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Converging Spherical Detonation Waves

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ABSTRACT

The problem of converging spherical detonation waves propagating through a gas with varying density is discussed. By neglecting the effect of variation of Q on the similarity exponent, both analytical and numerical solutions for motion of the detonation front have been obtained and are presented in graphical form.

1. INTRODUCTION

The problem of propagation of detonation waves in an inhomogeneous medium is of great interest in exploring the effects of explosions. Specially, atomic explosion problems are of great interest in S&T. During world war II, a good number of investigators studied the propagation of strong shocks. The well-known Taylor-Sedov solution, based on similarity and dimensional considerations, showed good agreement with experimental measurements of a shock trajectory only up to a few metres, but uniformly valid analytical solutions of the governing nonlinear partial differential equations together with moving boundary conditions could not be found. The problem of propagation of spherical shock wave through self-gravitating gases was taken up by several researchers. Kopal[1] and Sedov[2] tried to solve the problem with constant shock strength. Lidov[3] studied the case with variable shock strength. Purohit[4] studied the problem of self-similar homothermal flows of self-gravitating gas behind spherical shock wave in a nonuniform atmosphere. Rai and Gur[5] obtained analytical solutions for the problem of propagation of a strong MGD shock advancing into an inhomogeneous self-gravitating gas sphere. Kynch[6] and Taylor[7]obtained closed-form solutions for the flow variables behind a blast wave produced by a sudden explosion. The problem of propagation of a contracting spherical or cylindrical shock front into a uniform gas at rest was studied by Stanyukovich[8]. Welsh[9] and Nigmatulin[10] studied the propagation of a contracting detonation shock front into a uniform combustible gas. The main aim of the present investigation was to study the problem of converging spherical detonation waves through gas with varying density and releasing a constant amont of energy per unit mass of the gas. The following assumptions have been made to study the essential features of the detonation wave:

(a) The detonation front is a Chapman-Jouguet front i.e. It travels at sonic speed relative to the burnt gas, which determines the law of convergence[10].

(b) The detonation wave is strong, i.e. the values of pressure and internal energy in the undisturbed gas have been neglected in comparison with their values in the disturbed gas. Self-similar solutions are used to study the effects of various physical parameters in a region headed by a

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propagating detonation front. Density distribution in the undisturbed state of the detonation front ρ_1 is assumed to vary as an inverse power of radial distance from the centre of symmetry at time t, i.e.

$$\rho_1 = \rho_0 r_1^{-K} \tag{1}$$

where ρ_0 and K are constants.

The location of the spherical detonation front which is converging from infinity is given by:

$$r_2 = a_n(-t)^n \tag{2}$$

where a_n and n (<1) are constants.

2. BASIC EQUATIONS & SIMILARITY TRANSFORMATIONS

The basic equations for one-dimensional motion of a perfect inviscid gas are:

$$\frac{\delta\rho}{\delta t} + u\frac{\delta\rho}{\delta r} + \rho\frac{\delta u}{\delta r} + 2\frac{\rho u}{r} = 0 \tag{3}$$

$$\rho\frac{\delta u}{\delta t} + \rho u\frac{\delta u}{\delta r} + \frac{\delta P}{\delta r} = 0 \tag{4}$$

$$\frac{\delta P}{\delta t} + u\frac{\delta P}{\delta r} + \gamma P\frac{\delta u}{\delta r} + 2\frac{\gamma p u}{r} = 0 \tag{5}$$

where

u,p,ρ are the velocity, pressure and density of the gas and γ is the ratio of the specific heats. The boundary conditions on the detonation front can be expressed in the forms:

$$u_2 = \beta D \tag{6}$$

$$p_2 = \beta\rho_1 D^2 \tag{7}$$

$$\rho_2 = \frac{\rho_1}{(1-\beta)} \tag{8}$$

$$\beta = \frac{1}{\gamma+1}\left[1+\left\{1-2(\gamma-1)(\gamma+1)\frac{Q}{D^2}\right\}^{\frac{1}{2}}\right]$$

$$\frac{\alpha}{\gamma+1} \tag{9}$$

And D is the velocity of the detonation front. Suffixes 2 and 1 refer to conditions immediately behind and immediately ahead of the detonation front, respectively. Q denotes the heat release per unit mass of the gas.

Now, a similarity parameter $\eta = r/r_2$ is introduced to seek a solution of the form:

$$u = -\beta D U(\eta)$$

$$p = \beta\rho_1 D^2 \rho(\eta) \tag{11}$$

$$\rho = (\rho_1/(1-\beta))\, G(\eta)$$

Since the location of the detonation front is given by η=1, the boundary conditions (6) - (8) assume the following forms:

$$\left.\begin{aligned} U(1) &= -1 \\ P(1) &= 1 \\ G(1) &= 1 \end{aligned}\right\}$$

Applying the similarity transformation in Eqns (3)-(5) with the help of Eqns (10)-(12), we get:

$$G'\{\eta+\beta U\} - \beta(K-2)\frac{GU}{\eta} + \beta G U' = 0 \tag{14}$$

$$GU'\{\eta+\beta U\} - \frac{K(1-\beta)}{\eta}P + (1-\beta)P'$$

$$\frac{(n-1)}{n}GU = 0$$

$$P'\{\eta+\beta U\} + \gamma\beta\rho U' - \frac{2(n-1)}{n}P - \beta K\frac{PU}{\eta}$$

$$+\frac{2\gamma PU\beta}{\eta} = 0$$

where

G', U', and P', respectively denote the derivatives of G,U and P with respect to η.

From dimensional analysis, it is obvious that the detonation front must travel with a constant velocity, as the similarity exponent n is equal to one. But propagation of a converging detonation front with constant velocity is not possible, because as the front accelerates, its surface area diminishes, causing its velocity to increase towards the centre of symmetry, where it becomes infinite. So, determination of the law of convergence for the detonation front does not seem to be possible. Considering Eqn (9), $\alpha = 2$ corresponds to a shock wave without the release of energy and $\alpha = 1$ corresponds to the Chapman-Jouguet detonation regime, where the wave propagates along characteristics in the disturbed gas.

The extreme case $\alpha = 1$ is being investigated in the convergence process, which ensures the Chapman-Jouguet condition and the detonation front propagates along a characteristic. In this way, the law of convergence of the front is being saught as a characteristic dividing the disturbed and undisturbed media.

The one-dimensional equations of motion in Lagrangian variables are:

$$\rho_1 = \left(1+\frac{V}{r}\right)^2 \rho\left(1+\frac{\delta V}{\delta r}\right) \tag{17}$$

$$\rho_1 \frac{\delta^2 V}{\delta t^2} = -(1+V/r)^2 \frac{\delta P}{\delta r} \tag{18}$$

$$P/P_2 = (\rho/\rho_2)^\gamma \tag{19}$$

where

V is the displacement of the particle. The conditions on the detonation wave in the Chapman-Jouguet regime are:

$$u_2 = \frac{D}{\ldots 1} \tag{20}$$

$$P_2 = \frac{\rho_1 D^2}{\gamma+1} \tag{21}$$

$$\rho_2 = \left(\frac{\gamma+1}{\gamma}\right)\rho$$

Eqn (18) may be put in the following form:

$$\frac{\delta^2 V}{\delta t^2} = a^2(r, V, \delta V/\delta r)\, \delta^2 V/\delta r^2 + b(r, V, \delta V/\delta r)$$

Then, we can have the equations of the characteristics and conditions on them as:

$$dr = +a\,dt, \qquad dV_t = +\, a\,dV_r + b\,dt$$

where

$V_t = \delta V/\delta t$ and $V_r = \delta V/\delta_r$

Using Eqn (17) and the relations (20)-(22) in Eqn (19), we get:

$$P = \left(\frac{\gamma}{\gamma+1}\right)^\gamma \frac{\rho_1 D^2}{(\gamma+1)} \left\{\frac{1}{(1+V_r)^\gamma} \frac{1}{(1+V/r)^{2\gamma}}\right\}$$

Differentiating Eqn (25) and substituting in Eqn (18), we get:

$$\frac{\delta^2 V}{\delta t^2} = \frac{1}{\gamma+1}\left(\frac{\gamma}{\gamma+1}\right) \left[\frac{\gamma D^2}{(1+V/r)^{2(\gamma-1)}(1+V_r)^{\gamma+1}} \frac{\delta^2 V}{\delta r^2} + \frac{2\gamma D^2}{(1+V_r)^\gamma (1+V/r)^{2\gamma-1}} \left\{\frac{V_r}{r} - \frac{V}{r^2}\right\} \frac{2D}{(1+V_r)^\gamma (1+V/r)^{2(\gamma-1)}} \frac{dD}{dr}\right]$$

On equating Eqns (23) and (26)

$$a^2(r, V, V_r) = \left(\frac{\gamma}{\gamma+1}\right)^{\gamma+1} \frac{D^2}{(1+V_r)^{\gamma+1}(1+V/r)^{2(\gamma-1)}}$$

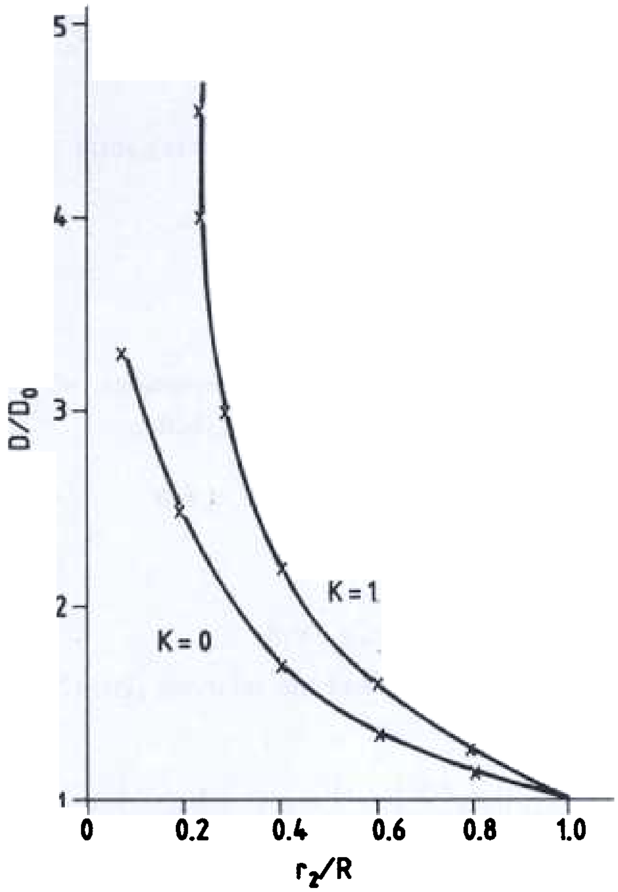

Figure 1. Variation of detonation front velocity

$$b(r,V,V_r) = \frac{1}{\gamma+1}\left(\frac{\gamma}{\gamma+1}\right)\left\{\frac{2\gamma D^2}{(1+V_r)^{\gamma+1}(1+V/r)^{2\gamma-1}}\right\}\left(\frac{V_r}{r} - \frac{V}{\;}\right)$$
$$\frac{2D}{(1+V_r)^{\gamma}(1+V/r)^{2(\gamma-1)}}\frac{dD}{dr}\Big\} \quad (28)$$

It is assumed that the detonation wave travels along a characteristic. Therefore.

$$a(r,o,V_{r_2}(r)) = a_2(r) = D(r) \quad (29)$$

From Eqns (27) & (29):

$$V_{r_2} = -\frac{1}{\text{[illegible]}} \quad (30)$$

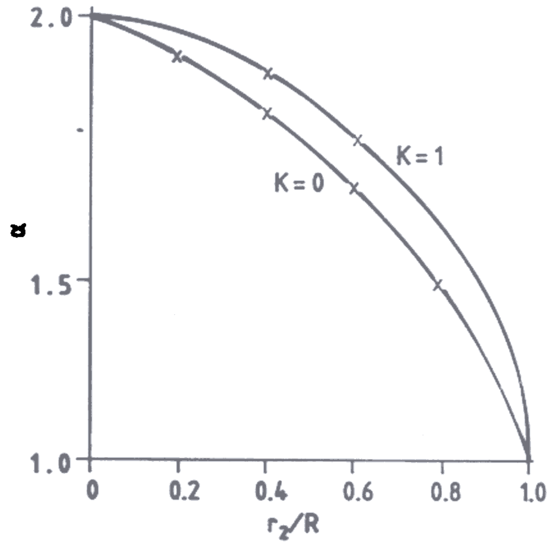

Figure 2. Variation of α

Also, from Eqn (28), we get:

$$b(r) = -\frac{2\gamma}{(\gamma+1)^2}\frac{D^2}{r} - \frac{2D}{(\gamma+1)}\frac{dD}{dr}$$

The equation of the characteristic which bounds the quiescent gas is:

$$u_{r_2} dt + u_{r_2} dr = du = 0$$
$$u_{t_2} + u_{r_2} a_2 = 0.$$

Using the fact that u_{r_2} is constant, on substituting Eqn (32) in Eqn (24), we get:

$$-u_{r_2} da_2 = b_2 dt$$

Considering Eqns (24), (29)-(31) and (33), we obtain:

$$|D| = \frac{A}{r_2^m}$$

where

$$m = \gamma / \gamma + 1$$

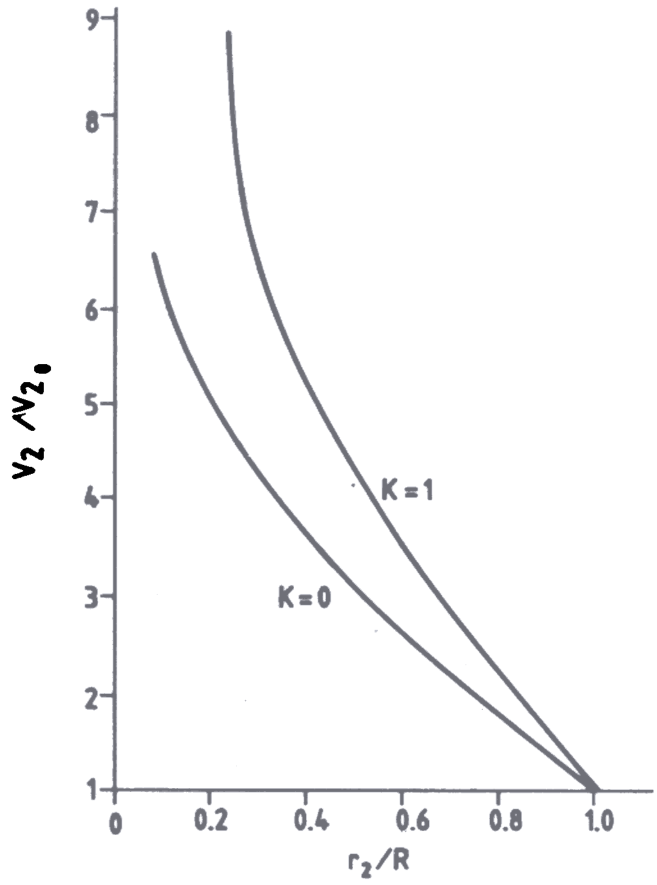

Figure 3. Variation of flow velocity

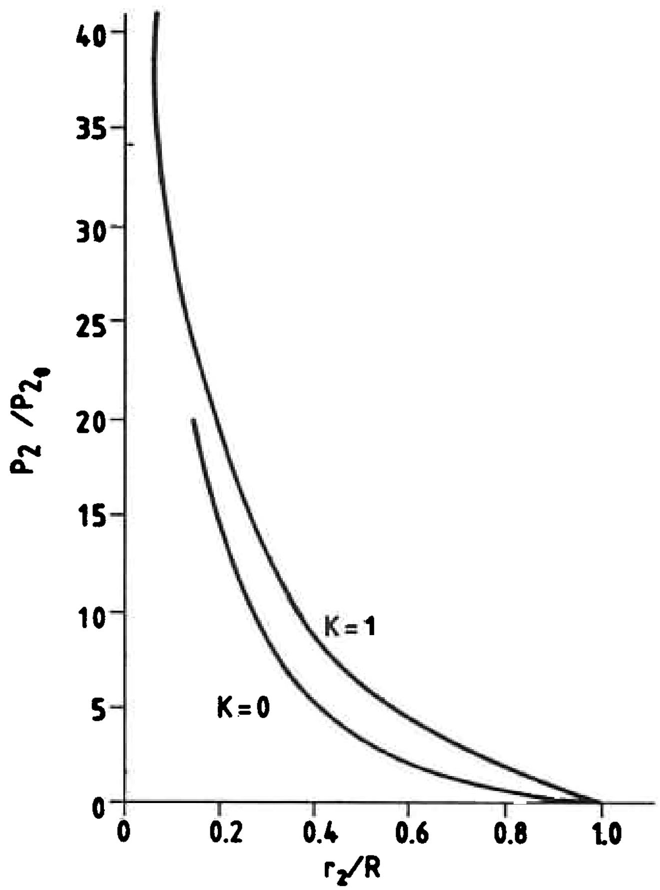

Figure 4. Variation of pressure

Integrating Eqn (34) with the condition that $r_2 = 0$ at t = 0, we get:

$$r_2 = a_n(-t)^n,\ n = 1/(m+1) \quad (35)$$

Thus, the similarity exponent n is determined in terms of K, of course, for a specific value of γ. After determining the value of n, the convergence law for the detonation front can be determined.

3. SOLUTIONS

Taking Q as constant in the convergence process [11], the following approach can be used to study non-self-similar problem of the converging detonation front started at a certain initial radius R and which has initiated its own motion at the Chapman-Jouguet velocity (α=1).

$$D_0^2 = 2(\gamma-1)(\gamma+1)Q_0 \quad (36)$$

With the help of Eqn. (34), the following relationship for the motion of the front can be obtained:

$$(D/D_0) = (R/r_2)^m \quad (37)$$

Substituting Eqn (37) into Eqn (9) and using Eqns (6)-(8), we obtain:

$$\alpha = 1 + \{1-(r_2/R)^{2m}\}^{1/2} \quad (38)$$

$$u_2/u_{2_0} = \alpha(R/r_2)^m \quad (39)$$

$$P_2/P_{2_0} = \alpha(R/r_2)^{2m} \quad (40)$$

$$(\rho_2/\rho_{2_0}) = \frac{\gamma}{\gamma-(\alpha-1)} \quad (41)$$

where u_{2_0}, P_{2_0} and ρ_{2_0} are the values of u, p and ρ in the Chapman-Jouguet detonation regime for the specified Q_0. Eqns (37)-(41) are the solutions in terms of radius of the front.

From Eqn (34), it is observed that m must be positive for the velocity of the detonation front to increase towards the collapsing point.

From Table 1, it is observed that the value of n

Table 1. Value of n for different values of γ and K

K	$\gamma = 2$	K	$\gamma = 3$
0			0.67067
0.1			0.6860
0.5			0.7550
1.0			0.8638
1.3			0.9402

increases considerably with increase in the value of K. This increases however is greater for $\gamma = 2$ than for $\gamma = 3$. D/D_0, $\propto$, u_2 / u_{2_0}, P_2 / P_{2_0} and ρ_2 / ρ_{2_0} have been calculated in terms of the front radius r_2/R for $\gamma = 2$, K =0,0.1. Graphs for the variations in detonation front velocity $\propto$, flow velocity, pressure and density have been plotted in Figs 1-4. It can be observed from the figures that due to varying initial density, there are changes in the detonation front velocity, flow velocity, pressure and density. This phenomenon may perhaps be due to the fact that the detonation is strong.

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