# Determination and Validation of Parameters for Riedel-Hiermaier-Thoma Concrete Model 

Yu-Qing Ding*, Wen-Hui Tang, Ruo-Qi Zhang, and Xian-Wen Ran<br>National University of Defense Technology, Changsha - 410 073, China<br>*E-mail: yqding_nudt@163.com


#### Abstract

Numerical modelling of the complex physical processes such as concrete structures subjected to highimpulsive loads relies on suitable material models appropriate for impact and explosion problems. One of the extensive used concrete material models, the RHT model, contains all essential features of concrete materials subjected to high dynamic loading. However, the application of the RHT model requires a set of material properties and model parameters without which reliable results cannot be expected. The present paper provides a detailed valuation of the RHT model and proposes a method of determining the model parameters for C40 concrete. Furthermore, the dynamic compressive and tensile strength function of the model formulation are modified to enhance the performance of the model as implemented in the hydrocode AUTODYN. The performance of the determined parameters of the modified RHT model is demonstrated by comparing to available experimental data, and further verified via simulations of physical experiments of concrete penetration by steel projectiles. The results of numerical analyses are found closely match the penetration depth and the crater size in the front surface of the concrete targets.


Keywords: Concrete, impact, Riedel-Hiermaier-Thoma, RHT model, model calibration

## 1. INTRODUCTION

Concrete is a kind of widely used structural material in civil and defence construction, and numerical simulations are an important tool in the investigation of the effects of blast and shock on concrete structure. Recent years, research has been conducted to develop efficient and accurate constitutive models ${ }^{1-4}$ to improve the fidelity of the numerical simulations. The Riedel-Hiermaier-Thoma (RHT) concrete model, as a coupled damage-viscoplasticity model, developed by Riedel ${ }^{5}$ is readily available to all users of the commercial hydrocode AUTODYN ${ }^{6}$. Over the last decade, numerous worldwide applications appeared in publications which deal with dynamic load cases such as projectile penetration, contact detonation, internal and external blast loading. However, the application of the RHT model requires a set of suitable model parameters without which reliable results cannot be expected. Moreover, the standard RHT model implemented in AUTODYN falls short in representing the concrete behaviour under the dynamic compression and tension loading. In present paper, the modifications of using two bilinear dynamic increase factor functions for the compressive and tensile strength were proposed through user codes. Furthermore, a method to determine the parameters for RHT concrete model was proposed, and the determined parameters of C40 concrete were validated by simulations of penetration test.

## 2. RIEDEL-HIERMAIER-THOMA CONCRETE MODEL

Riedel-Hiermaier-Thoma(RHT) concrete model couple an equation of state (EoS) that account for the porous compaction of concrete with the RHT strength model contains three limit surfaces in stress space which considering pressure, triaxiality and strain rate. The three surfaces respectively describe the elastic limit $Y_{\text {el }}$, failure $Y_{\text {fail }}$, and residual shear strength $Y_{\text {res }}$ of the damaged concrete under confined conditions.

### 2.1 Failure Surface

The failure surface, $Y_{\text {fail }}$, is defined as a function of the strength along the compression meridian $Y_{c}(p)$ multiplied by the factors $R_{3}(\theta)$ and $F_{\text {rate }}$ which has the form,

$$
\begin{gather*}
f\left(p, \sigma_{\mathrm{eq}}, \theta, \dot{\varepsilon}\right)=\sigma_{\mathrm{eq}}-Y_{\text {fail }}(p, \theta, \dot{\varepsilon})=\sigma_{\mathrm{eq}}-Y_{\mathrm{c}}(p) R_{3}(\theta) F_{\mathrm{rate}}(\dot{\varepsilon})=0  \tag{1}\\
Y_{\text {fail }}^{*}\left(p^{*}, \theta, \dot{\varepsilon}\right)=Y_{\mathrm{c}}^{*}\left(p^{*}\right) R_{3}(\theta) F_{\mathrm{rate}}(\dot{\varepsilon})  \tag{2}\\
Y_{\mathrm{c}}^{*}\left(p^{*}\right)=A \times\left[p^{*}-f_{\mathrm{tt}}^{*} F_{\mathrm{rate}}(\dot{\varepsilon})\right]^{N} \tag{3}
\end{gather*}
$$

where $\sigma_{\mathrm{eq}}$ is the equivalent stress, $p$ is the pressure, $\theta$ is the lode angle, $\dot{\varepsilon}$ is the equivalent strain rate, $A$ and $N$ are the two constants. All measures of hydrostatic pressure and deviatoric strength denoted with * are normalized over the uniaxial cylindrical compressive strength $f_{\mathrm{c}} . f_{\mathrm{ttt}}^{*}$ is the normalized
hydrostatic tensile pressure (compressive stresses defined positive).
$R_{3}(\theta)$ is used to describe reduced strength on shear and tensile meridians. The Lode angle $\theta$ describes stress triaxiality and depends on the third invariant $J_{3}$ of the stress tensor. The ratio $Q_{2}$ of tensile to compressive meridian decreases with increasing pressure. This effect is called 'brittle to ductile transition' and is described by,

$$
\begin{equation*}
R_{3}(\theta)=\frac{2\left(1-Q_{2}^{2}\right) \cos \theta+\left(2 Q_{2}-1\right) \cdot \sqrt{4\left(1-Q_{2}^{2}\right) \cos ^{2} \theta+5 Q_{2}^{2}-4 Q_{2}}}{4\left(1-Q_{2}^{2}\right) \cos ^{2} \theta+\left(1-2 Q_{2}\right)^{2}} \tag{4}
\end{equation*}
$$

$$
\begin{array}{ll}
\cos 3 \theta=\frac{3 \sqrt{3} J_{3}}{2 J_{2}^{3 / 2}}, & 0 \leq \theta \leq \frac{\pi}{3} \\
Q_{2}=Q_{2,0}+B_{Q} p^{*}, & 0.5 \leq Q_{2} \leq 1.0 \tag{6}
\end{array}
$$

where $Q_{2,0}$ is tensile to compressive meridian ratio, $B_{\mathrm{Q}}$ is brittle to ductile transition factor.

The term $F_{\text {rate }}$ accounts for the rate enhancement of strength for both compression and tension is expressed as a linear function of the strain rate in the logarithmic scale as follow,

$$
F_{\mathrm{rate}}(\dot{\varepsilon})=\frac{f_{\mathrm{cdttd}}}{f_{\mathrm{c} / t}}= \begin{cases}\left(\dot{\varepsilon} / \dot{\varepsilon}_{\mathrm{c} 0}\right)^{\alpha} & \text { for compression }  \tag{7}\\ \left(\dot{\varepsilon} / \dot{\varepsilon}_{\mathrm{t} 0}\right)^{\delta} & \text { for tension }\end{cases}
$$

where $f_{\mathrm{c} / \mathrm{t}}$ is the static uniaxial compressive and tensile strength, $f_{\text {cdtd }}$ is the dynamic uniaxial compressive and tensile strength, $\alpha$ and $\delta$ are the constants. The static strain rate was taken as $3.0 \times 10^{-5} \mathrm{~s}^{-1}$ for compression and $3.0 \times 10^{-6} \mathrm{~s}^{-1}$ for tension.

### 2.2 Elastic Limit Surface and Strain Hardening

The initial elastic surface $Y_{\text {el }}$ of the virgin material is derived from the failure surface $Y_{\text {fail }}$ using the ratio of elastic compressive and tensile stress over the respective ultimate strength $F_{\text {el }}\left(f_{\mathrm{c}, \mathrm{el}} / f_{\mathrm{c}}\right.$ and $\left.f_{\mathrm{t}, \mathrm{el}} / f_{\mathrm{t}}\right)$. The elastic surface is consistent with the porous equation of state towards higher pressures involving pore compaction using a parabolic cap function $F_{\text {cap }}$. The upper cap pressure is equal to Hugoniot elastic limit $p_{\text {el }}$ of the concrete material, and the lower pressure $p_{\mathrm{u}}$ for cap influence is set to $f_{\mathrm{c}} / 3$. The loading surface $Y_{\text {load }}$ described by Eqn (10) is scaled between $Y_{\text {el }}$ and $Y_{\text {fail }}$ controlled by the equivalent plastic strain. The plastic stiffness is specified by the hardening ratio $G_{\mathrm{el}} /\left(G_{\mathrm{el}}-G_{\mathrm{pl}}\right)$ which is equal to 2.0.

$$
\begin{align*}
& Y_{\mathrm{el}}^{*}=Y_{\text {fail }}^{*} \cdot F_{\text {cap }}(p) \cdot F_{\mathrm{el}}  \tag{8}\\
& F_{\text {cap }}(p)= \begin{cases}1 & \text { for } p \leq p_{\mathrm{u}}=3 / f_{\mathrm{c}} \\
\sqrt{1-\left[\left(p-p_{\mathrm{u}}\right) /\left(p_{0}-p_{\mathrm{u}}\right)\right]^{2}} & \text { for } p_{\mathrm{u}}<p \leq p_{\mathrm{el}} \\
0 & \text { for } p_{\mathrm{el}} \leq p\end{cases}  \tag{9}\\
& Y_{\text {load }}=Y_{\mathrm{el}}+\frac{\varepsilon_{\mathrm{eq}}^{\mathrm{pl}}}{\varepsilon_{\mathrm{eq}}^{\mathrm{pl}, \text { load }}}\left(Y_{\text {fail }}-Y_{\mathrm{el}}\right), \quad \varepsilon_{\mathrm{eq}}^{\mathrm{pl}, \text { load }}=\frac{Y_{\text {fail }}-Y_{\mathrm{el}}}{3 G}\left(\frac{G_{\mathrm{el}}}{G_{\mathrm{el}}-G_{\mathrm{pl}}}\right) \tag{10}
\end{align*}
$$

### 2.3 Damage Evolution and Residual Surface

When hardening states reach the ultimate strength of the concrete on the failure surface $Y_{\text {fail }}$, damage is accumulated during further inelastic loading controlled by plastic strain according to equation as below,

$$
\begin{equation*}
D=\sum \frac{\Delta \varepsilon_{\mathrm{p}}}{\varepsilon_{\mathrm{p}}^{\mathrm{f}}}, \quad \varepsilon_{\mathrm{p}}^{\mathrm{f}}=D_{1}\left(p^{*}-f_{\mathrm{tt}}^{*}\right)^{D_{2}} \geq \varepsilon_{\mathrm{f}, \min } \tag{11}
\end{equation*}
$$

where $\Delta \varepsilon_{\mathrm{p}}$ is the accumulated plastic strain, $\varepsilon_{\mathrm{p}}{ }^{\mathrm{f}}$ is the equivalent plastic strain at failure, $D_{1}$ and $D_{2}$ are the constants. At low pressure, a lower limit of the failure strain is set by introducing $\varepsilon_{\mathrm{f} \text { min }}$ to allow for a finite amount of plastic strain to fracture the material in order to suppress fracture from low magnitude tensile waves.

Under a multi-axial state of stress and existing confining pressure, the concrete retains a certain level of shear strength due to friction among crushed particles. The residual strength $Y_{\text {res }}$ of the fully damaged concrete is calculated from equation below. The strength $Y_{\text {frac }}$ is then interpolated from the strength values for the undamaged material $(D=0)$ at failure surface and the completely damaged material $(D=1)$ described below,

$$
\begin{align*}
& Y_{\mathrm{res}}=B \cdot\left(p^{*}\right)^{M}  \tag{12}\\
& Y_{\mathrm{frac}}=D \cdot Y_{\mathrm{res}}+(1-D) \cdot Y_{\mathrm{fail}}
\end{align*}
$$

### 2.4 Equation of State

In the RHT model, for pressures between the initial pore crush pressure $p_{\mathrm{el}}$ and compacted pressure $p_{\text {comp }}$, the $P-\alpha$ model is employed as follows,

$$
\begin{align*}
& p=f\left(\rho_{\text {matrix }}, e\right) \xrightarrow{\text { porous }} p=f(\alpha \rho, e)  \tag{13}\\
& \alpha=1+\left(\alpha_{0}-1\right)\left[\left(p_{\text {comp }}-p\right) /\left(p_{\text {comp }}-p_{\text {el }}\right)\right]^{n}
\end{align*}
$$

where $\alpha_{0}$ is the initial porosity, $n$ is the exponent constant.
For the compaction state, EoS has the form described below,

$$
\begin{equation*}
p=A_{1} \mu+A_{2} \mu^{2}+A_{3} \mu^{3}+\left(B_{0}+B_{1} \mu\right) \rho_{0} e \tag{14}
\end{equation*}
$$

and for the tension state,

$$
\begin{equation*}
p=T_{1} \mu+T_{2} \mu^{2}+B_{0} \rho_{0} e \tag{15}
\end{equation*}
$$

where $\mu=\rho / \rho_{0}-1, \rho_{0}$ is the initial density, $A_{1}, A_{2}, A_{3}, B_{0}, B_{1}, T_{1}$ and $T_{2}$ are the parameters for polynomial EoS.

## 3. DETERMINATION OF PARAMETERS

### 3.1 Strength Parameters

The C40 concrete specimen was made of Portland 42.5 cement, general river sand, limestone aggregate and tap water with a composition of $1: 1.38: 2.67: 0.41$ by weight. The initial density of concrete was $2.35 \mathrm{~g} / \mathrm{cm}^{3}$. Cubic specimens was 150 mm in length for uniaxial compression experiment and splitting experiment were prepared and cured for 28 days before the experiment according to Chinese standard GB/T 50081-2002. Both uniaxial compression and splitting experiments were conducted on the instron-1346 material testing system machine. The cubic uniaxial compressive strength $f_{\mathrm{cu}}$ was 40 MPa , the splitting tensile strength $f_{\text {tp }}$ was 3.1 MPa, and the shear modulus $G$ was 14.5 GPa derived from the elastic modulus $E$ and poisson's ratio $v$ obtained from the
uniaxial compression experiment by $G=E / 2(1+v)$. Since the size for concrete specimen of Chinese standard is different with other countries (e.g. Germany, cylinder with 150 mm in diameter and 300 mm in height ${ }^{7}$ ), the uniaxial compressive strength of the same mix proportion concrete will be different by reference to different standards. The conversion coefficient between uniaxial cylindrical and cubic compressive strength is set to 0.8 , and the uniaxial tensile strength $f_{\mathrm{t}}$ is approximately equal to $f_{t p}$ by referring Chinese standard GB 50010-2002. Consequently, the uniaxial cylindrical compressive strength $f_{\mathrm{c}}$ was set to 32 MPa , the tensile strength ratio $f_{\mathrm{t}} / f_{\mathrm{c}}$ was equal to 0.1. In addition, the shear strength $f_{\mathrm{s}}$ was defined by strength ratio $f_{s} / f_{\mathrm{c}}$ which setting to 0.18 , together with the parameters $f_{\mathrm{c}, \mathrm{el}} / f_{\mathrm{c}}$ and $f_{\mathrm{t}, \mathrm{el}} / f_{\mathrm{t}}$ which setting to 0.53 and 0.7 respectively ${ }^{8}$.

In the appendix of GB 50010-2002, several characteristic strengths of concrete under static loading were provided for determine the parameters of certain concrete failure criterion. The characteristic strengths including hydrostatic tensile strength $f_{\text {ttt }}=-0.09 f_{\mathrm{c}}$, uniaxial compressive strength $f_{\mathrm{c}}\left(p^{*}\right.$ $=1 / 3, \tau_{\text {oct }}^{*}=1$ ), bi-axial compressive strength $f_{\mathrm{cc}}\left(p^{*}=0.853\right.$, $\tau_{\text {oct }}^{*}=0.603$ ), triaxial compressive strength $f_{\text {txcl }}\left(p^{*}=4.3, \tau^{*}\right.$ oct $=2.0)$ and $f_{\mathrm{txc} 2}\left(p^{*}=5.5, \tau^{*}{ }_{\text {oct }}=3.3\right)$ were applied to determine the failure strength parameters $A, N, Q_{2,0}$ and $B_{\mathrm{Q}} \cdot \tau_{\text {oct }}^{*}$ is the normalized octahedral shear stress.

In the condition of concrete under static compressive loading, the rate enhancement factor $F_{\text {rate }}$ is equal to $1.0, \theta$ is equal to $60^{\circ}$, the Eqn (2) becomes as below,

$$
\begin{equation*}
Y_{\text {fail }}^{*}\left(p^{*}, \theta\right)=(3 / \sqrt{2}) \times \tau_{\mathrm{oct}}^{*}=A \times\left[p^{*}-f_{\mathrm{ttt}}^{*}\right]^{N} \times R_{3}(\theta) \tag{16}
\end{equation*}
$$

The parameters $A$ and $N$ were obtained by three characteristic strengths $f_{\mathrm{ttt}}, f_{\mathrm{c}}$ and $f_{\mathrm{txc} 2}$, then the equations can be established as,

$$
\left\{\begin{array}{l}
A \times(1 / 3+0.09)^{N}=1  \tag{17}\\
A \times(5.5+0.09)^{N}=(3 / \sqrt{2}) \times 3.3
\end{array}\right.
$$

We get $A=1.929, N=0.764$. In the condition of concrete under tensile loading, $\theta$ is equal to $0^{\circ}$. By plugging $A$ and $N$


Figure 1. Experimental concrete shear strength response and RHT model prediction.
into the Eqn (16), the parameters $Q_{2,0}$ and $B_{\mathrm{Q}}$ were obtained by two characteristic strengths $f_{\mathrm{cc}}$ and $f_{\mathrm{txc} 1}$, then the equations can be established as,

$$
\left\{\begin{array}{l}
1.929 \times 0.9433^{0.764} \times\left(Q_{2,0}+0.853 B_{Q}\right)=(3 / \sqrt{2}) \times 0.603 \\
1.929 \times 4.39^{0.764} \times\left(Q_{2,0}+4.3 B_{Q}\right)=(3 / \sqrt{2}) \times 2.0
\end{array}\right.
$$

We get $Q_{2,0}=0.69, B_{\mathrm{Q}}=0.0048$. Fig. 1, illustrates the prediction of the RHT model for the compression (CM) and tension meridians (TM) using the Eqn (16). It is noted that MRHT means using the modified failure strength parameters, while RHT means using the default failure strength parameters in AUTODYN material database ${ }^{6}$. It is obvious that the meridian curves plotted by the modified failure strength parameters are more close to the experimental data ${ }^{9}$.

Over the past few decades a great amount of experiments have been carried out on the behaviour of concrete specimens under high rates of uniaxial compressive loading as well as tensile loading. A thorough bibliography of the abundant experimental data can be found ${ }^{10,11}$. A summary of the available experimental data is presented in Figs. 2 (a) and (b) for the cases


Figure 2. Variation of dynamic increase factor with strain rate for concrete: (a) uniaxial compression, (b) uniaxial compression.
of compressive and tensile loading respectively, expressing the relationship between the DIF (dynamic increase factor, the ratio of the dynamic to static strength) and the strain rate. In the standard RHT model as implemented in AUTODYN, the DIF is determined by the parameter $\alpha$ and $\delta$ for compression and tension respectively, see Eqn (7). As seen in Fig. 2, for two values of $\alpha$ and $\delta$, the original DIF cannot be chosen in a way that fits the experimental data. Therefore, a user-defined DIF rectify the dynamic compressive and tensile strength to improve the behaviour of the model referring to Leppanen ${ }^{12}$. The proposed stepwise linear DIF model for compression ${ }^{7}$ and tension ${ }^{12}$ are described by equations below separately,

$$
\begin{align*}
& \operatorname{DF}_{\mathrm{c}}(\dot{\varepsilon})= \begin{cases}\left(\dot{\varepsilon} / \dot{\varepsilon}_{\mathrm{c} 0}\right)^{\alpha_{\mathrm{c}}} & \text { for } \dot{\varepsilon} \leq 30 s^{-1} \\
\beta_{\mathrm{c}}\left(\dot{\varepsilon} / \dot{\varepsilon}_{\mathrm{c} 0}\right)^{1 / 3} & \text { for } \dot{\varepsilon}>30 s^{-1}\end{cases}  \tag{19}\\
& \operatorname{DIF}_{\mathrm{t}}(\dot{\varepsilon})= \begin{cases}\left(\dot{\varepsilon} / \dot{\varepsilon}_{\mathrm{t} 0}\right)^{\delta_{\mathrm{t}}} & \text { for } \dot{\varepsilon} \leq 1 s^{-1} \\
\beta_{\mathrm{t}}\left(\dot{\varepsilon} / \dot{\varepsilon}_{\mathrm{t} 0}\right)^{1 / 3} & \text { for } \dot{\varepsilon}>1 s^{-1}\end{cases} \tag{20}
\end{align*}
$$

where subscript c and t represent compression and tension respectively, the static strain rate is taken as $3.0 \times 10^{-5} \mathrm{~s}^{-1}$ for compression and $1.0 \times 10^{-6} \mathrm{~s}^{-1}$ for tension. $\alpha_{\mathrm{c}}$ and $\beta_{\mathrm{c}}$ were equal to 0.014 and 0.012 respectively ${ }^{7} . \delta_{\mathrm{t}}=0.031$ and $\beta_{\mathrm{t}}=0.015$ were obtained based on the experimental data. It is obvious that the implemented DIF fit the experimental data well.

Because of the residual surface can so far not be measured $^{8}$, the residual strength parameters $B$ and $M$ cannot be determined by experimental data. However, the parametric studies, conducted by leppanen ${ }^{12}$, indicate that the simulation results were reasonable and fit the experiment results well by setting $B$ and $M$ equal to 1.50 and 0.70 , respectively.

### 3.2 Damage Parameters

The uniaxial cyclic loading and unloading experiments were performed on cylindrical concrete specimens with 50 mm in diameter and 100 mm in height, and the repeated experiments were conducted to ensure the reliability of experimental results. The representative stress-strain curve was illustrated in Fig. 3. An assumed failure surface was defined from the test results, indicating a total loss in strength at an axial strain of $\varepsilon_{x}$. During the loading process, the elastic strain $\varepsilon_{\mathrm{xe}}$ and the volumetric strain $\mu$ can be neglected due to the early curvature of modulus and the low pressures $\left(0-f_{\mathrm{c}} / 3\right)$ that occur respectively, as shown in Fig.3. By assuming $\varepsilon_{\mathrm{xe}}=\mu=0$, the equivalent plastic strain at failure $\varepsilon_{\mathrm{p}}{ }^{\mathrm{f}}$ was equal to $\varepsilon_{\mathrm{x}}$. Initiating the damage curve at $f_{\mathrm{ttt}}{ }^{*}=-0.09$ and satisfying $\varepsilon_{\mathrm{p}}^{\mathrm{f}}$ at $\bar{p}^{*}=1 / 6$ (average from $p^{*}=0$ to $p^{*}=1 / 3$ ), the constant $D_{1}$ was obtained by the Eqn (21),

$$
\begin{equation*}
D_{1}=\frac{\varepsilon_{p}^{f}}{\left(\bar{p}^{*}-f_{\mathrm{tt}}^{*}\right)^{D_{2}}} \tag{21}
\end{equation*}
$$

where $\varepsilon_{\mathrm{p}}{ }^{\mathrm{f}}$ was equal to 0.013 obtained from the experiment, and $\varepsilon_{\mathrm{f}, \text { min }}, D_{2}$ was set to default value ${ }^{6}$.

### 3.3 Equation of State Parameters

In order to derive the EoS parameters of the large scale heterogeneous mixture material such as concrete, it is necessary to decompose the concrete into smaller scale homogeneous
components mortar and aggregate, to measure the hugoniot properties separately. A hugoniot mixing rule ${ }^{13}$ based on the mass-weighted contribution of each component to density $\rho$, bulk sound speed $c_{B}$, slope $s$ of the shock particle velocity and the Grüneisen parameter $\gamma$ were applied as below,

$$
\begin{equation*}
\rho_{0}=\sum m_{i} \rho_{0 i}, c_{\mathrm{B}}=\sum m_{i} c_{\mathrm{Bi}}, s=\sum m_{i} s_{i}, \gamma=\sum m_{i} \gamma_{i} \tag{22}
\end{equation*}
$$

The mortar specimens were made of the same proportion as concrete specimens just removing the aggregates, and cured for 28 days prior to the experiments. The density of mortar specimen was measured to be $2.10 \mathrm{~g} / \mathrm{cm}^{3}$. Plate impact experiments were performed using one stage light gas gun facility with a bore diameter of 57 mm and an impact velocity ranging from $190 \mathrm{~m} / \mathrm{s}$ to $500 \mathrm{~m} / \mathrm{s}$, to obtain stress levels ranging from 0.5 GPa to 2 GPa . Fig. 4 shows the plate configuration just before impact. The geometry of target and projectile, restricted by the bore diameter of 57 mm , was investigated by pre-test numerical analysis to prevent release wave effects during experimental data acquisition. The projectile consists of a 7 mm thick flyer mounted on the front of a sabot which both made of 2024 aluminium alloy. Sequential pin-shorting method was used to measure impact velocities and tilt was fixed to be less than 1 mrad by means of an adjustable specimen mount. The target consists of three 4.8 mm thick mortar specimens in the


Figure 3. The stress-strain curve from uniaxial cyclic loading and unloading experiment.


Figure 4. Configuration of planar impact experiment.
impact direction, and the target device was aligned to accuracy below 1 mrad , longitudinal stress measurements were taken by embedding piezoresistive manganin gauges between ties of the target material using a low viscosity epoxy adhesive. Fig. 5(a) illustrates some representative stress wave profiles as obtained from the gauges. The shock velocity $D$ in mortar specimen can be obtained from the thickness of the middle specimen $L$ and the time duration $\Delta t$ measured by the two gauges. The detail experimental results were listed in Table 1. By using the impedance matching relations at boundaries between different materials ${ }^{13}$, the hugoniot of mortar obtained is shown below in Fig. 5(b). The relationship between shock wave velocity $D$ and the particle's velocity after shock wave $u$ is $D=3.24+1.15 u(\mathrm{~km} / \mathrm{s})$, that is, $c_{\mathrm{B} 1}=3.24 \mathrm{~km} / \mathrm{s}, \mathrm{s}_{1}=1.15$. The hugoniot parameters of limestone were $c_{\mathrm{B} 2}=3.40 \mathrm{~km} / \mathrm{s}$, $s_{2}=1.54$ according to Ahrens ${ }^{14}$. In addition, the Dugdale and Macdonald's approximation ${ }^{13}$ for the Grüneisen coefficient $\gamma$ is applied, $\gamma=2 s-1$. By using the Eqn (22), based on the composition of concrete, the hugoniot parameters of concrete were obtained: $c_{\mathrm{B}}=3.32 \mathrm{~km} / \mathrm{s}, s=1.34, \gamma=1.68$. Furthermore, the parameters of the polynomial equation of state can be obtained by the equations as follows,

$A_{3}=\rho_{\mathrm{s} 0} c_{\mathrm{B}}^{2}\left[2(s-1)+3(s-1)^{2}\right]$

$$
B_{0}=B_{1}=\gamma
$$

(a)
where $\rho_{\mathrm{s} 0}$ is the compacted density of concrete which setting to $2.75 \mathrm{~g} / \mathrm{cm}^{3}$ by reference to Riedel ${ }^{8}$. We get $A_{1}=T_{1}=30.3 \mathrm{GPa}$, $A_{2}=44.1 \mathrm{GPa}, A_{3}=31.1 \mathrm{GPa}, B_{0}=B_{1}=1.68$. The parameter $T_{2}$ was set to zero ${ }^{6}$.

The porous soundspeed $c_{\mathrm{p}}$ was $2950 \mathrm{~m} / \mathrm{s}$ measured by ZBL-U520 non-mental ultrasonic testing device. In addition, $p_{\text {el }}, p_{\text {comp }}$ and $n$ were set to $2 / 3 f_{\mathrm{c}}, 6 \mathrm{GPa}$, and 3 respectively ${ }^{8}$. These values are assumed to be identical for all practicable concrete.

## 4. NUMERICAL SIMULATION OF PENETRATION INTO CONCRETE

For testing the predictive quality of the model parameters in the simulation of impact processes, the results of numerical simulation were compared to the penetration tests reported by Hansson ${ }^{15}$. In the experiments the steel projectiles with an ogive nose of caliber-radius-head (CRH) 3.0, a length of 225 mm and a diameter of 75 mm were fired with zero attack angle into massive cylindrical concrete targets with a diameter of 1.6 m and a length of 2.0 m . The total mass of the projectile was 6.28 kg . The steel material had the following properties: bulk modulus 159 GPa , shear modulus 81.8 GPa , and yield stress 792 MPa . The compressive strength of 150 mm cubic concrete was about 40 MPa . The impact velocity and penetration depth were measured to be $485 \mathrm{~m} / \mathrm{s}$ and $655 \mathrm{~mm}-660 \mathrm{~mm}$ from two

(b)

Figure 5. The results of plate impact experiment: (a) The output of the gauges from experiment with impact velocity of $0.332 \mathrm{~km} / \mathrm{s}$, (b) Hugoniot data for mortar specimen: shock velocity $D$ vs particle velocity $u$.

Table 1. Experimental parameters and hugoniot data for mortar

| Shot <br> No. | Specimen <br> thickness $(\mathbf{m m})$ | Impact <br> velocity $(\mathbf{k m} / \mathbf{s})$ | Time <br> duration $(\boldsymbol{\mu s})$ | Shock <br> velocity $(\mathbf{k m} / \mathbf{s})$ | Particle <br> velocity $(\mathbf{k m} / \mathbf{s})$ | Pressure <br> $(\mathbf{G P a})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1\# | 4.77 | 0.500 | 1.31 | 3.641 | 0.335 | 2.558 |
| 2\# | 4.79 | 0.458 | 1.33 | 3.602 | 0.307 | 2.325 |
| 3\# | 4.80 | 0.420 | 1.35 | 3.556 | 0.283 | 2.110 |
| 4\# | 4.83 | 0.332 | 1.39 | 3.475 | 0.225 | 1.640 |
| 5\# | 4.71 | 0.273 | 1.36 | 3.463 | 0.185 | 1.343 |
| 6\# | 4.74 | 0.262 | 1.38 | 3.435 | 0.177 | 1.280 |
| 7\# | 4.88 | 0.196 | 1.43 | 3.413 | 0.133 | 0.950 |



Figure 6. Numerical simulation of penetration into concrete target: (a) model set-up and discretization, (b) numerical and experimental results.

Table 2. The parameters for C40 concrete of RHT model

| $\boldsymbol{f}_{\boldsymbol{c}}(\mathbf{M P a})$ | $\mathbf{G}(\mathbf{G P a})$ | $\boldsymbol{f}_{\boldsymbol{s}} / \boldsymbol{f}_{\boldsymbol{c}}$ | $\boldsymbol{f}_{\boldsymbol{t}} / \boldsymbol{f}_{\boldsymbol{c}}$ | $\boldsymbol{f}_{\mathrm{c}, \mathrm{el}} / \boldsymbol{f}_{\boldsymbol{c}}$ | $\boldsymbol{f}_{\mathrm{t}, \mathrm{el}} / \boldsymbol{f}_{\boldsymbol{t}}$ | $\boldsymbol{A}$ | $\boldsymbol{N}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 32 | 14.5 | 0.18 | 0.10 | 0.53 | 0.7 | 1.929 | 0.764 |
| $Q_{2,0}$ | $B_{\mathrm{Q}}$ | $\alpha_{\mathrm{c}}$ | $\beta_{\mathrm{c}}$ | $\delta_{\mathrm{t}}$ | $\beta_{\mathrm{t}}$ | $B$ | $M$ |
| 0.69 | 0.0048 | 0.014 | 0.012 | 0.031 | 0.015 | 1.50 | 0.70 |
| $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\varepsilon_{\mathrm{f}, \text { min }}$ | $\rho_{0}\left(\mathrm{~g} / \mathrm{cm}^{3}\right)$ | $c_{\mathrm{p}}(\mathrm{m} / \mathrm{s})$ | $\rho_{\mathrm{s} 0}\left(\mathrm{~g} / \mathrm{cm}^{3}\right)$ | $p_{\mathrm{el}}(\mathrm{MPa})$ | $P_{\text {comp }}(\mathrm{GPa})$ |
| 0.051 | 1.0 | 0.01 | 2.35 | 2950 | 2.75 | 21.3 | 6.0 |
| $n$ | $A_{1}(\mathrm{GPa})$ | $A_{2}(\mathrm{GPa})$ | $A_{3}(\mathrm{GPa})$ | $B_{0}$ | $B_{1}$ | $T_{1}(\mathrm{GPa})$ | $T_{2}(\mathrm{GPa})$ |
| 3 | 30.3 | 44.1 | 31.1 | 1.68 | 1.68 | 30.3 | 0 |

shots, respectively. The dimension of the crater produced was also measured and the diameter was about 800 mm . A numerical model of the experiments was set up which schematically illustrated in Fig. 6(a). The concrete targets were discretized in a $2.5 \times 2.5 \mathrm{~mm}$ Lagrangian mesh applying the RHT model parameters determined in the previous sections exhibited in Table 2. The projectile was also modelled in a Lagrangian mesh with a mesh size of between $7.0 \times 3.75 \mathrm{~mm}$ in the rear part and $8.4 \times 3.75 \mathrm{~mm}$ in the projectile nose applying the material model Steel 4340 from the AUTODYN material library. The experiments were simulated and the penetration depths were determined after the projectile had stopped. The numerical and experimental results generally show a good agreement as shown in Fig. 6(b).

## 5. CONCLUSIONS

The Riedel-Hiermaier-Thoma (RHT) material model takes account of many important features of concrete under high-impulsive loading. Concrete has a great variety of strength grades, but only 35 MPa and 135 MPa concrete parameters are provided in AUTODYN material database. Moreover, the standard RHT model implemented in AUTODYN falls short in representing the concrete behaviour under the dynamic compression and tension loading. Therefore, the modifications of using two bi-linear dynamic increase factor functions for the compressive and tensile strength are proposed through user codes. With the modifications, the RHT model is found to behave more realistically in modelling the concrete behaviour in compression as well as in tension. Furthermore, the present paper proposes a method to determine the RHT model parameters, and the RHT model parameters for C40 concrete
were obtained. Numerical simulation of penetration of concrete targets by steel projectile is conducted to further evaluation of the performance of the modified RHT model using the determined parameters in real applications. The numerical simulation results were in appreciable good agreement with experimental results.

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## CONTRIBUTORS



Mr Yuqing Ding obtained his Master's degree from National University of Defense Technology, China, in 2009. He is pursuing PhD at NUDT. His research areas are engineering mechanics and the vulnerability of targets.


Dr Wenhui Tang obtained his PhD from National University of Defense Technology, China, in 1995. At present, he is Professor at Institute of Engineering Physics, NUDT. His research interests are explosion mechanics, the vulnerability of targets, and shock wave physics.


Mr Ruoqi Zhang is Professor at Institute of Engineering Physics, NUDT. His research interests are in shock wave physics, explosion mechanics and explosion protection.


Dr Xianwen Ran obtained his PhD from National University of Defense Technology, China, in 2006. He is working as a Lecturer at Institute of Engineering Physics, NUDT. His research interests are explosion mechanics, the vulnerability of targets, and shock induced phase transition.

