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Bond Growth under Temperature Gradient

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ABSTRACT

Grain and bond growth for dry snow are determined by the distribution of temperature and temperature gradient in the snow matrix. From the standpoint of particle approach and based on cubic packing structure, a bond growth model has been developed for TG metamorphism. The paper highlights the importance of bond formation and its effect on snow viscosity and finally on the rate of settlement. This is very important for developing a numerical snowpack model if microstructure is considered to be a basic parameter. A few experiments have been carried out to validate bond formation under temperature gradient.

NOMENCLATURE

φ	Porosity of the model for bond growth
a	Grain radius
l	Neck length
b	Bond radius
h	Distance between the grains
d_c	Calculated grain size
d_i	Initial grain size
d_o	Grain growth in time t
N	Porosity of the model for grain growth
γ	Experimental constant
q_i	Heat transfer rate
A_g, A_b, A_p, A	Cross-sectional area of grain, bond, pore and average cross-sectional area
k_i, k_a, k_s	Conductivity of ice, air and snow.
T_1,T_2	Bottom and top snow temperatures
$\theta_1, \theta_2 \ldots. \theta_6$	Temperatures as shown in Fig.2
$(TG)_{micrograin}$, $(TG)_{microbond}$, $(TG)_{micropore}$,	Temperature gradients between grains, along the neck and between the pores
ρ_i,ρ_s	Density of ice and snow.
D	Diffusion coefficient of water vapour into air (2.02×10^{-5} m²/s)
P	Saturation water vapour pressure over an ice surface at. temperature I(K)
P_0	Saturation water vapour pressure over an ice surface at triple point (611 N/m²)
L	Latent heat of sublimation (2.838×10^6 J/kg)

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1. INTRODUCTION

The evolution of snowpack through numerical models has always been a challenging task, as it evolves through different processes of densification, metamorphism, melt-water percolation, drifting snow, etc. Metamorphism is one of the important processes, and if not considered correctly, can result in an unrealistic picture of temperature distribution and densification.

Earlier work on temperature gradient (TG) metamorphism dealt only with the rate of grain growth for different temperature and temperature gradient conditions. However, Kojima[1] and Akitaya[2] have observed formation of columns of snow grains during depth hoar development, as shown in Fig. 1. Formation of columns of faceted/depth hoar grains is a result of the bond growth at the contact points.

Kojima concluded, through his experiments on mechanical properties of depth hoar, that the coefficient of compressive viscosity for depth hoar is larger than that for fine-grained compact snow, but the mechanical strength of depth hoar against a shear force is much smaller than that for fine-grained compacted snow. Kojima's conclusion can be explained later from the experimental evidence of columnar structure formation, as seen during the experiment depicted in Fig. 1.

Akitaya observed that the cup crystals having a large crystal body and a thin joint at the base individually are linked vertically, forming a column. He explained that the larger strength of depth hoar against a vertical force is due to the vertical columnar structure of skeleton-type depth hoar. At the same time, it can be said that if the force exceeds a certain value which produced a stress larger than the yield stress of ice in the ice joint of depth hoar, destruction of the vertical column, which has no other support, proceeds very quickly. Akitaya also observed that for some special kind of depth hoar (polar and others), the hardness is more than the hardness of fine-grained compact snow with the same density. The increase in hardness can also be explained from the bond growth phenomenon.

The mechanism of bond formation through temperature gradient is essentially important to understand, because it increases the snow viscosity and lowers the settlement rate. At the beginning, when snow is fresh, viscosity is relatively very low compared to the viscosity of older snow. The low viscosity of fresh snow results in a high settlement rate. After a snowfall, a steep temperature gradient may develop in the snowpack; the snowcover then evolves under temperature gradient metamorphism, resulting in faceted or depth hoar grains within a few days. This type of layers comprising faceted or depth hoar grains settles very slowly during the growth period, but the settlement rate can increase if melt is produced due to warm temperatures or melt water percolation. The melt–water changes the grain shape from faceted to rounded one, resulting in close pack snow structure and speedy settlement.

Taking this background into account, a bond growth model for temperature gradient has been developed. To represent the snow structure, a simplified geometry of cubic packing has been assumed, as shown in Fig. 2. A few experiments have also been carried out to validate bond formation during TG metamorphism. However, the experimental results are yet to be analysed to test the model results. This paper highlights the need to understand the bond growth phenomenon under TG metamorphism and its importance in developing snowpack model.

2. MODEL DEVELOPMENT

Snow can be viewed from two entirely different standpoints–first as a continuum and second as a particulate medium. However, the metamorphic processes that profoundly affect mechanical properties are examined from the standpoint of particulate medium.

To model bond growth from snow geometry, as shown in Fig. 1, a cubic packing of snow grains has been assumed to represent the snow structure, as shown in Fig. 2. Under this model, spherical grains have been considered to simplify the model calculations. These grains are connected through bonds with other grains of the same size. These grains are separated by a distance h. T_1 and T_2 can be assumed as two input temperatures, as shown in Fig. 2.

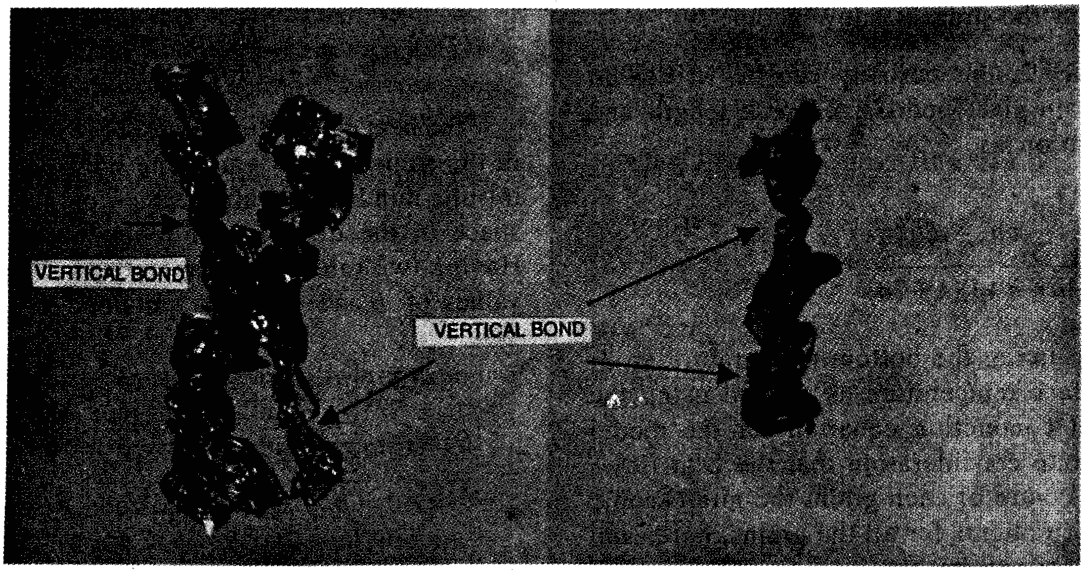

Figure 1. Vertical columns of temperature gradient snow resulting from bond formation

It can be concluded from Fig. 2 that all the bonds perpendicular to temperature gradient will not grow, because they take part both as a vapour source and sink at the same time. Therefore, mass evaporation from the top surface of the bond will be approximately equal to the mass deposition from down below. Bonds parallel to temperature gradient will grow with time. It is also possible that a few bonds not taking part during mass transfer may remain isolated throughout the process of metamorphism. Such isolated bonds and bonds perpendicular to temperature gradient are not

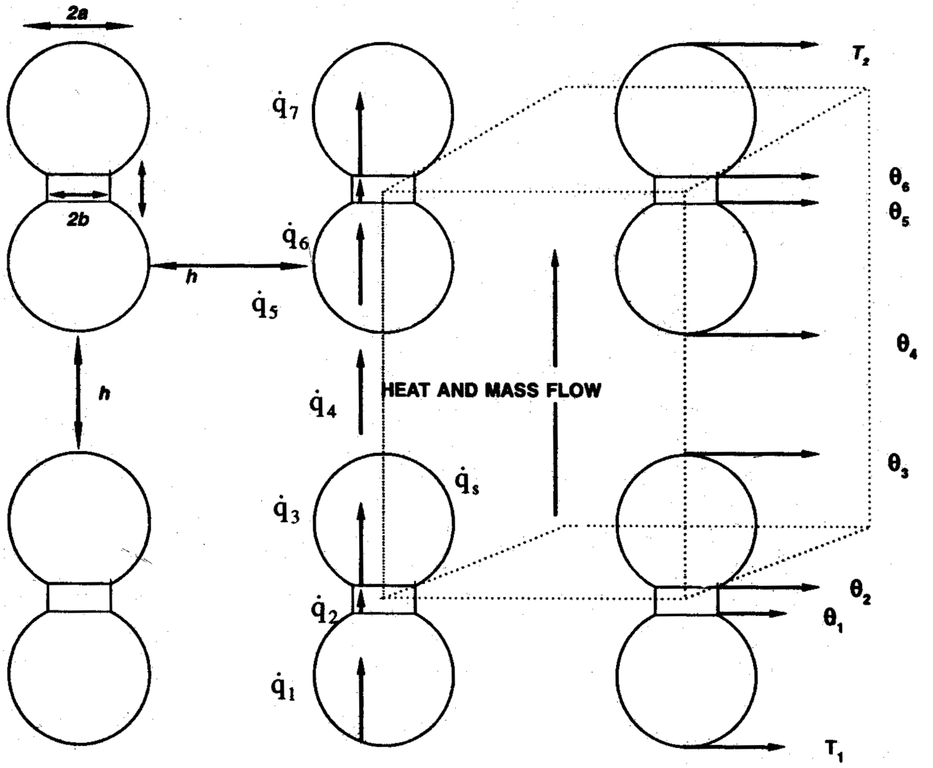

Figure 2. Cubic packing of snow grains for bond growth

considered in the present snow geometry.

From Fig. 2, the porosity can be written in terms of grain size, bond size, neck length and grain separation as

$$\varphi = 1 - \frac{\frac{8}{3}\pi a^3 + \pi l b^2}{(4a+h+l)(h+2a)^2} \tag{1}$$

Also, assuming the bottom temperature as T_1 and the surface temperature as T_2 and using the Fourier heat conduction equation for the model and taking into consideration that the heat flows through the centre of each grain, the microscopic temperature gradients for all the grains, pores and neck geometry can be obtained as

$$\partial q_1 = k_i A_g \frac{T_1 - \theta_1}{2a} \partial t \tag{2}$$

$$\partial q_2 = k_i A_b \frac{\theta_1 - \theta_2}{l} \partial t \tag{3}$$

$$\partial q_3 = k_i A_g \frac{\theta_2 - \theta_3}{2a} \partial t \tag{4}$$

$$\partial q_4 = k_a A_g \frac{\theta_3 - \theta_4}{h} \partial t \tag{5}$$

$$\partial q_5 = k_i A_g \frac{\theta_4 - \theta_5}{2a} \partial t \tag{6}$$

$$\partial q_6 = k_i A_b \frac{\theta_5 - \theta_6}{l} \partial t \tag{7}$$

$$\partial q_7 = k_i A_g \frac{\theta_6 - T_2}{2a} \partial t \tag{8}$$

Assuming snow as a bulk continuum, the Fourier heat conduction equation can be written as

$$\partial q_s = k_s A \frac{T_1 - T_2}{8a + 2l + h} \partial t \tag{9}$$

where

$$A = \left(A_g + A_b + A_p\right)/3$$

The average temperature gradient for the model can be written as

$$(TG)_{mean} = \frac{T_2 - T_1}{8a + 2l + h} \partial t \tag{10}$$

The heat conduction Eqns (2)-(8) can be solved in the same way as was done by Yoshida[3] for an infinite number of grains, each separated by a pore space in the direction of temperature gradients. Having two reference temperatures, T_1 and T_2, the values of θ_1 and θ_2 can be obtained as

$$\theta_1 = \frac{T_2 + \left[\frac{A_g}{A_b}\frac{l}{2a} + \frac{2a}{h} + \frac{k_i}{k_a}\frac{h}{2a} + 4\right] T_1}{\frac{2a}{h} + \frac{k_i}{k_a}\frac{h}{2a} + \frac{A_g}{A_b}\frac{l}{2a} + 5} \tag{11}$$

and

$$\theta_2 = \theta_1 \left[1 + \frac{A_g}{A_b}\frac{l}{2a}\right] - \left[\frac{A_g}{A_b}\frac{l}{2a}\right] T_2 \tag{12}$$

Therefore, temperature gradients for the grain bond and pore are:

$$(TG)_{micrograin} = \frac{8a + 2l + h}{2a\left[\frac{2a}{h} + \frac{k_i}{k_a}\frac{h}{2a} + \frac{A_g}{A_b}\frac{l}{2a} + 5\right]} (TG)_{mean} \tag{13}$$

$$(TG)_{microbond} = \frac{A_g}{A_b} (TG)_{micrograin} \tag{14}$$

and

$$(TG)_{micropore} = \frac{k_i}{k_a} (TG)_{micrograin} \tag{15}$$

Using Eqn (9) the values of temperature gradient along the grains and necks can be directly written as

$$(TG)_{microbond} = \frac{k_s}{k_i}\frac{A}{A_b} (TG)_{mean} \tag{16}$$

$$(TG)_{micropore} = \frac{k_s}{k_a}\frac{A}{A_g} (TG)_{mean} \tag{17}$$

$$(TG)_{micrograin} = \frac{k_s}{k_i}\frac{A}{A_g} (TG)_{mean} \tag{18}$$

Snow conductivity can be obtained from the conductivity model of Brown[4] and the results can be compared with the actually obtained microscopic temperature gradients from Eqns (13) and (14).

Using Fick's law of diffusion for mass transfer along the neck, one has:

$$\dot{m} = D\frac{dC}{dZ} \tag{19}$$

where D is the diffusion coefficient of water vapour into air from the ice surface, and dC/dZ is the vapour concentration gradient along the neck.

The question here is how the vapour will be redistributed over the neck surface and how mass evaporation will take place from the neck. This is important, because both deposition and evaporation take place at the same time. But certainly, the rate of mass deposition is more compared to the rate of evaporation due to higher microscopic temperature gradient for the neck than for the grain. The microscopic temperature gradient can be obtained either from Eqns (13) and Eqns (14) or from Eqns (16) and (18), depending upon the input parameters available, and thus there is continuous evaporation of mass from the grains just below the neck. To obtain the rate of bond growth, the net mass deposition at the neck should be taken into consideration.

From Fig. 2, the average surface area available for mass evaporation from the grain surface is:

$$2\pi a^2 - \pi b^2$$

and this mass will be deposited over the neck area, whose surface area is $2\pi bl$.

Using Eqns (19) mass flux can be written as

$$\dot{m} = \frac{DPL}{R^2T^3}(TG)_{microbond}\left(2\pi a^2 - \pi b^2\right) \tag{20}$$

where $P = P_0 \exp(0.0857(T\text{-}273))$

and the rate of bond growth is:

$$\dot{b} = \frac{\dot{m}}{2\pi bl\rho_i} = \frac{DPL}{R^2T^3}(TG)_{microbond}\left(\frac{a^2}{bl} - \frac{b}{2l}\right)\frac{1}{\rho_i} \tag{21}$$

where $b < a$.

It has been observed that bond growth under temperature gradient is highly dependent on snow density and, therefore, an empirical density function in the bond growth equation can be introduced to make it density-dependent. As a first case, for simplification, linear dependence on density has been used. Subsequently, true values will be used as new results become available.

$$\dot{b} = \frac{\dot{m}f(\rho_s)}{2\pi bl\rho_i} \tag{22}$$

The results from Eqn (22) are presented in Fig. 3.

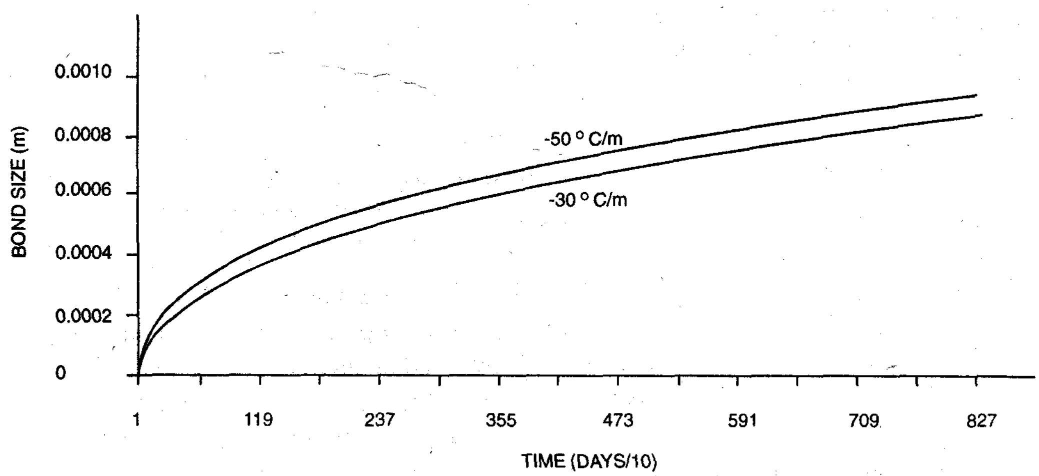

Figure 3. Increase in bond size with time for different mean temperature gradients at –30 °C/m and -50 °C/m

As it is obvious that grains are also growing with time, the growth factor should be included in the bond growth rate. The grain growth model for this purpose is taken from Satyawali[5] to predict the bond growth correctly. The equation for grain growth suggested by Satyawali[5] is:

$$d_c = d_i + d_o n^r \tag{23}$$

From Eqns (22) and (23), it can be concluded that increase in grain and bond size will also change the microscopic temperature gradient, which will further change the mass transfer rates. This aspect is now taken care of.

3. MODEL RESULTS

The above model gives interesting results in respect of increase in bond size (Fig. 3). There is instantaneous increase in bond size at the beginning, but with further increase in time, the bond tends to grow rather slowly. This can be understood from Fig 4 where bond growth rate has been obtained for three different temperatures as (–1.0 °C, – 5.0 °C and –10 °C) while keeping the mean temperature gradient at –30 °C/m. The growth rate is very high when grains are just in contact, but then decreases and tends to reach the minimum value. When grains are just in contact, most of the heat transfer takes place through air medium. This increases the temperature difference at the neck, resulting in high temperature gradient. When the bond grows, considerable heat flows through the bond due to its high conductivity, which results in low microscopic temperature gradient and also, thereafter, low bond growth rate. This explains, bond growth rate reaches a maximum value and then starts decreasing subsequently.

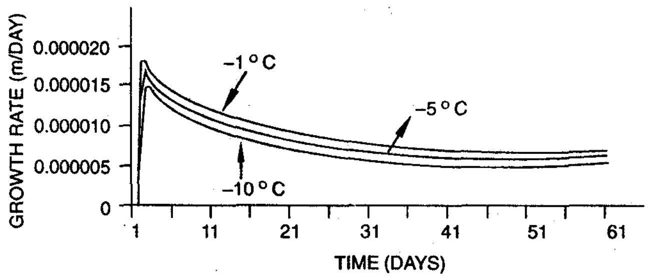

Figure 4. Bond growth rate as a function of time for different temperatures.

In Fig. 5, microscopic temperature gradient at the bond has been shown for a given grain size 0.002 m and mean temperature gradient –30 °C/m. When grains are just in contact, microscopic temperature gradient starts with a very high value as –870 °C/m, then decreases and finally tends to become

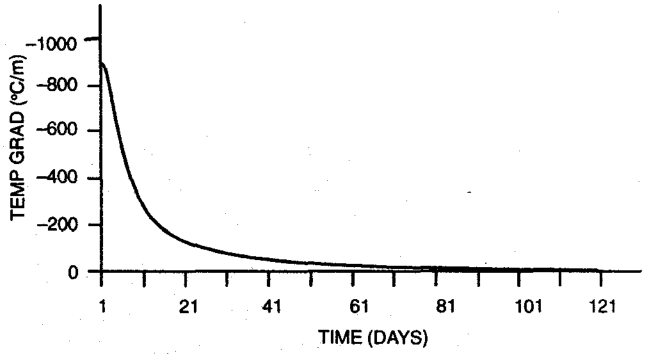

Figure 5. Microscopic temperature gradient at the neck. As the bonds grow, temperature gradient tends to decrease with time.

zero. The mass flux is calculated from this microscopic temperature gradient to obtain the bond growth rate. This mass flux rate is two to three orders higher compared to equi-temperature conditions.

The explanation given above and the model results for bond growth look contradictory to the observed fact of the weakening of snow under

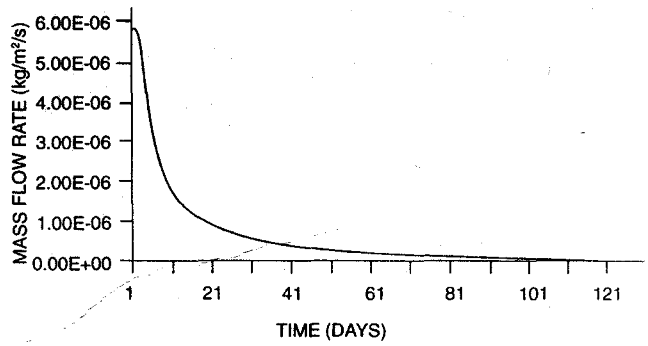

Figure 6. Rate of mass transfer as a function of time for given set of temperature, temperature gradient initial grain size and neck length.

temperature gradient, as one knows that bond formation under equitemperature condition increases the strength of snow and under temperature gradient, snow becomes loose. This can be explained from the proposed geometry, as shown in Fig. 2. It is obvious that vertical bonds will grow as a result of high temperature gradient at the neck. Finally, snow will form a columnar structure due to bond formation, as shown in Fig. 1. Although in a natural snowcover, horizontal bonds may also grow if they are in favourable position, such geometry is not considered in this model.

The columnar structure of snow under temperature gradient has also been observed by Akitaya[1]. Snow under temperature gradient can be visualised as a columnar structure placed parallel to each other and having no bonds between them, resulting in a weak snow structure. If only vertical bonds are considered, the number of bonds per unit volume will be very less compared to the number of bonds for fine-grained snow of the same density Fig. 6. The number of bonds during temperature gradient metamorphism should, therefore, remain fairly constant once they are formed. Therefore, it is clear that the number of bonds per unit volume is important factor as compared to the size of bond to describe the snow strength. These are some unresolved questions and the concept must be validated through experimental results from vertical and horizontal surface sections.

4. EXPERIMENTAL WORK

The theory described above and the model results quantify the rate of bond growth under temperature gradient snow. But it still requires validation on the basis of experimental results. Three snow samples were analysed for this purpose and described in terms of their density, grain size distribution and horizontal and vertical thick sections. Each snow block was kept for the metamorphic experiment and it was sampled five times at equal time intervals. In this way, one can obtain the bond growth rate from the vertical section. This work was shared with Schneebeli, *et. al*[6]. and Pielmeier[7].

The first snow sample taken on 19 January 1998 was wind packed, fine-grained and destructive metamorphosed. It was placed in an experimental device kept inside the cold room. The floor of the device and the cold room housing the device were maintained at –2 °C and –11 °C, respectively. The second snow sample was prepared on 09 March 1998 from the same location. It was produced by sieving fresh snow. The third sample was taken from the test field at Weissfluhjoch (2600 m) on 25 March 1998 and subjected to temperature gradient on 15 April 1998 after sieving in the same manner as in Experiment 2. temperature gradient for these samples was kept as –33 °C/m with average snow density as 250 kg/m^3 and average temperature as –7.0 °C (Approx.).

Columnar structures of faceted grains were observed from the temperature gradient sample, also mentioned by Schneebeli, *et. al*[6]. from the same work. These structures were connected through the necks. These structures were not very strong and though the grains from the sample were taken out carefully, a few columnar structures broke during analysis. The bonds were clearly visible in the structure. As the sample was very fresh, it was clear that bonds were not formed before the experiment, but it was very difficult to consider the alignment of these bonds in snow. For simplification, only vertical bonds have been considered in the present study. Horizontal bonds if present, act as sink or source grains during the process of TG metamorphism, and therefore, they diminish quickly.

5. CONCLUSIONS

Metamorphism is one of the important processes, which, if not considered correctly, can result in an unrealistic picture of temperature distribution and densification. Considering all these aspects, a bond growth model for snow under temperature gradient has been developed. This paper describes the practical need to consider the bond growth for snowpack model development. It also supports the observations made by Kojima and Akitaya. It has been concluded that bond formation mechanism during temperature gradient is essentially important, as it has a direct relationship with snow viscosity. An explanation has also been given for the weakness of snow under temperature gradient. Model results for bond growth rate are demonstrated and a few experiments have been designed to validate bond formation. However, experimental results to test the validity of the model have not been analysed so far.

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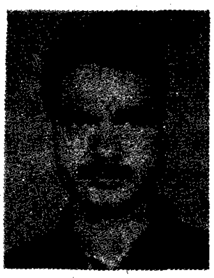

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