation. The wall admittance,  $Y_w$  will, in general, have different values on the walls along Y-axis than on those along X-axis. With these assumptions the cavity model has been modified and the results of the proposed model are compared with the original cavity model for a rectangular patch antenna.

### 2. THEORY

Figure 1 shows a rectangular patch antenna of length  $l$ , width  $w$ , on a substrate of thickness  $h$  and dielectric constant  $\epsilon_r$ . For electrically thin substrate

(i.e.  $h \ll \lambda_0$ ) the Z-directed electric field will be independent of Z, under the patch. The modes will be  $TM_{mn}$  so that

$$E_z(x, y) = \sum_m \sum_n A_{mn} e_{mn}(x, y) \quad (1)$$

where,  $A_{mn}$  are the mode amplitude coefficient and  $e_{mn}$  are the orthonormal electric field mode vectors, expressed as

$$e_{mn}(x, y) = \chi_{mn} (e^{-jK_m x} + R_m e^{jK_m x}) (e^{-jK_n y} + R_n e^{jK_n y}) \quad (2)$$

where  $R_m$  and  $R_n$  are the reflection coefficients along the boundaries parallel to X and Y-axes, respectively. Using normalisation condition for  $e_{mn}(x, y)$ , it is found that

$$\chi_{mn} = \frac{\delta_m \delta_n}{2\sqrt{lw}} \quad (3)$$

$$\delta_p = \left( 1 + \cos p\pi \times \frac{\sin k_p x}{k_p x} \right)^{-1/2} \quad (4)$$

with  $p = m$  when  $x = w$  and  $p = n$  when  $x =$

$$k_m = \frac{m\pi}{w_e} \text{ and } k_n = \frac{n\pi}{l_e}$$

where  $l_e$  and  $w_e$  are the effective length and width, respectively due to the fringing field<sup>3</sup>, expressed as

$$l_{eq} = l + 2\delta_l$$

$$\delta_l = 0.412h \frac{(\epsilon_{eff} + 0.3)(\frac{w}{h} + 0.264)}{(\epsilon_{eff} - 0.258)(\frac{w}{h} + 0.813)}$$

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{10h}{w}\right)^{-1/2}$$

By interchanging  $l$  with  $w$  in the above equation, the effective width can be calculated.

## 2.1 Determination of Reflection Coefficient

Maxwell's equation states

$$\vec{H} = \frac{j}{\omega\mu_0} \nabla \times \vec{E} = \frac{j}{\omega\mu_0} \left( \hat{x} \frac{\partial E_z}{\partial y} - \hat{y} \frac{\partial E_z}{\partial x} \right) \quad (5)$$

From this equation, the  $\vec{H}$  field on the

(i) Y-surface at  $y = 0$  and  $y = 1$  will be of the form

$$H_y = \frac{-j}{\omega\mu_0} \frac{\partial E_z}{\partial x} \quad (6)$$

(ii) X-surface at  $x = 0$  and  $x = w$  will be of the form

$$H_x = \frac{-j}{\omega\mu_0} \frac{\partial E_z}{\partial y} \quad (7)$$

The impedance boundary condition on these surfaces is

$$\vec{H} = Y_w (\hat{n} \times \vec{E}) \quad (8)$$

For the above equation

$$\begin{aligned} \text{at } y=0, \hat{n} &= -\hat{y}; \quad y=1, \hat{n} = -\hat{y} \\ x=0, \hat{n} &= -\hat{x}; \quad x=w, \hat{n} = -\hat{x} \end{aligned}$$

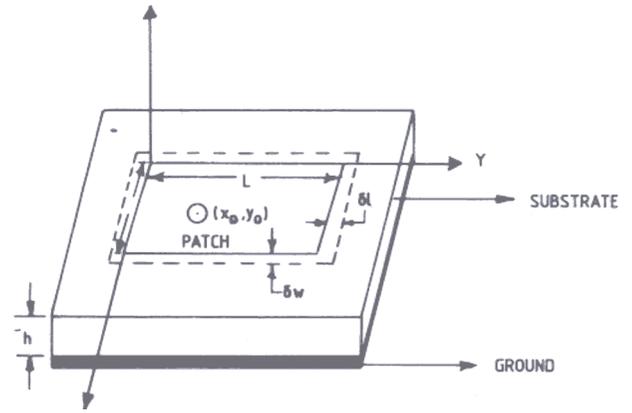


Figure 1. Coax-fed microstrip antenna with coordinate system used.

Equating the  $mn^{th}$  terms in the series for  $H_x$  from Eqns (7) and (8) at  $y = 0$  and  $y = 1$ , one gets

$$\frac{k_n}{\omega\mu_0} [1 - R_n] = -Y_w [1 + R_n] \quad (9)$$

$$\frac{k_n}{\omega\mu_0} [e^{-jk_n l} - R_n e^{jk_n l}] = Y_w [e^{-jk_n l} + R_n e^{jk_n l}] \quad (10)$$

Dividing Eqn (10) by Eqn (9) and solving for  $R_n$ , one gets

$$R_n = e^{-jk_n l} \quad (11)$$

Similarly,

$$R_m = e^{-jk_m w} \quad (12)$$

## 2.2 Determination of Modal Coefficient

For a coaxial feed, considering the effect of Z-directed current  $I_0$  on the probe of small circular cross-section of diameter  $d$  at  $(x_0, y_0)$  the coefficient of each electric mode vector are

$$A_{mn} = \frac{j \omega \mu_0}{k_d^2 - k_{mn}^2} \iiint J \cdot e_{mn}^* dv \quad (13)$$

Here,  $J$  is the current density. To account for the losses, it is assumed that  $k_d^2 = \omega^2 \mu_0 \epsilon_{eff} (1 - j\delta_{eff})$  where  $\delta_{eff}$  is the effective loss tangent.

For compatibility in definitions, we also take  $k_{mn}^2 = \omega_{mn}^2 \mu_0 \epsilon_0 \epsilon_{eff} = k_m^2 + k_n^2$ .

Using basic calculus with simple algebraic manipulations, Eqn (13) reduces to

$$A_{mn} = \frac{j\omega I_0}{\epsilon_0 \epsilon_{eff} [\omega^2 (1 - j\delta_{eff}) - \omega_{mn}^2]} e_{mn}^*(x_0, y_0) \times G_{mn} \quad (14)$$

with

$$G_{mn} = \frac{\sin(k_m d/2)}{\dots} \times \frac{\sin(k_n d/2)}{\dots} \quad (15)$$

and  $I_0$  is the Z-directed current on the feeding probe.

### 2.3 Determination of Input Impedance

Now by substituting Eqn (14) in Eqn (1)

$$E_z(x, y) = \frac{j\omega I_0}{\epsilon_0 \epsilon_{eff}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{e_{mn}(x, y) \times e_{mn}^*(x_0, y_0)}{\omega^2 (1 - j\delta_{eff}) - \omega_{mn}^2} \times G_{mn}$$

The voltage at the feed point is given by

$$V_{in} = - \int_0^h E_z dz = -h E_z(x_0, y_0) \quad (17)$$

$$\frac{-j\omega I_0 h}{\epsilon_0 \epsilon_{eff}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{e_{mn}(x, y) \times e_{mn}^*(x_0, y_0)}{\omega^2 (1 - j\delta_{eff}) - \omega_{mn}^2} \times G_{mn} \quad (18)$$

Using the expressions for  $R_m$  and  $R_n$ , the input impedance is calculated as

$$Z_{in} = \frac{j16\omega h}{\epsilon_0 \epsilon_{eff}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{|\chi_{mn}|^2 \cos^2 \left[ \frac{m\pi}{w_e} (x_0 - \frac{w}{2}) \right] \cos^2 \left[ \frac{n\pi}{l_e} (y_0 - \frac{l}{2}) \right]}{\omega_{mn}^2 - (1 - j\delta_{eff}) \omega^2} \quad (19)$$

The resonant resistance in the dominant  $TM_{01}$  mode is

$$R_r = \left| \frac{2hG_{01}}{\dots} \right| \left\{ \cos \frac{\pi(y_0 - \frac{l}{2})}{l_e} \right\}^2 \times \left\{ 1 - \frac{\sin(\frac{\pi l}{l_e})}{\frac{\pi l}{l_e}} \right\}^{-1} \quad (20)$$

### 3. RESULTS & DISCUSSION

The resonant resistance is calculated using both old cavity model and the proposed cavity model. The results are compared with the measured values<sup>4</sup> in Table 1. The present method gives accurate results for the resonant resistance in comparison to the old method in all the cases. Resonant resistance are mostly within an error of 6 per cent. The observed discrepancy can be explained in the following manner:

When the physical dimensions of the antenna are small, the tolerance effect is high<sup>5</sup>, i.e., the feed

**Table 1. Comparison of input resistance (experimental and theoretical results) of printed rectangular patches for both old and proposed methods**

$\epsilon_r$	$h$ (mm)	$l$ (mm)	$w$ (mm)	$Y_0$ (mm)	$d$ (mm)	Measured <sup>4</sup>		Old model <sup>2</sup>			Proposed model		
						$f_r$ (GHz)	$R_r$ ( $\Omega$ )	$f_r$ (GHz)	$R_r$ ( $\Omega$ )	Error (%)	$f_r$ (GHz)	$R_r$ ( $\Omega$ )	Error (%)
10.2	1.27			6.5	1.19	2.26	335	343	+2.33	2.31	-1.79		
10.2	1.27			3.2	1.19	4.43	339	389	+14.75	4.49	+5.84		
10.2	2.54			6.5	2.38	2.18	363	394	+8.50	2.29	+0.08		
2.22	0.79			4.0	2.42	3.92	136	136	0.0	3.92	-0.59		
2.22	0.79			2.0	2.42	7.56	152	153	+0.65	7.61	-0.39		
2.22	1.52			4.0	4.66	3.82	119	153	+28.57	3.82	+27.2		
2.22	1.52			2.0	4.66	7.72	69	147	+113.0	7.55	+108.0		

position may be slightly offset giving an H-plane error. This can result in excitation of stronger higher order modes. These higher order modes are responsible for shift in resonant resistances in all the cases, and the shifts are prominent as the physical dimensions are reduced increasing the H-plane error.

For antennas with dielectric substrate which are electrically thick and/or have high dielectric constant, excitation of surface wave is strong. This is also responsible for higher discrepancies between the theoretical and the experimental results.

#### ACKNOWLEDGEMENT

This work is done as a part of the project sponsored by the Deptt. of Electronics, Government of India. The authors express their gratitude to the sponsor.

#### REFERENCES

1. LO, Y. T.; Solmon, D. & Richard, W. F. Theory and experiments on microstrip antennas. *IEEE Trans. Antennas Propag.*, 1979, **27** (2), 137-45.
2. Carver, K. R. Practical analytical techniques for the microstrip antenna. Proceedings of the Workshop on Printed Circuit Antenna Technique, New Mexico State University, Las Cruces, October 1979, pp. 7/1-20.
3. Bhartia, P.; Rao, K.V.S. & Tomar, R.S. Millimeter wave microstrip and printed circuit antennas. Artech House Inc., Norwood, 1991.
4. Schaubert, D.; Pozar, D. & Adrian, A. Effect of microstrip antenna substrate thickness and permittivity: comparison of theories with experiment. *IEEE Trans. Antennas Propag.*, 1989, **37**, 677-82.
5. Mishra, R.K. & Milligan, T. Cross-polarisation tolerance requirements of square microstrip patches. *IEEE Antennas Propag. Mag.*, 1996, **38** (2), 56-58.

#### Contributors



**Dr RK Mishra** obtained his PhD from University College of Engineering, Burla, Sambalpur University, in 1992. He has been working as lecturer at the Department of Electronic Science, Berhampur University, Orissa, since 1991. Presently, he is executing a major technology development project sponsored by the Department of Electronics. He has published more than 25 research papers in national/international journals. His areas of research include microwaves, computer-aided design in microwaves, application of artificial neural networks in microwaves, active and passive patch antennas in free space and plasma medium. He is a life member of the Indian Society of Telecommunication Engineers (India), a member of the Institute of Electronics and Telecommunication Engineers and IEEE (USA).

**Mr GK Patra** obtained his MSc (Electronics) and post-graduate diploma in computer applications (PGDCA) from Berhampur University in 1994 and 1995, respectively. Presently, he is working as Scientist at the National Aerospace Laboratory, Bangalore. His areas of research include development of computer-aided design models for patch antennas.



**Mr A Patnaik** obtained his MSc (Electronic) and post-graduate diploma in computer applications (PGDCA) from Berhampur University in 1993 and 1994, respectively. He has been working as a research fellow at the Department of Electronic Science, Berhampur University since 1995. His areas of research include application of artificial neural network to microwaves, particularly to patch antennas and computer-aided design for patch antennas. He has published more than eight research papers in national/international journals. He is a student member of IEEE (USA).



**Mr SK Dash** obtained his MSc (Physics) from the Utkal University, in 1991. Presently, he is working at the Department of Electronic Science, Berhampur University for his PhD. His areas of research include patch antennas in free space and plasma medium. He is a life member of the Indian Society of Telecommunication Engineers.