

Technique for Exterior Expansion Measurement During Autofrettage for Constant Fatigue Life

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ABSTRACT

In an autofrettage process a given thick cylinder is subjected to such a pressure which gives a specified depth of elasto-plastic boundary. The outside diameter expansion during autofrettage process is a function of depth of autofrettage. To obtain a specified depth of elastic-plastic interface, the applied autofrettage pressure increases in direct proportion to the proof stress of the pressure vessel material. Although this increases the load bearing capacity of the barrel, resulting in enhanced factor of safety, this increases the maintenance cost of a hydraulic autofrettage plant. To assure quality, product safety and manufacturing economy, an optimal autofrettage pressure is defined. The paper proposes that the minimum autofrettage pressure is the pressure at which the pressure exterior expansion curve intersects line of constant factor of safety. At higher values of 0.2 per cent proof stress of tube material autofrettage based on line of constant factor of safety will result in a reduction in fatigue life. The point of intersection of exterior expansion curve with line of constant fatigue life has been defined as the optimal pressure because the specifications of factor of safety and fatigue life are simultaneously achieved. The proposed process design based on above concept has been validated using finite element simulation and empirical post-autofrettage measurements. The verification of the shakedown condition for reverse yielding due to the Bauschinger effect (Huang's model) and fatigue life has also been satisfied.

Keywords: Pressure exterior expansion curve, factor of safety, Bauschinger effect, Huang's model, shakedown condition, fatigue life, element death

NOMENCLATURE

a, a_1	Inside bore diameter of autofrettage forging in mm	n_{2f}	Ratio of autofrettage boundary diameter to inside diameter at S_{y2} proof stress value for finished tube
a_f, a_{1f}	Inside diameter of finished tube in mm	n_R	Ratio of reverse yield diameter to inside diameter of autofrettage forging
C^m	Von Mises correction factor	p	Autofrettage pressure in MPa
D_o	Outside diameter of autofrettage forging in mm	p_{sd}	Pressure at shakedown condition
E_o	Young's modulus in MPa	r	Radial coordinate of the section of tube
k	Generalised variable representing the ratio of outside to inside diameter of a tube	u_1	Exterior expansion in mm at S_{y1} proof stress value
k_f	Outside to inside diameter ratio of finished tube	u_2	Exterior expansion in mm at S_{y2} proof stress value
k_1	Ratio of autofrettage forging outside diameter to finished inside diameter	S_{1y}, S_{2y}, S_y	0.2% proof stress
MSP_1	Maximum safe pressure for yield stress 1	S_{ry}	Reverse yield stress
MSP_2	Maximum safe pressure for yield stress 2	ϵ	Strain
m_R	Slope of plastic stress strain curve for post reverse yield plastic deformation	ϵ_p	Equivalent plastic strain due to autofrettage
n	Ratio of autofrettage boundary diameter to inside diameter of autofrettage forging	σ_d	Difference between the yield stress in tension and compression
n_f	Ratio of autofrettage boundary diameter to inside diameter of finish tube	ϵ_y	Yield strain
n_1	Ratio of autofrettage boundary diameter to inside diameter at S_{y1} proof stress value for autofrettage forging	ξ	S_{1y}/S_{2y}
n_{1f}	Ratio of autofrettage boundary diameter to inside diameter at S_{y1} proof stress value for finished machined tube	ν	Poisson's ratio
n_2	Ratio of autofrettage boundary diameter to inside diameter at S_{y2} proof stress value for autofrettage forging		

1. INTRODUCTION

The high pressure vessels are autofrettaged to achieve higher fatigue life and to withstand higher firing pressure. This is because the residual compressive hoop stress in

the plastically deformed zone causes a very significant reduction in stress intensity factor for the cracks present at the bore. Due to the presence of compressive residual stress field in the plastically deformed zone the hoop stress due to applied pressure is lower than a non-autofrettage tube of similar dimensions and material. Thus an autofrettaged tube can withstand higher firing pressure as compared to a non-autofrettaged tube for same thickness of cross section. Alternately for the same safe service pressure, an autofrettaged tube will have a thinner cross sectional area as compared to a non-autofrettaged tube. A barrel can be autofrettaged by hydraulic or by swage process. In a hydraulic autofrettage process the tube is subjected to a hydraulic pressure which causes plastic deformation in the cross section of the tube up to the designed depth. The depth of autofrettage can be determined by measuring the expansion of the outer diameter of the tube and is generally referred to as the exterior expansion of the tube. The minimum autofrettage pressure as given by Rheinmetall¹, is the pressure at which the curve representing the variation of pressure versus exterior expansion intersects the line of constant degree of autofrettage. Here the degree or the depth of autofrettage is defined as the ratio of the autofrettage boundary diameter to the bore diameter of the autofrettage tube forging. The autofrettaged tube, after the application of autofrettage pressure, is subjected to accelerated strain ageing treatment. In case of a hydraulic autofrettage process the tube is subjected to a test pressure which is equal to the autofrettage pressure. A correctly autofrettaged tube shows an elastic response when subjected to the test pressure. If the autofrettage of a tube is carried out as the procedure outlined by Rheinmetall¹, it will have the following advantages:

- (i) The tube always achieves the minimum factor of safety for a given range of 0.2% proof stress.
- (ii) The tube always has a designed degree of autofrettage after finish machining even if the value of 0.2% proof stress exceeds the upper specification limit.
- (iii) The factor of safety increases with the increase in 0.2% proof stress.

The above autofrettage process design is likely to have the following disadvantages:

- (i) There is a loss of compressive residual stress due to reverse yielding of the tube section when the autofrettage pressure is withdrawn. The reverse yielding can be due to Bauschinger effect if the ratio of outer diameter to bore diameter at a given section is between 1.2 and 2.2. If the above diameter ratio is more than 2.2 the reverse yielding of the section of tube will be due the combined effect of Bauschinger effect and very high compressive hoop stress due to a high diameter ratio. The loss of compressive residual stress also increases with the depth of autofrettage. As such if the degree of depth of autofrettage is fixed for a given range of proof stress values for the material of the tube, the above process will not be able to achieve an optimum

value of compressive residual stress at the bore. Here the value of autofrettage can be considered to be optimal if it give specified maximum safe pressure and fatigue at minimum applied autofrettage pressure. Now if the depth of autofrettage is kept constant for a given range of 0.2% proof stress there will at least 10%-20% increase in autofrettage pressure if the 0.2% stress for the tube is close to upper specification limit.

- (ii) The minimum autofrettage pressure significantly increases with the increase in proof stress value. A 10% increase in proof stress can results in 20% increase in applied autofrettage pressure. This affects the maintenance economy of a hydraulic autofrettage plant. Based on the method outlined by Bhatnagar², the maintenance cost implications will be £100 per tube for a 20% increase in autofrettage pressure.
- (iii) In case of an in-process breakdown of the autofrettage plant it is often beneficial to prescribe a duplex autofrettaging in which the tube is subjected to a accelerated ageing treatment followed by re-autofrettaging. There is a 10% to 15% increase in applied autofrettage pressure if the duplex autofrettage is prescribed. This affects the maintenance economy of a hydraulic autofrettage plant³.

The above drawbacks can be overcome if it is possible to suggest a minimum possible autofrettage pressure that gives an acceptable factor of safety and fatigue life. Since the depth of autofrettage can be measured by means of exterior expansion, therefore the hydraulic autofrettage of a tube will result in its testing of the design parameters. The objective of this paper is to describe the design of an autofrettage process that gives an acceptable fatigue life and improves the operational and maintenance economy. The proposed process design also has the advantage of continuously monitoring the stability of exterior expansion during the autofrettage process. The paper seeks to validate the concept by determining shakedown condition, fatigue lifetimes and bore measurement for an actual tube.

The experimental setup for the measurement of exterior expansion is shown in Fig. 1. In the setup the expansion of outer diameter of the tube to be autofrettaged is measured by means LVDT sensors and the pressure is measured by a piezoelectric transducer.

2. AUTOFRETTAGE PROCESS DESIGN

Based on the review and drawbacks of the current practices in the autofrettaging outlined in the introduction, it was considered that the design of an autofrettage process must be based on the following considerations:

- (i) The applied autofrettage pressure should be a continuous function of 0.2% proof stress value. In some autofrettage process designs the applied autofrettage pressure is prescribed based on the sub-ranges of the specified range of 0.2% proof stress.
- (ii) The process design must minimise the applied



Figure 1. The experimental setup for pressure exterior expansion measurement.

autofrettage pressure and thereby reduce the plant maintenance. At the same time the finished tube must conform to the design and manufacturing standards of a thick tube.

The process must minimise the loss of compressive residual stresses by reducing the degree of autofrettage if the proof stress of the material is higher than the minimum specified limit. If the proof stress value of the forging is within the range and higher than the minimum specified value, then the depth of the reverse yielding zone can be minimised by minimising the autofrettage pressure to achieve same fatigue life and factor of safety.

In this work it has been assumed the material defects have been controlled within specified limits by suitable nondestructive testing methods. The selection of material and the condition of heat treatment has been made by the designer. Since the autofrettage process design lies in the purview of the detailed design phase therefore the proposed work pertains to the design of autofrettage process for the purpose of implementation of autofrettage process at the manufacturing work.

To achieve the above objectives an attempt was made to draw the lines of constant factor of safety on the pressure exterior expansion curves. The development of a set of equations for lines of constant factor of safety is obtained by equating maximum safe pressure, for a finished tube, at the minimum specified proof stress of the material to the maximum safe pressure corresponding to the values of proof stress incremented in steps of 1% of its minimum value. The definitions of maximum safe pressure and factor of safety are

- **Maximum Safe Pressure :** This is the maximum service pressure at which an autofrettaged and finish machined tube will yield up to the same autofrettaged boundary as created by the application of autofrettage pressure if the effect of compressive residual stress is neglected.
- **Factor of Safety :** This is the ratio of maximum safe pressure to the maximum service pressure.

In the proposed approach the line of constant factor of safety are obtained by equating the factor safety at minimum specified limit of 0.2% proof stress to the

factor of safety at the maximum specified value of 0.2 % proof stress. In this way the applied pressure will gets cancelled out on both sides and the equation of constant maximum safe pressure is obtained. Since the maximum safe pressure cannot be interpreted without the specification of applied pressure so the equation for the line of constant maximum safe pressure has been named as line of constant factor of safety. The factor of safety is a non-dimensional form of maximum safe pressure with respect to the applied pressure. The solution of resultant equations gives the ratio of yield boundary diameter to bore diameter $\left(\frac{D_n}{D_i}\right)$ corresponding to the proof stress values. The ratio of yield boundary diameter to bore diameter can be used to calculate the autofrettage pressure.

The equations of lines of constant factor of safety for pressure exterior expansions curves are given as: MSP at S_{yi}

$$MSP_i = S_{yi} \times C_m \left(\ln n_{if} + \frac{k_f^2 - n_{if}^2}{2k_f^2} \right) \quad (1)$$

where index i and has represents i^{th} material and has been given member 1 and 2.

$$C_m = 0.22k + 0.78 \quad k < 1.5$$

$$C_m = 1.11 \quad k \geq 1.5$$

where C_m is the correction factor for Von Mises yield criteria^{4,5} where k is a generalized variable representing the ratio of outside diameter to inside diameter of a tube.

If the factor of safety is to remain constant then the maximum safe pressure at any value of 0.2% proof stress, let us say S_{y2} , must remain the same. The minimum value of autofrettage pressure and the depth of autofrettage are known for the minimum value of 0.2% proof stress S_{y1} . In this work the S_{y1} and S_{y2} are the upper and lower specification limits of 0.2% proof stress respectively. The depth of autofrettage, let us say n_{2p} , can be obtained by equating Eqn. (1) with index 1 to Eqn. (1) with index 2. The resultant equation is given by Eqn. (2)

$$\ln\left(\frac{n_{1f}}{n_{2f}}\right) + \frac{(\xi - 1)}{2} - \frac{n_{1f}^2 \xi - n_{2f}^2}{2k_f^2} = 0 \quad (2)$$

where $\xi = \frac{S_{y1}}{S_{y2}}$

The autofrettage boundary diameter ratios corresponding to the autofrettage forgings which have proof stress values corresponding to S_{y1} and S_{y2} are given by a generic Eqn. (4).

$$n_i = n_{if} \times \frac{a_f}{a} \quad (4)$$

The exterior expansions corresponding to n_1 and n_2 are given by Eqn. (5)

$$u_i = \frac{C_m S_{yi} n_i^2 D_0}{E k^2} \quad (5)$$

The Eqns. (1)-(5) can be combined to obtain following

equation for obtaining the value of exterior expansion u_2 , for any other value of proof stress within the specified range of 0.2% proof stress

$$\ln \left(\frac{\left(\frac{a}{a_f} \right)^{\xi-1} \left(\frac{u_1 E k^2 k^2}{C_m S_{y1} D_0} \right)^{\frac{\xi}{2}}}{\left(\frac{u_2 E k^2}{C_m S_{y2} D_0} \right)^{\frac{1}{2}}} \right) - \frac{a^2 u_1 E k^2 \xi}{a_f^2 C_m S_{y1} D_0} - \frac{a^2 u_2 E k^2}{a_f^2 C_m S_{y2} D_0} + \frac{\xi-1}{2} = 0 \quad (6)$$

The set of Eqns. (2), (4), and (6) together with Eqns. (7) and (8) represent the curve of constant factor of safety

$$p = C_m S_{y2} \left\{ \ln n_2 + \frac{k^2 - n_2^2}{2k^2} \right\} \quad (7)$$

$$p = C_m S_{y2} \left\{ \ln \left(\frac{a^2 u_2 E k^2}{a_f^2 C_m D_0 S_{y2}} \right) + \frac{k^2 - \left(\frac{a^2 u_2 E k^2}{a_f^2 S_{y2} D_0} \right)}{2k^2} \right\} \quad (8)$$

Equations were applied to a sample case study for a tube of having a yield stress at 960 MPa for 30CrNiMo8 steel. The outside to inside diameter ratio of 1.94 was used and the autofrettage boundary diameter to inside diameter ratio was 1.795.

3. RESULTS AND DISCUSSION

The pressure versus exterior expansion curve with lines of constant factor of safety has been shown in Fig. 2. The applied autofrettage pressure decreases with the increase in proof stress of the forging along the lines of constant factor of safety. Thus, the lines of constant factor of safety for selection of applied autofrettage gives a maximum of 5% saving in autofrettage pressure. The adoption of lines of constant factor of safety should be practicable for the case of a forging with no lengthwise variation of proof stress. In practice there is a permissible variation in proof value along the length of the tube. As such the finished tube may not comply with the designed factor of safety in the regions where the proof stress value is very slightly lower than the minimum specified value. If the point along the length of the tube at which the exterior expansion is being measured has proof stress value close to the maximum limit and the proof stress value in the nearby regions is close to the minimum value of proof stress then the value of autofrettage pressure, given by the line of constant factor of safety on the pressure exterior expansion curve, will not give the correct factor safety for the nearby regions. This problem has been circumvented by permitting a 2% residual error in solution of Eqn. (8). A further investigation for the fatigue life times evaluation for autofrettage pressure given by the line denoted as y_1 in Fig. 2 led to the conclusion that fatigue life of a tube remains constant along these line. In the Fig.2 y_1 represents the minimum value of 0.2% proof stress for each line of constant fatigue life. For the purpose of comparison the line of constant factor of safety has

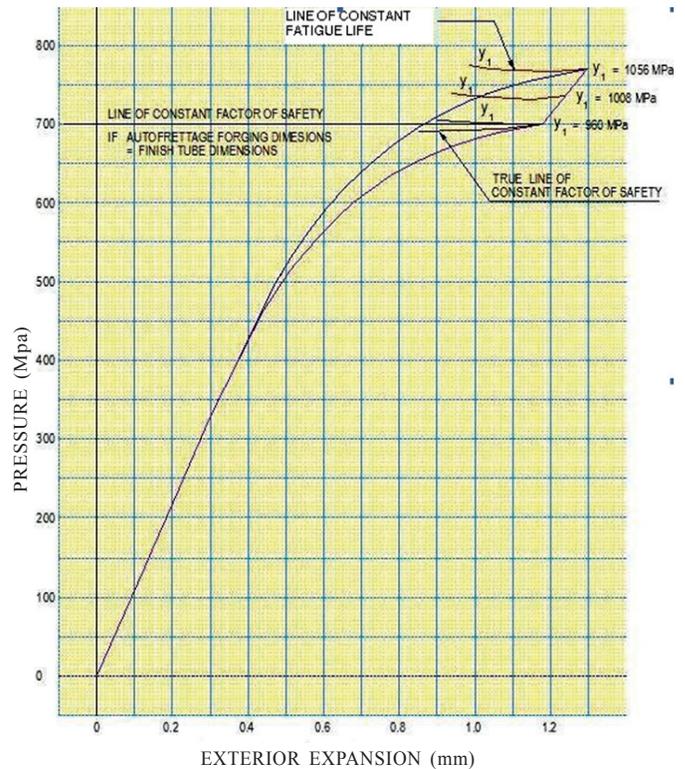


Figure 2. Curve representing the variation of pressure versus exterior expansion of outer diameter. (also called as the pressure exterior expansion) with line of constant factor of safety and lines of constant fatigue life.

been shown for $y_1 = 960$ MPa. Thus the variation of the autofrettage pressure for given range of 0.2% proof stress will take place along the line of constant factor of safety if the line of constant factor of safety is used for the estimation of autofrettage pressure. The same is also true for the line of constant fatigue life. Thus, new line defined as the line of constant fatigue life is obtained which gives a slight increase in autofrettage pressure with the increase in proof stress value. This concept has been validated in the following subsections.

3.1 Validation of the Concept

The concept of lines of constant factor of safety was validated by examining the following aspects

- (i) By shakedown analysis with the inclusion of Bauschinger effect.
- (ii) Actual measurement.
- (iii) Fatigue life study.

3.1.1 Validation of Shakedown Condition

The shakedown pressure has been worked out by considering the Huang's Bauschinger effect factor model given by Huang⁵ and Parker⁸, *et al.* as shown in Fig.3. The shakedown pressure has been derived by considering the pressure required to make the tube yield at the plastic loading-elastic unloading and elastic loading-elastic unloading boundary. In this case the unloading has been considered equal to the applied autofrettage pressure. The determination of shakedown pressure

involves following steps:

- (1) Calculation of net residual shear stress at the plastic loading–elastic unloading and elastic loading–elastic unloading boundary.
- (2) Determination of yield stress due to plastic loading of the tube.
- (3) The sum of shear stresses due to step (1) and step (2) must be equal to zero.

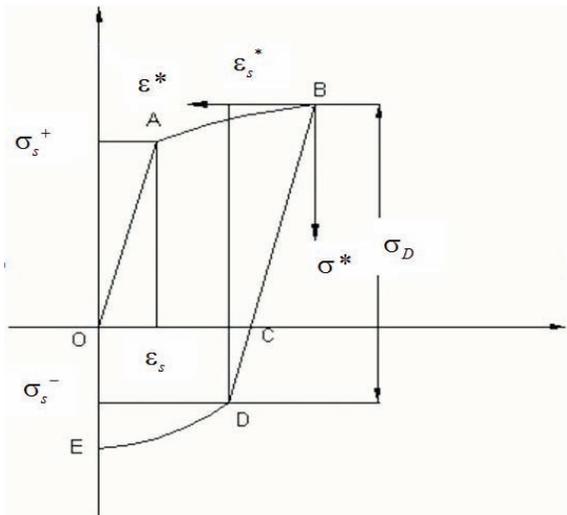


Figure 3. Variation of true stress vs strain for Huang’s model⁵.

Following the above procedure based on equations outlined by Chakraborty⁴, the equation of shakedown pressure is given as:

$$(n) + \frac{S_y - A_1}{\sqrt{3}B_1} \left(\frac{n_f^2}{k_f^2} - 1 \right) - \frac{S_y}{\sqrt{3}} \left(\frac{n_f^2}{k_f^2} - 1 \right) - \frac{\sigma_D}{\sqrt{3}} \left(\frac{n_{yr}}{n_f} \right)^2 + \frac{S_y}{2} \quad (9)$$

In the above the equation the symbols are described with reference to Fig. 3. For loading phase in linear elastic regime:

$$\sigma = E\varepsilon \quad (10)$$

For strain hardening regime A-B

$$\sigma = A_1 + A_2\varepsilon^{B_1} (\varepsilon \geq \varepsilon_y) \quad (11)$$

In the above equation A_1 can be interpreted as the elastic limit, A_2 is constant for plastic curve for loading and B_1 is the plastic strain index and ε and ε_y stands for strain and yield strain respectively. The values of parameters of elastoplastic curve are given as follows:

A1(MPa)	A2(MPa)	B1	Sry(MPa)	n_f	k_f
918.2	9157.0	1.0	1420.0	1.6828	1.725

The other symbols used in the equations are common to the previous equations and so the definition is also the same. The lines of constant factor of safety have been developed based on an isotropic hardening model. The Shakedown pressure obtained using Eqn. (11) is given in the table below:

Pressure	Eqn. (11)	Isotropic hardening model
p_{sd} in MPa	564	580

The value of maximum safe pressure predicted by Huang’s model is in good agreement with the isotropic hardening model. If the strain hardening and strain aging, due to accelerated ageing treatment are taken into account the value of S_y will be more than 960 MPa. This will render the values of MSP to be more conservative. Thus the lines of constant factor of safety comply with the shakedown condition even when the Bauschinger effect is considered. The equations derived above consider the machining of inside diameter followed by outside diameter of an autofrettage forging. This results in maximum possible loss of residual hoop stresses due to machining. In this way the value of pressure shakedown condition is estimated at the lowest limit.

3.1.2 Finite Element Simulation of Autofrettage Process

The calculations for shakedown condition based on the Bauschinger effect consideration were based on a constant Bauschinger effect factor. In actual case the value of ε_p varies from a maximum value at the bore to a minimum value at the autofrettage boundary. This results in a variation in the Bauschinger effect factor in a layer wise manner. This aspect has been investigated by finite element simulation assuming a bilinear kinematic hardening model using ANSYS software^{2,7}.

The finite element solution of the autofrettage and reverse yielding of a tube was developed using axisymmetric 4 noded quadrilateral elements, with 2x2 integration points, for a solid section of having inside radius of 75 mm and outside radius of 145.5 mm. The choice of diameter ratio of 1.94 is helpful in achieving an appreciable amount of reverse yielding since it is close to 2.2. The advantage of FEM solution is that it can take into account the variable over strain and apply variable degree of Bauschinger effect for reverse yielding. The material properties were : Young’s modulus (E) is 201000 MPa; Poisson ratio is 0.3 v; 0.2% proof stress is 960 MPa; Modulus of plasticity is 1950 MPa.

The asymmetric finite element mesh along with boundary conditions is shown in Fig. 4. The solution is based on Newmark-Beta predictor corrector method⁸.

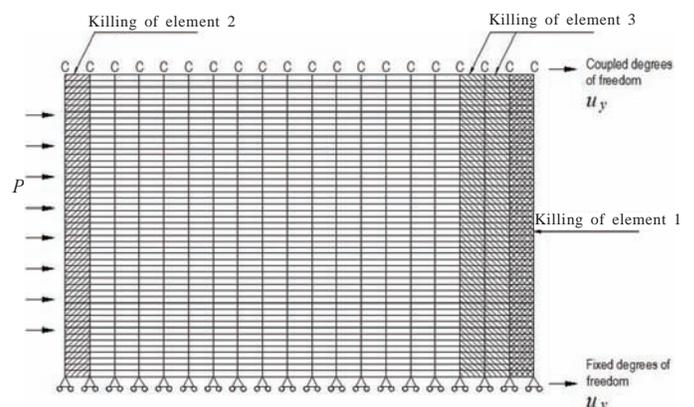


Figure 4. Finite element mesh and simulation of autofrettage and post autofrettage machining by killing of elements in sequence numbered as 1-2-3.

The post autofrettage machining was simulated by first killing the innermost column of elements followed by outermost column of elements and finally the two outermost columns of elements. In this way the machining of autofrettage forging in following sequence was simulated.

Outside machining → Inside machining → Outside machining².

The above sequence of the post autofrettage machining is the best manufacturing practice as it results in minimum loss of compressive residual stress due to machining and good concentricity. The element death formulation used by⁷ software is based on. Here, the killing of elements implies that the elements are deactivated by multiplication of element stiffness by a constant reduction factor^{2,9-11}, which is of the order of 10^{-6} .

The elastic response was observed during the re-pressurisation simulation of the autofrettaged tube up-to initial autofrettage pressure. The tube also gave elastic response at maximum safe pressure after post autofrettage machining simulation. Finite element simulation validated the shakedown condition for lines of constant factor of safety. The plots of variation of residual hoop stress for true lines of constant factor of safety and lines of constant factor of safety with 2 % residual error in solution of Eqn. (8) are shown in the Fig. 5. Maximum and minimum value of compressive residual stress corresponding to maximum and minimum value of a typical 0.2 % proof stress values for a gun tube can be determined. The curves show the variation of compressive residual stresses along the thickness of the tube section. From Fig. 5 it can be seen that if the autofrettage pressure is determined as per constant line of factor of safety there will be a

reduction of 10 % in compressive residual stress if the 0.2 % proof stress at the upper specification limit. In general if a heat treated and machined tube has 0.2 % proof stress it is quite possible that there may some hard spots along the length of tube where the proof stress may exceed the upper specification limit. Therefore if the autofrettage process is designed based on line of constant factor of safety there is possibility of reduction in compressive residual stress below 10 % value. The depth of autofrettage will be less than the designed autofrettage depth. These two things will result in reduction maximum safe pressure and reduction fatigue life. The thickness of the tube section after post autofrettage machining is 58.0 mm. Thus, if the tube is autofrettaged using the line of constant fatigue life, the fatigue life is obtained to be same at higher value of proof stress even if the residual hoop stress is lower than the residual hoop stress corresponding to the minimum value of proof stress. This is a natural outcome as the fatigue limit is a function of proof stress and so the fatigue life is a function of fatigue limit and residual hoop stress.

3.1.3 Fatigue Lifetimes Calculations

The effectiveness of an autofrettage process also needs to be evaluated in terms of the fatigue lifetimes of the autofrettaged tubes. The fatigue life in this case refers to repeated number of firing pressure cycles to which a cracked eroded or non eroded tube can be subjected such that the initial crack at the bore grows to the critical crack size. The fatigue lifetimes calculations are based on software developed by¹¹⁻¹³. The calculations presented in this paper are intended to only demonstrate the variation of fatigue life along the line of constant factor of safety. The input data for fatigue lifetimes calculations is as follows:

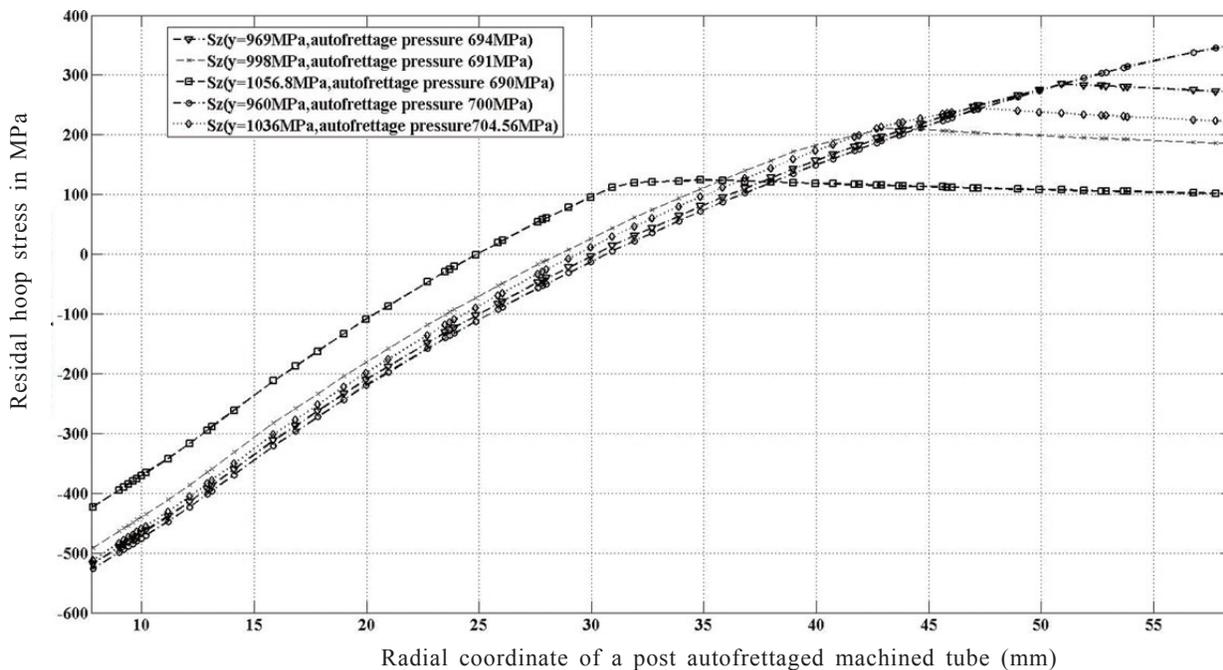


Figure 5. Finite element simulations of variation of residual hoop stress along the radius of the cross section of the barrel if the autofrettage pressure is obtained by the intersection of pressure versus exterior expansion curve and line of constant factor of safety at different values of yield stress (S_y).

0.2% proof stress (MPa)	Fracture toughness K_{IC} (MPa m ^{1/2})	No of cracks and grooves	% of tube thickness autofrettages	Correction Bauschinger effect	Depth of groove (mm)	Initial crack length (mm)
S_y min=960 S_y max=1056	131	1	85% at Min proof value of 0.2% proof stress	Simple Bauschinger corrections	3.0	0.2

and 48 % at $S_y = 1056$ MPa (depth as measured from the inside diameter along the semi minor axis of the groove).

The Paris law equation is given as :

$$\frac{da}{dN} = C(\Delta K)^n \tag{12}$$

where a is the crack length, N is the number of cycles C is the Paris coefficient, n is the Paris law index and ΔK is the change in stress intensity factor. The values of Paris law constants are given as, $C = 6.52 \times 10^{-12}$ and $n = 3$.

The groove geometry is shown in Figs. 6 and 7. The calculations show that along the lines of constant factor of safety there is 10 % reduction in fatigue life. The fatigue life at $S_y = 960$ MPa has been found to be 116 cycles and 2003 cycles for bore condition of with and without erosion groove respectively. Here, a load cycle is defined as the pressurization of a tube to the maximum permissible pressure followed by unloading to zero pressure. The fatigue life at $S_y = 1056$ MPa falls to 98 cycles and 1835 cycles for the condition of bore with and without erosion groove respectively. If a 2% residual error is permitted in the accuracy of solution of the equation for line of constant factor of safety at $S_y = 1056$ MPa the degree of autofrettage increases to 54 %. The fatigue lifetimes obtained for this case are 108 cycles and 1935 cycles for bore condition of with and without erosion groove respectively. The variation in fatigue lifetimes is within 10 % because the compressive residual stress at the bore is almost same therefore the stress intensity factor for the same crack geometry is same. Since the initial crack length for both the cases is same therefore fatigue life remains almost same for both cases and so the concept of line of constant fatigue life is valid. Thus, the new line obtained can be defined as

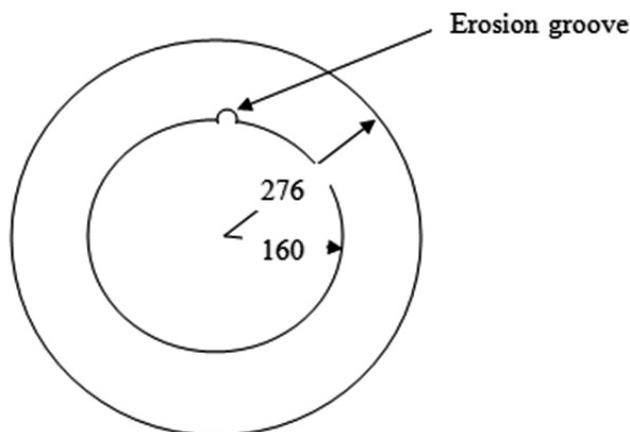


Figure 6. Schematic of tube with erosion groove for fatigue life time estimation.

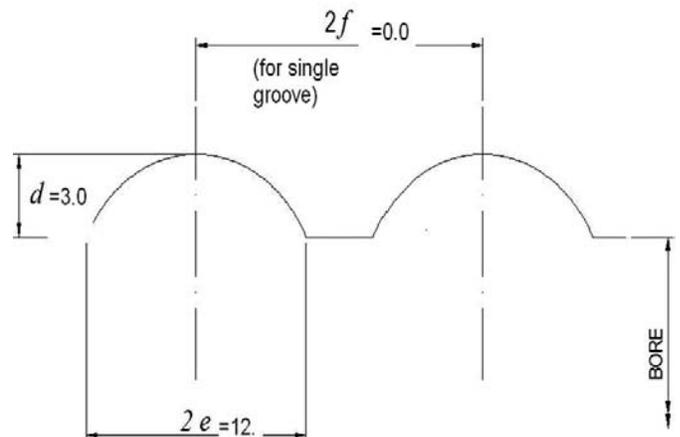


Figure 7. Generalised erosion groove geometry of the bore of the tube for fatigue life times calculations.

the line of constant fatigue life. If a pressure exterior expansion curve of an autofrettage forging intersects the line of constant fatigue life the corresponding test pressure verifies factor of safety, fatigue life and the safety of the barrel against the flaws which are smaller than the scale of ultrasonic standard.

Thus an autofrettage process design proposed above results in a comprehensive improvement in the effectiveness of a hydraulic autofrettage process.

3.1.4 Validation by Actual Measurements

The solution was also validated by comparing the permanent set at the bore as obtained by the FEM solution with the value obtained by the equation given by Newhall¹⁴. The latter equation has been found to give a bore deformation in very good agreement with the measured value. The FEM solution gives the permanent bore deformation after autofrettage as 0.48 mm and the calculated value using equation from Newhall¹⁴ is 0.528 mm. The measured value of permanent bore strain is 0.5 mm. The actual measurements indicate that the isotropic hardening behavior is in better agreement having 5.6 % error as compared to FEM. These measurements were done for 20 tubes of the production lot and the standard deviation was found to be 0.0023 mm.

The FEM model also closely agrees with the actual measurement. This is because if the tube wall thickness ratio and degree of autofrettage are chosen for optimal results the Bauschinger effect is minimised. Since the Bauschinger effect decreases radially from bore towards the elasto-plastic boundary therefore a thin reverse yielding boundary is obtained and the value of bore deformation is in fair agreement with the Newhall

model¹⁴. The FEM results are in close agreement with the actual bore measurements because the FEM model is able to account for variable Bauschinger effect. Since the predicted permanent bore deformation is within the acceptable limits of permissible errors and so both the model can be considered to be valid. Thus, the concept was considered to be validated both theoretically and experimentally.

4. CONCLUSION

The concept of selection of autofrettage pressure based on lines of constant factor of safety was derived by equating the maximum safe pressure for minimum value of yield stress to the given value of yield stress. The investigation was further extended to introduce the concept of constant fatigue life for all the value of yield stress in the specified range of yield stress for the tube material. It was found that if the residual error in prediction of maximum safe pressure for the line of constant factor of safety is 10 % the line of constant fatigue life is obtained. The line of constant fatigue life ensures that the factor of safety and fatigue line are not affected by the axial variation of yield stress for a given tube. The concept has been validated by finite element simulation of ANSYS by considering the loss of compressive residual stress due to Bauschinger effect and machining of the tube. The fatigue life of the tube has been verified for cracked and eroded geometry of tube. The concept gives good operational economy and the application of test pressure leads to the verification of the fatigue life and factor of safety of the tube. The concept is particularly useful for the hydraulic autofrettage because during hydraulic autofrettage also subjects the barrel to pressure testing against any flaw that might have escaped ultrasonic law detection.

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