Defence Science Journal, Vol 48, No 3, July 1998, pp. 277-285


# Prediction of Fatigue Life of Gear Subjected to Varying Loads

D. Hanumanna

*Combat Vehicles Research & Development Establishment, Chennai* - 600 054.

and

S. Narayanan and S. Krishnamurthy

*Indian Institute of Technology, Chennai* - 600 036.

**ABSTRACT**

Structural members and components of a vehicle during service are subjected to varying loads which are random in nature. For structural members subjected to loads of constant amplitude, it is possible to describe the load with explicit mathematical relationship, and thereby, the life span can be estimated. Whereas, for structural members subjected to varying loads with time, there is no satisfactory method to estimate their life span. This paper describes a method for the estimation of life span of a gear in the gear box of a fighting vehicle subjected to fluctuating loads. For this purpose, it is assumed that the load spectrum corresponds to Gaussian (normal) distribution, and the life has been worked out by applying linear cumulative damage theory.

## NOMENCLATURE

| | |
|---|---|
| $H_i$ | Number of exceedings of prefixed stress level, $\sigma_i$ |
| $\sigma_i$ | Prefixed stress level |
| $\sigma$ | Normal stress |
| $\sigma_{max}$ | Maximum stress level |
| $\sigma_{min}$ | Minimum stress level |
| $\sigma_{max_i}$ | Maximum prefixed stress level |
| $\sigma_{min\,i}$ | Minimum prefixed stress level |
| $\overline{H}$ | Size of frequency spectrum as given by the total number of cumulative cycles |
| $\overline{\sigma}_{max}$ | *Characteristic maximum stress value |
| $\overline{\sigma}_{min}$ | *Characteristic minimum stress value |
| $\overline{\sigma}_o$ | *Characteristic stress amplitude |
| $\overline{R}$ | *Characteristic stress ratio |
| $p,q$ | Factors describing the type of distribution |
| $h_i$ | Number of cycles at each stress level as per standard spectrum |
| $\sigma_o$ | Failure stress at $10^3$ cycles |
| $\sigma_e$ | Failure stress at $10^6$ cycles |
| $D$ | Damage |
| $S$ | Stress amplitude |
| $N$ | Life in number of cycles |
| $Z_1$-$Z_9$ | Gear numbers |

## 1. INTRODUCTION

An armoured fighting vehicle (AFV) has to negotiate different terrains in the actual field conditions. The systems fitted in such a vehicle are therefore subjected to fluctuating loads that are

Received 20 November 1995, revised 05 December 1997

* These are the values which are exceeded by a relative frequency of once in a million cycles.

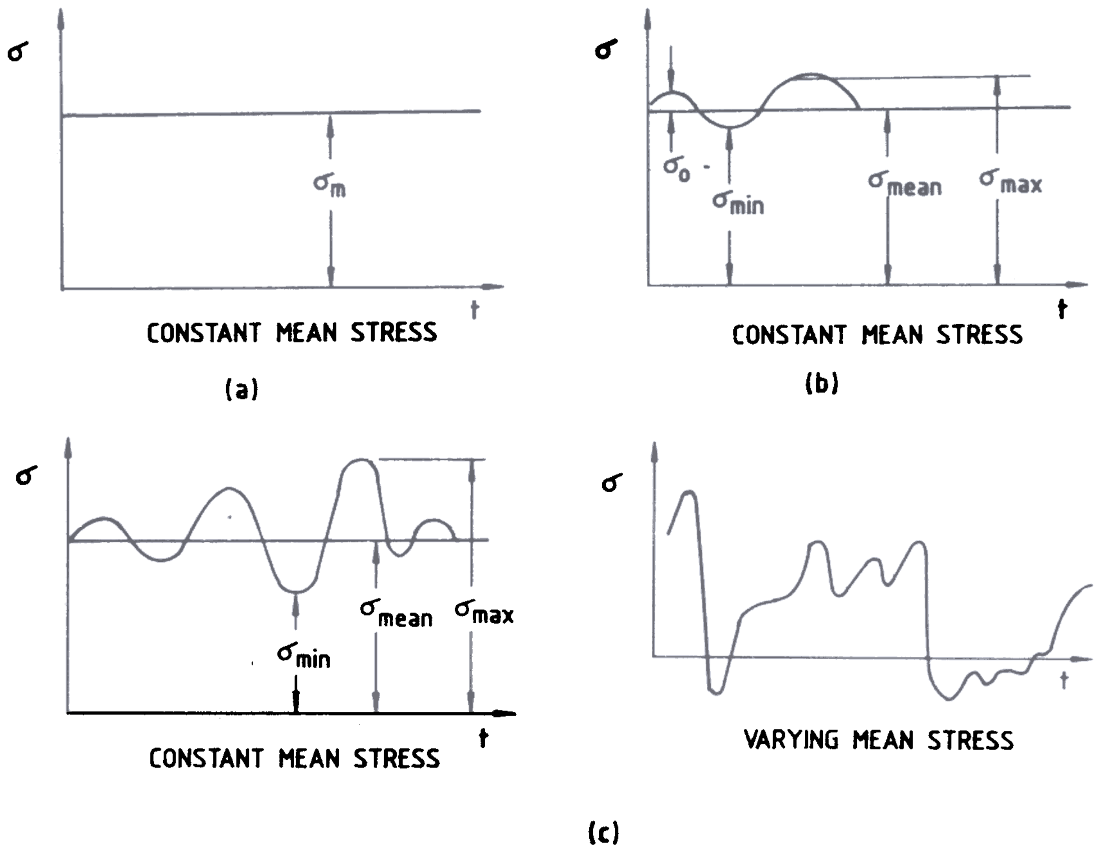


**Figure 1. Classification of stress time history: (a) static stress, (I ) periodically varying stress, and (c) irregularly varying stress**

random in nature. During the service of an AFV, the pinion of a reduction gear box engaged with the ring gear of the turret is subjected to fluctuating loads of varying amplitudes with time. The various gears in the gear train of the gear box are also subjected to varying loads. It is therefore difficult to estimate the life of these gears as these are not amenable to mathematical description. The best method for determining the life of these gears is to conduct prototype testing in which the actual stress time history (STH) is reproduced. This type of testing, called the random fatigue testing, is costly and time-consuming. However, STH itself is not repetitive in nature in actual service life as the STH obtained on one occasion need not be the same as obtained on another occasion. Assuming that the load spectrum during the service conditions follows Gaussian distribution, the cumulative damage theory[2] (Miner's rule) has been applied for predicting the life of a gear in the gear box of an AFV.

## 2. CLASSIFICATION OF STRESS TIME HISTORIES

The stress time histories are classified[3] as (i) deterministic and (ii) random Fig. 1. The simplest case is the one shown in Fig.1(a) where the stress is constant and does not vary with time. In other cases, the stresses are time-dependent. Figure 1(b) shows periodically varying stress. The irregularly varying stress as shown in Fig. 1(c) is the result of incremental stresses superimposed over the basic stress. The basic stress need not be constant and may also change with time.

## 3. CUMULATIVE FREQUENCY DISTRIBUTION

The cumulative frequency distribution (CFD) represents an approximate description of STH.

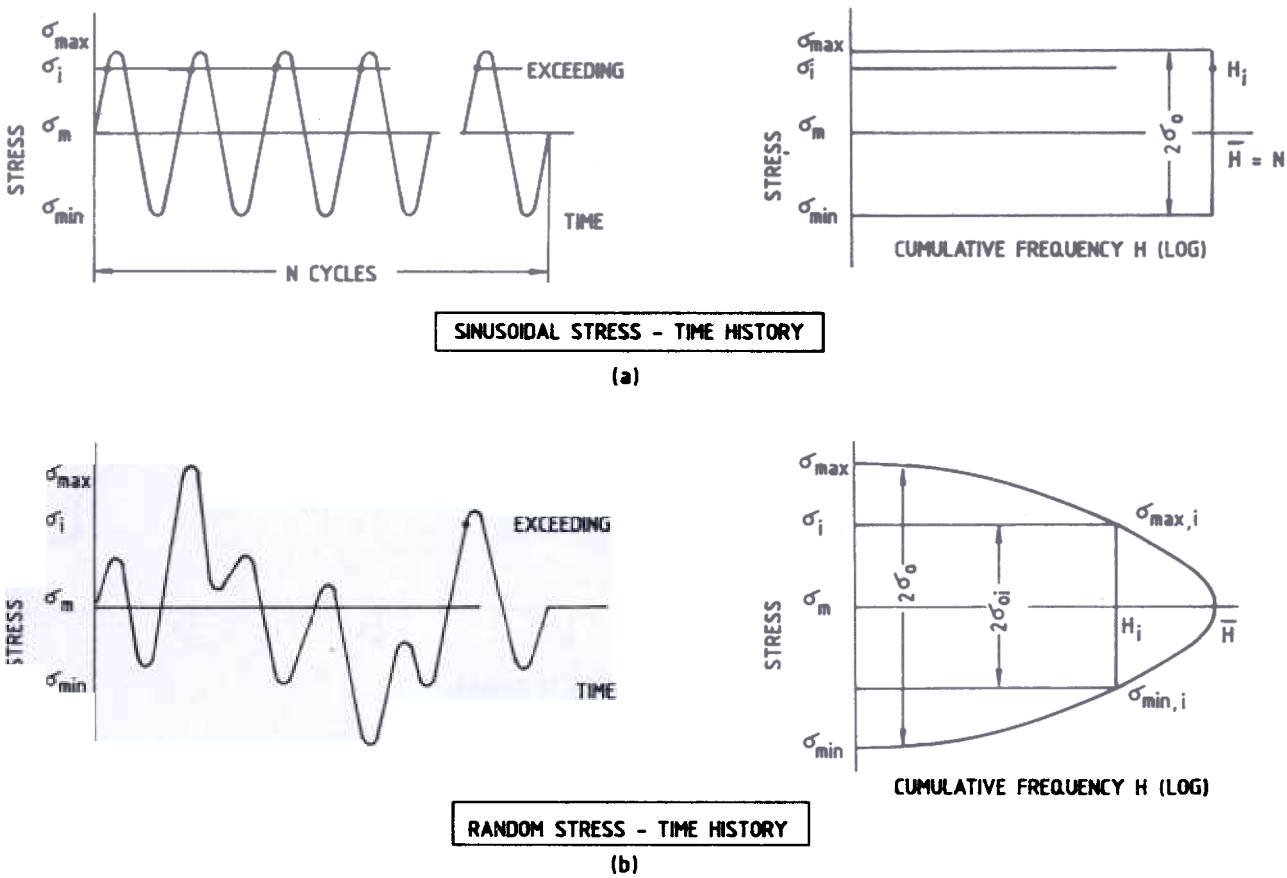


**Figure 2. Stress time history and cumulative frequency distribution**

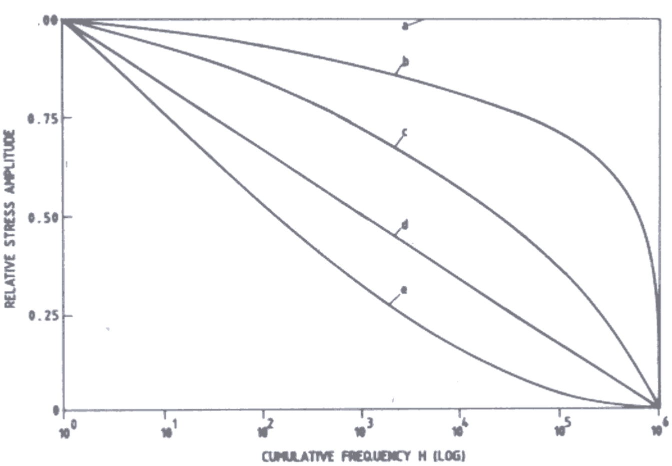


**Figure 3. Typical amplitude distributions plotted on relative scale.**

These are determined from long-term measurements of the service stresses using suitable instrumentation. In the simplest case, the number of exceedings, $H_i$ for several prefixed stress levels, $\sigma i$ were counted. For a sinusoidal STH life that was applied in a constant amplitude *S-N* (stress amplitude *vs* cycles to failure) test Fig. 2(a), a sequence of $N$-cycles yields $H_i = H$ exceedings, for each stress level between $\sigma_{max}$ and $\sigma_{min}$. Outside this stress range, $H_i$ was zero. Thus, on the semi-logarithmic plot, CFD of a sinusoidal STH appears rectangular Fig. 2(a). The CFD for an irregular STH is produced in the same way Fig. 2(b). It indicates that within the operation period under consideration, stress cycles occur that are equal to or exceed the stress limits, $\sigma_{max}$, and $\sigma_{min}$. This can be seen from the rectangle indicated in Fig. 2(b). Hence, the size of the distribution, $H$ shows the total number of small, medium and high stress cycles that replace the irregular STH. As the size of the distribution is proportional to the operation time considered, it allows for the direct conversion of the number of cycles (or zero-level crossings), as obtained in a variable amplitude fatigue test, into the fatigue life in terms of miles,

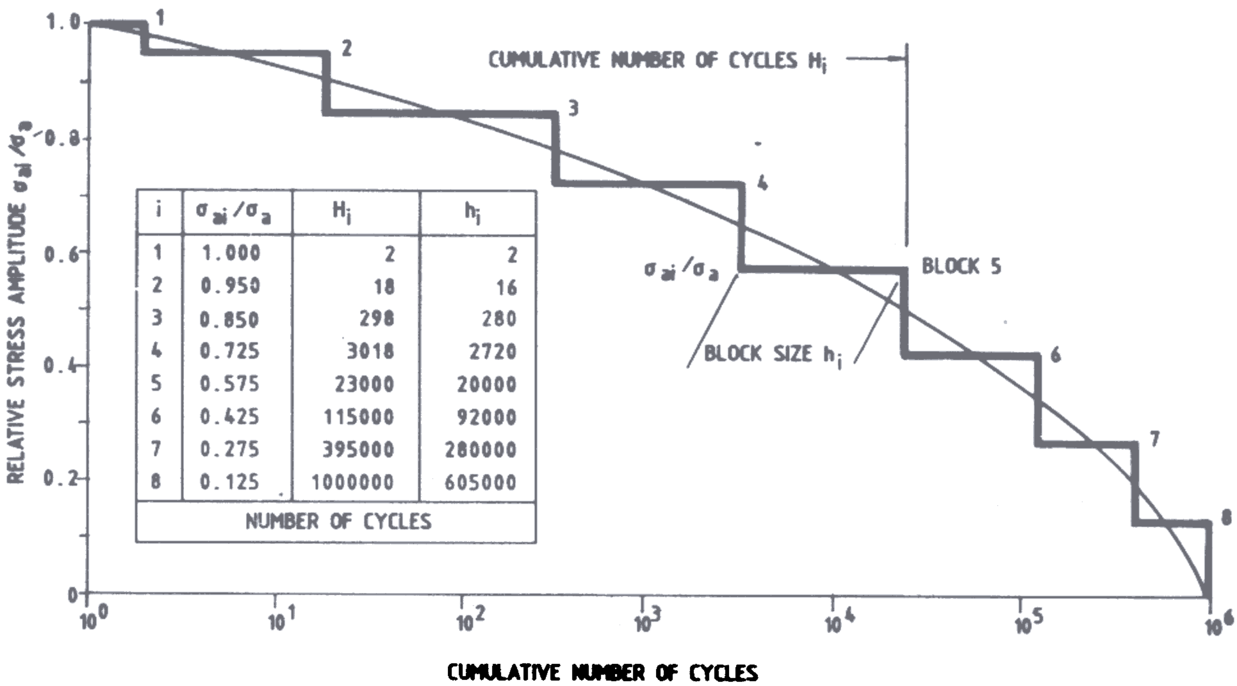


**Figure 4. Standard load-spectrum according to Gassener**

hours, etc. The shape of the distribution is equivalent to a statistical distribution function and indicates the relative frequency of high and low stress amplitudes.

## 4. TYPICAL AMPLITUDE DISTRIBUTIONS

During investigations, several typical amplitude distributions were often observed (Fig. 3). Besides the rectangular distribution (curve a, Fig. 3) which is the extreme type, the distribution curve (curve c, Fig. 3) can be regarded important. It represents the distribution corresponding to stationary Gaussian process and applies to STH of elastic structures, as produced by runway roughness, gusts or sea waves under uniform service conditions. Uniform service conditions are given, for instance, when a vehicle is driven over a homogeneous surface roughness. Since a similar distribution to curve (curve c in Fig. 3) is often obtained for STH caused by individual events, this type of distribution curve proves to be the most suitable for fundamental fatigue investigations under variable amplitude loading.

**Table 1. Values of torque and bending stress on each gear in gear train**

| Gear | Module | Pitch circle diameter (mm) | Face width (mm) | Gear pair | Geometric factor | Bending stress (MPa) |
|---|---|---|---|---|---|---|
| Z1 | 4 | 44 | 20.0 | Z1-Z2 | 0.21 | 84.54 |
| Z2 | 4 | 126 | 20.0 | Z2-Z1 | 0.22 | 93.96 |
| Z3 | 2 | 38 | 20.0 | Z3-Z4 | 0.31 | 144.75 |
| Z4 | 2 | 34 | 16.8 | Z4-Z5 | 0.29 | 188.22 |
| Z5 | 2 | 106 | 22.1 | Z5-Z4 | 0.40 | 101.58 |
| Z6 | 3 | 51 | 50.0 | Z6-Z7 | 0.29 | 343.59 |
| Z7 | 3 | 135 | 44.4 | Z7-Z6 | 0.39 | 292.28 |
| Z8 | 6 | 120 | 64.0 | Z8-Z9 | 0.32 | 135.96 |
| Z9 | 6 | 168 | 67.0 | Z9-Z8 | 0.35 | 121.84 |

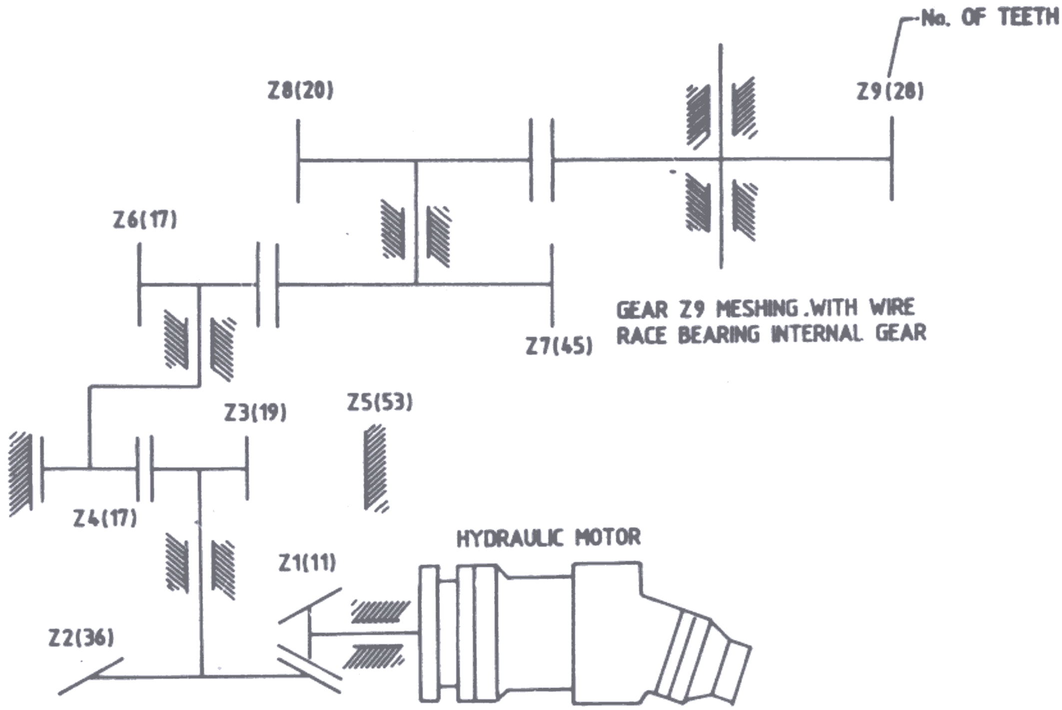


**Figure 5. Kinematic diagram of armoured fighting vehicle's traverse gear box**

The curves (d) and (e) in Fig. 3 represent distributions as obtained under nonuniform service conditions. These may be explained by a superposition of several successive stationary Gaussian processes of different stress intensities and of short duration. For example, the variation of the stress intensity results from running a vehicle with variable speed on roads of different surface roughness.

For fatigue considerations, it has been found from comparative test series, to be quite sufficient if those distributions were synthesised by just a few distributions of type (c) of different size and stress intensity, for, the cumulative damage of the latter compares well to the damage of the original

**Table 2. Damage calculation based on Gaussian cumulative frequency distribution**

| Relative stress amplitude $\sigma_{ai}/\sigma_a$ | Bending stress (MPa) | Cumulative number of cycles ($H_i$ at each stress level as per standard spectra) | Number of cycles ($h_i$ at each stress level as per standard spectra) | Number of cycles for failure from *S-N* curve | Partial damage |
|---|---|---|---|---|---|
| 1.000 | 800 | 2 | 2 | $10^3$ | 0.002 |
| 0.9.950 | 760 | 18 | 16 | $2.2 \times 10^3$ | 0.007273 |
| 0.850 | 680 | 298 | 280 | $7.5 \times 10^3$ | 0.0373 |
| 0.725 | 580 | 3018 | 2720 | $2.8 \times 10^4$ | 0.971 |
| 0.575 | 460 | $23 \times 10^3$ | $20 \times 10^3$ | $2.0 \times 10^5$ | 0.1000 |
| 0.425 | 340 | $11.5 \times 10^4$ | $9.2 \times 10^4$ | $\infty$ | 0 |
| 0.275 | 220 | $3.95 \times 10^5$ | $28 \times 10^5$ | $\infty$ | 0 |
| 0.125 | 100 | $1 \times 10^6$ | $60.5 \times 10^5$ | $\infty$ | 0 |

Total damage: 0.243673

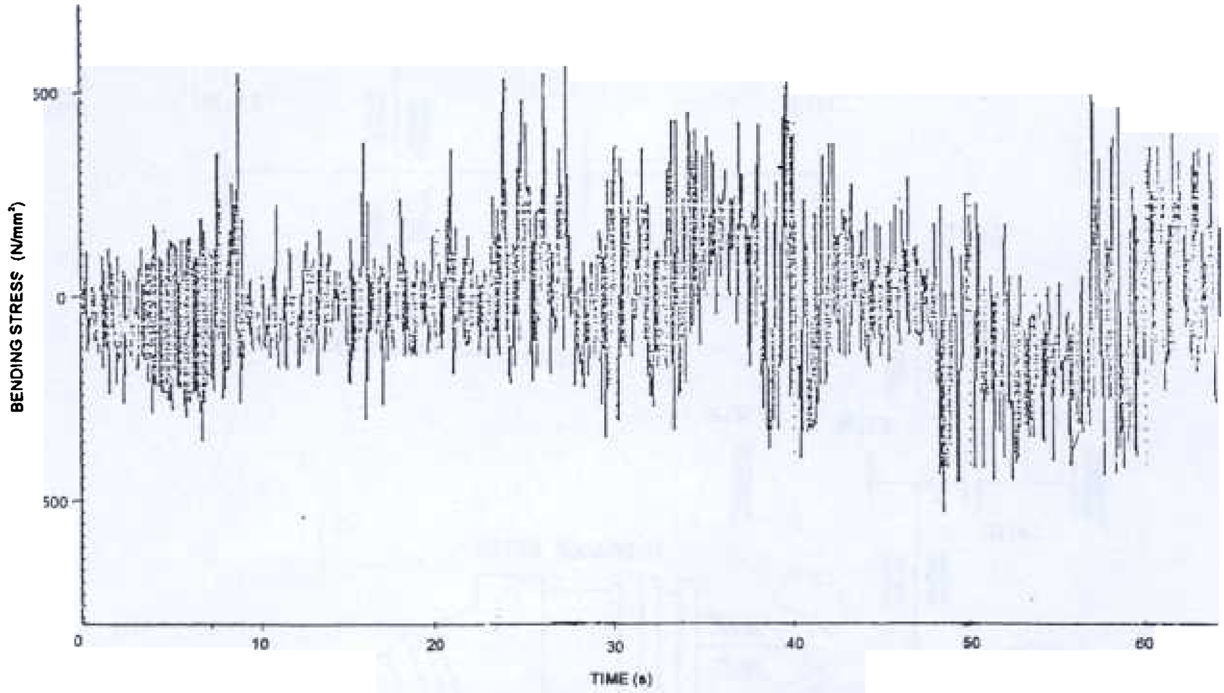


Figure 6. Stress time history (cross country run)

distribution. So the distribution of type (d) and (e) may be treated without the need for special time-consuming tests.

Another group of typical distributions is indicated by curve (b) in Fig. 3. In this distribution, the stress amplitude always exceeds a certain level while the amount by which this level exceeds follows the Gaussian distribution curve (c) in Fig. 3.

## 5. LOAD SPECTRUM & PREDICTION OF FATIGUE LIFE

For the members subjected to variable loading in service like aircraft structures, Gassener[4] suggested a programme test on the member instead of tests under constant amplitude loading. The STH under consideration was simulated by fitting a stepped curve to the steady curve of CFD, each step corresponding to a number of cycles with constant amplitude. The 'blocks' of cycles were applied to the test specimen in a certain sequence. Gassener, *et al.* have suggested a standard load spectra divided into eight steps. This spectrum curve corresponds to the well-known Gaussian (normal) distribution (Fig. 4). This spectrum was used to estimate the life of a component subjected to random loading. The cumulative damage theory, called the Miner's rule, was used to calculate the partial damage of the component at each stress level. From the stepped spectrum as shown in Fig. 4, the damage was calculated at each step and summed up to obtain the total damage. Since the spectrum was for a total of $10^6$ number of cycles, the probable life $L$ of the member is given by Eqn (1) where $D$ is the total damage:

$$L = 10^6/D \tag{1}$$

## 6. TRAVERSE GEAR BOX

The kinematic diagram of a reduction gear box is shown in Fig. 5. The gear box is coupled to a hydraulic rotary actuator and an electrohydraulic servo valve. The output pinion of the gear box is engaged with the ring gear of the turret of an AFV. The actuator is a high speed hydraulic motor, the speed and direction of which is regulated by the hydraulic fluid controlled by a servo valve. The opening of servo valve depends upon the spool movement actuated by the feedback signals of the rate Gyros, depending upon the disturbances of terrain encountered by AFVs to ensure that rotating

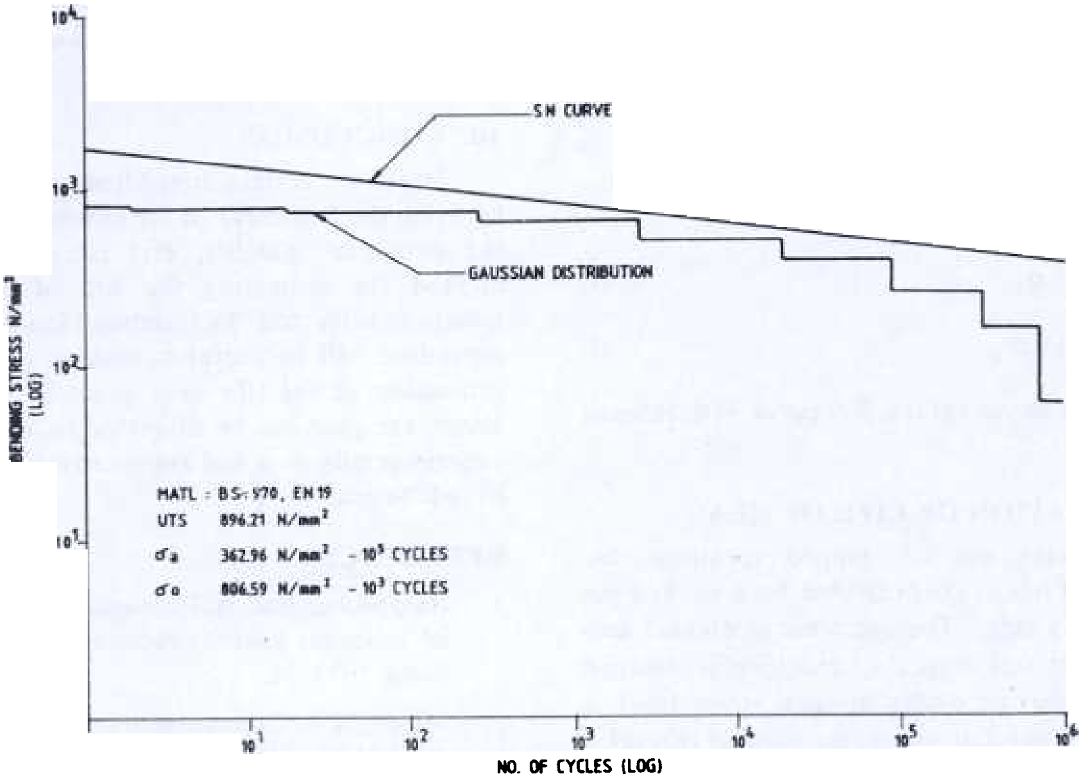


**Figure 7.** ***S-N*** **curve with stepped Gaussian distribution**

mass remains a stable platform. While doing so, the pinion engaged with the ring gear is subjected to fluctuating loads which are random, depending upon terrain/disturbance.

## 7. TORQUES & STRESSES ON GEAR

Based on a value of torque transmitted by the pinion, the torque on each gear was arrived at. Similarly, the bending stress acting on each gear was calculated (Table 1). From Table 1, it can be seen that the gear Z6 is subjected to maximum stress as compared to other gears.

The actual torque transmitted by the pinion (Z9) of the gear box was measured experimentally on different terrains using suitable instrumentation. A typical STH as experienced by gear Z6 is shown in Fig. 6. The maximum stress and relative stress amplitudes, as per the Gaussian distribution have been arrived at.

## 8. PREDICTION OF *S-N* CURVE

As the gear under study is made of *EN*-19 steel, the *S-N* curve for this material has been worked out. This fatigue curve, generally is a straight line between $10^3$ to $10^6$ cycles on a log-log plot. From the test data, it was observed that the failure stresses at $10^3$ and $10^6$ cycles were related to the ultimate tensile strength (UTS) and for 50 per cent probability of survival. These are given by

$$\sigma_o = 0.9\, \sigma_{ut} \tag{2}$$

$$\sigma_e = 0.5\, \sigma_{ut} \tag{3}$$

If the UTS of material is known, the probable *S-N* curve for the test conditions can be developed and fatigue life corresponding to any stress level may be obtained. The situations encountered in practice are usually different from test conditions. Under these conditions, the *S-N* curves are affected. By applying suitable correction factors, the probable value of $\sigma_o$ and $\sigma_e$ may be obtained.

These correction factors are load factor, size factor and surface factor. At $10^6$ cycles, the load and surface factors are taken as unity and the size factor is taken as 0.9, as the gear under study is small. At $10^3$ cycles, all the three factors are assumed as unity. After applying the correction factors, the stresses $\sigma_o$ and $\sigma_e$ are given below:

$$\sigma_o = 0.9\ \sigma_{ut} \tag{4}$$

$$\sigma_e = 0.45\ \sigma_{ut} \tag{5}$$

With the above values, *S-N* curve is developed (Fig. 7).

## 9. ESTIMATION OF LIFE OF GEAR

By making use of stepped spectrum, the estimation of life of gear (Z6) has been worked out using Miner's rule. The spectrum is divided into eight steps. At each step, the stress level is obtained and the number of cycles at each stress level is found. From the *S-N* curve, the number of cycles for failure at each stress level is found. From these two values, the partial damage at each stress level is derived. The results are shown in Table 2. The total damage for $10^6$ cycles calculated is used for estimating the life of gear.

Total damage for $10^6$ cycles $= 0.243673$

Total life of gear $= 1/0.243673$
$= 4.1026 \times 10^6$ cycles.

The life of gear obtained as above in terms of number of cycles can be converted into life in terms of number of hours of operation.

## 10. CONCLUSION

Based on certain simplified assumptions and by using the *S-N* curve of the structural material of the structural member, this paper outlines the method for estimating the life of a structural member subjected to random loading. This procedure will be useful in making a preliminary estimation of the life of a gear. To validate the above, the gear can be subjected to load-spectrum experimentally in a test rig to arrive at the actual life of the gear.

## REFERENCES


1. Ram Mohan Rao, A. Estimation of the fatigue life of randomly loaded structures. *J. Aeronau. Soc. India,* 1974, **26**.
2. Miner, M.A. Cumulative damage in fatigue. *J. Appl. Mech.*, 1945, **12**.
3. Gassener, E. & Haibach, E. Testing procedure for the design and life estimation of fatigue sensitive structures; structural safety and reliability, edited by A, Freudenthal. Pergamon Press, New York, 1972.
4. Gassener, E. Effect of variable load and cumulative damage on fatigue in vehicle and aircraft structures. Proceedings of the International Conference on Fatigue of Metals, Institution of Mechanical Engineers, London. 1956. p. 304

**Contributors**

**Mr D Hanumanna** obtained his BE (Mech.) from Sri Venkateswara University and MTech in Design of Machine Tools from Regional Engineering College, Warangal. Presently, he is working as Additional Director at the Combat Vehicles Research & Development Establishment, Avadi. He has been involved in inspection, integration, design and development of armoured fighting vehicles for Indian Army for more than two decades. Currently he is the incharge of the main battle tank - *Arjun* programme. His areas of research are hydraulic controls and fatigue failure of components under random loading.

**Dr S Narayanan** obtained his BE (Mech.) from University of Chennai, in 1967 and PhD (Aeronautical) from Indian Institute of Technology (IIT), Kanpur, in 1975. He had been teaching at IIT, Chennai and IIT, Kanpur since 1973. Presently, he is working as Professor at IIT, Chennai in the areas of vibration, random vibration, nonlinear vibration, acoustics and noise control. He has published about 70 papers in national/international journals and conferences. He is Fellow of the Indian National Academy of Engineering, the Aeronautical Society of India, and Alexander Van Hurbolit post doctoral Fellow at the Technical University, Berlin. He is also coauthor of book on Applications of Random Vibration.

**Dr S Krishnamurthy** obtained his MS (Mech.) from Jawaharlal Nehru Technological University, Hyderabad and PhD from Indian Institute of Technology (IIT), Chennai. He is currently Associate Professor at IIT, Chennai. He has been teaching since 1968. His areas of research include surface durability and failure analysis of gears. He is a member of the Institution of Engineers (India), Fluid Power Society of India and Treasurer of Association for Machines and Mechanisms.