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SHORT COMMUNICATION

Burn Rate Modelling of Solid Rocket Propellants

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ABSTRACT

A generalised model of burning of a solid rocket propellant based on kinetics of propellant has been developed. A complete set of variables has been formed after examining the existing models. Buckingham theorem provides the functional form of the model, such that the existing models are the subcases of this generalised model. This proposed model has been validated by an experimental data.

NOMENCLATURE

A	Frequency factor in Arrhenius equation
r_b	Burn rate
P	Pressure
ρ	Thermal conductivity
Q	Heat release
E	Activation energy
R_o	Universal gas constant
T	Temperature
C	Heat capacity
Z_s	Pre-exponential factor
n	Pressure index
a	Burn rate coefficient
w	Rate of generation of species
e	Density
g	Gas
s	Solid

1. INTRODUCTION

Burn rate plays a significant role in the study of solid rocket propellants. Two approaches (experimental and theoretical) are followed to study the burn rate behaviour of propellants. In the experimental approach, data is generated either by burning propellant strands or by testing small grains, whereas in the theoretical approach, burn rate is predicted following mathematical models. In these models, two methods followed are: Chemist's approach and Aerothermochemist's approach. In the Chemist's approach, chemical reactions and reaction rates are considered and burn rates are predicted ignoring physical processes. However, in Aerothermochemist's approach, considerations are mainly on the physical processes like diffusion, mass, heat, momentum transfer, etc. which predominates the rocket propellant combustion. This work is aimed at developing a model that

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incorporates the parameters of chemical kinetics and physical processes.

2. BURN RATE PREDICTIONS

2.1 Mathematical Models

Several models based on theoretical aspects have been proposed. N. Kubota & T. Masamoto[1] have their model based on flame structure of double base propellants and arrived at the following equation:

$$r_b = p\left[\frac{\lambda g Q g \mathrm{E}^2 Z g \exp(-Eg \quad RTg)}{\rho_s^2 C_s^2 (T_s - T_o - \frac{Q_s}{C_s}}\right]^{1/2} \tag{1}$$

Assuming the regression at the surface to be one step, Arrhenius-type reaction, the burn rate could be expressed as

$$r_b = Z_s \exp(-E_s \quad RT_s) \tag{2}$$

Zeldovich derived burn rate equation by considering that the burning is controlled by a condensed phase reaction as

$$r_b = \left|\frac{2 Q_s R_o T_s \lambda A \exp(-E_s \quad RT_s)}{\rho_s E_s C_s^2 (T_s - T_{o)}^2}\right]^{1/2} \tag{3}$$

I Shih Tseng & Vigor Yang[2] derived the burn rate equation as

$$r_b = \frac{\beta \alpha_s A_c \exp(-\beta)}{(1 - \frac{T_o}{T} - \frac{Q_s}{}}^{1/2} \tag{4}$$

Victor K Bulgakov, *et al*[3]. in their study on erosive burning have derived the burn rate equation

$$r_b = \left|\frac{\lambda T_s^2 R_o A \exp\left(\frac{-E_s}{R_o T_s}\right)}{E_s \rho_s (C_s (C_s (T_s - T_o) - \frac{Q_s}{2}}\right.^{1/2} \tag{5}$$

2.2 Empirical Equations

Several empirical equations have been proposed[4-8]. Pressure-burn rate relationship is given in the equation as

$$r_b = ap^{n} \tag{6}$$

and for all practical purposes, Vieille's law is being used in propulsion technology.

3. MATHEMATICAL ASPECTS

The burn rate formulae discussed above and the physics of combustion wave structure clearly indicate that the burn rate r_b is to be related to the parameters of chemical kinetics and physical processes.

The dimensional matrix of exponents are given in Table 1.

Table 1. Matrix of exponents

*	r_b	λ	$\dot{w}$	Q_g	C_s	ρ_s	T'	P	E	R	T_s
L	1	1	-3	2	2	-3	0	-1	2	2	0
M	0	1	1	0	0	1	0	1	0	0	0
T	-1	-3	-1	-2	-2	0	0	-2	-2	-2	0
θ	0	-1	0	0	-1	0	1	0	0	-1	1

* Fundamental quantities

The parameters like density of propellant, pressure, temperature and reaction rate that can be measured easily and play dominant role in burn rate, are chosen as controlling parameters and the fundamental quantities (length, mass, temperature) can be expressed in terms of these four variables.

The model contains eleven variables to connect themselves as a physical process to express burn rate as

$$f(r_b, \lambda, \dot{w}, Q_g, C_s, \rho_s, T', P, E, R, T_s) = 0 \tag{7}$$

Since four fundamental quantities of mass, length, time and temperature appear to describe these variables, Buckingham's non-dimensional analysis theorem suggests that seven non-dimensional groups may be formed to describe the physical process as

$$f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) = 0 \quad (8)$$

where

π_i's are non-dimensional groups defined as

$$\pi_1 = r_b \sqrt{\frac{\rho_s}{p}}, \pi_2 = \frac{\lambda \dot{w} T'}{p_2}, \pi_3 = \frac{Q\rho_s}{P}, \pi_4 = \frac{C_s \rho_s T'}{P}$$

$$\pi_5 = \frac{E\rho_s}{p}, \pi_6 = \frac{R\rho_s T'}{p}, \pi_7 = \frac{T_s}{T'}$$

The corresponding multimonomial law of burning is expressed as

$$\pi_1 = \pi_2^{a_1} \pi_3^{a_2} \pi_4^{a_3} \pi_5^{a_4} \pi_6^{a_5} \pi_7^{a_6} \quad (9)$$

where each of the exponent is unknown.

Based on the above equations, the assumed data of burn rate models has been reported in Table 2.

Table 2. Assumed values of indices in different models

Eqn No.	a1	a2	a3	a4	a5	a6
1	0.5	0.5	-1.0	0.0	0.0	
3	0.5	0.0	-0.5	-0.5	-0.5	
4	0.5	0.0	0.5	0.5	-0.5	
5	0.5	0.0	-0.5	-0.5	0.5	

4. PROPOSED BURN RATE MODEL

All the models (Table 2) suggest that the burn rate equations have been locally assumed to be explicitly independent of pressure p. Then equating the indices of p to zero, we get

$$0.5 - 2\,a1 - a2 - a3 - a4 - a5 = 0$$

A rearrangement of non-dimensional groups with a5 eliminated from the above results leads to

$$r_b = \sqrt{RT'} \left(\frac{\lambda \dot{w}}{\rho_s^2 R^2 T'}\right)^{a_1} \left(\frac{Q}{RT'}\right)^{a_2} \left(\frac{C_s}{R}\right)^{a_3} \left(\frac{E}{RT'}\right)^{a_4} \left(\frac{T_s}{T'}\right) \quad (10)$$

The values of different constants[1] used and the data on burn rate for calculations are given in Tables 3 and 4. The indices of the above empirical law have been evaluated from the experimental data given in Table 4. The surface temperatures at different pressures have been computed by iteration using Eqn (2) and the proposed burn rate equation.

Table 3. Values of different constants

Constants	Values
C_p	0.37 cal/g/ k
C_g	0.37 cal/g/ k
e	1.54 g/cm
E_i	14000 cal/mole
Z_i	3600 cal/s
Q_g	100 cal/g
Q_s	300 cal/g

Table 4. Burn rate data on CMDB propellant formulations

Propellant composition Particle size				Para-meters	Pressure (kg/cm^2)				
DNC	CL	AP	AP		35	50	70	90	120
60	40		-	r (mm/s)	5.0	5.9	7.1	8.2	9.8
				T_s (°K)	630	642	658	670	690
50	40	10	150	r (mm/s)	5.6	6.2	7.5	8.7	10.2
				T_s (°K)	641	659	665	690	700

Table 5. Calculated values of indices in different models

Eqn No.	a1	a2	a3	a4	a5	a6
1	0.47				0.00	
3	0.43				-0.39	
4	0.57				0.45	
5	0.61				-0.40	

Table 5 shows that a2, a3, a4, and a6 are reasonably close to 0.5, -1.0, -0.5, 1.0, respectively, whereas the values of a1 varies from 0.43 to 0.61. The formula of burn rate in terms of a1 is

$$r_b = \left(\frac{\lambda_s R_o \dot{w}}{\rho_s^2 C_s^2}\right) \frac{\sqrt{Q}}{E} \frac{T_s}{T'} \quad (11)$$

The computed values of a1 for five formulations in composite modified double-base (CMDB) propellant are shown in Table 6.

Table 6. Computed values of a1 for five propellant formulations

Propellant formulations	Pressure (kg/cm^2)	Burn rate (mm/s)	Surface Temp.(^oK)	a1
	35	5.0	630	0.432
	70	7.1	649	0.461
	140	10.9	676	0.441
II	35	3.2	606	0.424
	70	5.1	631	0.432
	140	8.7	663	0.422
III	35	6.0	640	0.418
	70	8.1	658	0.419
	140	12.2	684	0.416
	35	9.3	666	0.413
	70	11.8	683	0.406
	140	16.9	706	0.441
V	35	11.3	679	0.407
	70	14.3	695	0.407
	140	19.5	717	0.408

The above analyses indicates that the proposed burn rate model comprises three terms. In combustion mechanism, these terms represent chemical kinetics, chemical potential and significant temperature ratio (T_s/T) relevant to predict the burn rate. Chemical potential appears to be the same for all the rocket propellant formulations, and thus, change in this term does not affect burn rate to any appreciable extent. The temperature ratio T_s/T and the chemical kinetics, both play significant role in controlling the burn rate. T is, in turn, governed by the surface temperature T_s. The factor T which is the temperature just beneath the burning surface is found to affect the burn rate appreciably. The chemical kinetics which contains a species generation rate is found to affect the burn rate considerably. The value of the exponent a1 is governed by the rate of chemical reaction and thus, the parameter a1 is a function of the propellant. The proposed burn rate equation has been validated for five propellant formulations belonging to CMDB class of propellant indicating the consistent values of the parameter a1.

5. CONCLUSION

A detailed examination of the burn rate characteristics was made for different models and a unified model has been formulated in terms of three groups, namely chemical potential, chemical kinetics and significant temperature ratio. This model can be used to predict burn rates for a different CMDB class of propellant. This model is of interest in getting burn rate data for diverse propellant formulations incorporating nitramine, ammonium perchlorate and *Al* particularly, in view of the coincidence between the experimental and the burn rate predicted by this model at different pressures.

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REFERENCES

Kubota, N. & Masamoto, T. Flame structures and burn rate characteristics of CMDB propellants. 16th International Symposium on Combustion. Combustion Institute, Pitsburgh, 1977. pp. 1201-09.

2. I. Shih; Tseng & Vigor, Yang. Combustion of a double-base homogeneous propellants in rocket motor. *Combustion and Flame*, 1994, **96**, 325-42.

3. Victor, K. Bulgakov, *et al.* Numerical studies of solid propellants erosive burning. *J. Prop. Power*, 1983, **6**, 812-18.

4. Penner, S.S. Chemical rocket propulsion and combustion research. Garden and Breach Science Publishers, New York, 1962. pp. 132-37.

5. Rice, O.K. Theory of burning of rocket powders.1955. USRD Report No. 5574.

6. Voss Eldbe G., *et al.* Chemical kinetics and physical processes in composite solid propellant combustion. Atlantic Research Corp., 1967. Report. No. 66307.

7. Rastogi, *et al.* Pressure dependence of burning rate of composite solid propellants. *AIAA Journal*, 1978, **16**, 534-36.

8. Barrer, M., *et al.* Rocket propulsion. Elsevier Publishing Company, UK, 1960.

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