Defence Science Journal, Vol 49, No 4, August 1999, pp. 291-298

REVIEW PAPER

Recent Developments in Tolerancing Techniques

Kamlesh Purohit and C.S. Sharma
Jai Narain Vyas University, Jodhpur-342 001.

ABSTRACT

Mechanical tolerances are specified in conjunction with part geometry, material type and other technical specifications during the design of mechanical components and assemblies. These tolerances ensure the expected design function, and provide guidelines for the manufacture of designed parts. However, assigning proper tolerances for a design is a non-trivial task. This paper reviews the recent developments in the field of tolerancing techniques, particularly with reference to the use of computers, cost tolerance relationship and tolerance design for quality improvement of the product.

1. INTRODUCTION

Engineering drawings have traditionally been used for design documentation and for effective communication between individuals. It is a graphical representation of the shape and size of a product with the use of dimension which maintains the design integrity of the part during manufacturing. Because of the limitation of machine accuracy, parts are manufactured within certain range of dimensions rather than with specific quantities. Appropriate tolerances defined in the design phase can certainly reduce the difficulties in manufacturing. In order to ensure that the design requirements are interpreted unambiguously, industry has developed a methodology for geometric dimensioning and tolerancing. However, the allocation of tolerance is still a key issue in controlling the product quality and decreasing the manufacturing cost. In this paper, a review of the recent developments in tolerancing technique is presented.

2. TOLERANCE ALLOCATION

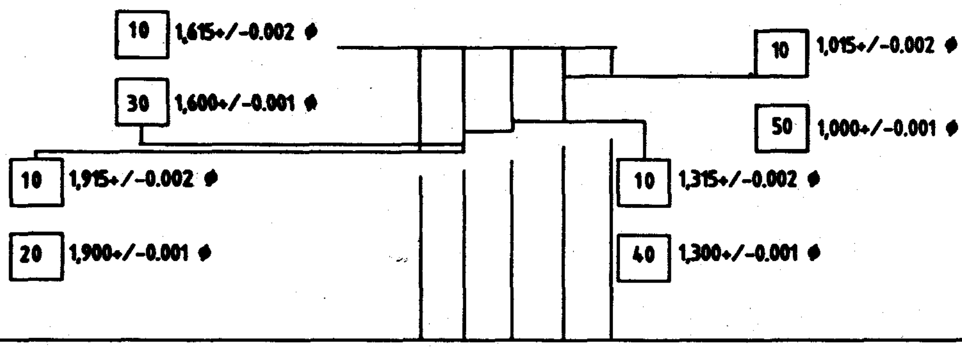
2.1 Manual Tolerance Allocation through Tolerance Chart

The task of transforming raw material into components satisfying the specific blueprint dimension is carried out by a process plan. The feasibility of the sequence of process is checked via a tolerance chain commonly known as tolerance chart. These charts were introduced as a mean of aiding the planning of manufacturing process[1-3] in the earlier 1950s. A tolerance chart is a graphical representation of the dimensions of a work-piece through a sequence of machining operations. Starting from a blank chart, the process planner first sketches a work-piece and copies the relevant dimensions to the chart based on the information provided by engineering blueprint. Next, he assigns a machine tool as well as a pair of datum surface and machined surface for each operation in accordance with the machining schedule. Subsequently, he goes through a series of calculations before a proper

Received 03 June 1999

arrangement of process tolerance and stock removal have been reached. This methodology is usually based on trial and error approach. The tolerance range of the resultant blueprint dimensions is the result of addition of tolerance ranges associated with individual cuts, which contribute to it. This accumulation is commonly referred to as tolerance stack-up.

Figure 1 shows manually-developed tolerance chart for a steel plug developed by Wade[4,5]. In this chart, a line represents machining cuts with an arrow at one end and a dot at the other end. A balance dimension is the algebraic sum of two or more cuts and has a dot at both ends. A heavy black line drawn between two machined surfaces represents a blueprint dimension. The resultant dimension represents the actual value obtained after machining. He has reported a detailed procedure for the development of



LINE NO.	OPE. NO.	REV. NO.	M/C USED	MACHINED TO MEAN	MACHINED TO +/-tol	A	B	C	D	E	BALANCE DIM MEAN	BALANCE DIM +/-tol	LINES INVOLVED	STOCK REMOVAL MEAN	STOCK REMOVAL +/-tol
1	10		W&S	0.979	0.003									SOLID	
2	10			1.994	0.003									SOLID	
3	10			3.003	0.003									SOLID	
4											1.009	0.006	3-2		
5											0.024	0.006	3-1		
6	10			4.031	0.010									SOLID	
7											1.025	0.013	6-3		
8	20		MONAR	1.008	0.004								7-8	0.020	0.017
9											2.017	0.010	4-8		
10											3.032	0.010	6-8		
11											4.011	0.007	3-8		
12	30		NORTON	1.000	0.001								8-12	0.008	0.005
13											1.017	0.011	9-12		
14											2.032	0.011	10-12		
15	40		NORTON	1.000	0.003								13-15	0.017	0.014
16											2.000	0.004	12+15		
17											1.032	0.014	14-15		
18	50		NORTON	1.000	0.015								17-18	0.032	0.029
19											3.000	0.019	16+18		
20	60			4.000									11-20	0.011	0.008
21					0.001						3.000	0.002	20-12		
22											2.000	0.005	20-16		
23											1.000	0.020	20-19		
24															
25															
26															
27															
28															
29															
30															
31			BLUE	PRINT							RESULTANT				
32			4.000	0.005							4.000	0.001	20		
33			1.000	0.020							1.000	0.020	23		
34			2.000	0.009							2.000	0.005	22		
35			3.000	0.002							3.000	0.002	21		

Figure 1. Manual tolerance chart

tolerance chart.

2.2. Computer-Aided Tolerance Charting

With the development of CAD/CAM, several groups of software personnel[6-8] have developed software packages to automate the tolerance chart. The software developed by Ahluwalia and Karolin[9] is based on tolerance chain technique. It has been developed in segments to facilitate its integration to CAD/CAM system. It requires a part drawing, a raw material drawing, rough routing information, a blueprint and raw material dimensions and tolerances as input. The output of the software is in the form of (i) stock removal for each machining operation, and (ii) minimum process tolerance. The computer-aided tolerance chart output is a graphical plot of tolerance chart. It can also be used as an aid in developing CAPP scheme. The assignment of tolerances is based on random gauss rather than systematical searching. The constraints involved in tolerance charts have not been explicitly specified.

The application of CAD methodology to tolerance charting has identified two main aspects: (i) identification of dimensional chain, and (ii) tolerance control. Dimensional chains associated with a tolerance chart not only constrain the dimension change among operations but also affect the accumulation of tolerances. Identification of these chains, therefore, becomes the first step for automated tolerance charting. Tang and Davies[10] have developed an interactive computer-aided dimensional planning (CADP) system that uses matrix-tree chain approach for tracing each tolerance chain. In this approach, a matrix is used to represent a tolerance chart, each element in the matrix relates to an item in tolerance chart. It is necessary that the working dimensions defined in the matrix are in the correct order for each operation and the sequence of cuts must be shown in the matrix in the same order as they are taken on the machine for that operation, so that the calculation for the tolerance accumulations can be made correctly.

The CADP software is written in VAX FORTRAN. The software graphically shows the dimensions, the surface machined and the surface measured at every stage of manufacturing of a component. It also shows the stock removal for every cut taken. It can determine maximum allowable tolerances on working dimensions so as to reduce the production cost. The only limitation of CADP is that it does not provide support to the plotter.

Irani[11], *et al.* suggested that the designer can represent tolerance relationship using a graph instead of a traditional tolerance tree. These graphs show the degree of functional relationship between the surfaces and the manufacturing operations. Based upon this hypothesis, a graphical theoretic representation for the tolerance chart has been developed. Subsequently, the allocation of tolerances is optimised using linear programming model where the objective function seeks to minimise the cumulative slack over all schematics. The constraints attached to this objective function can involve tolerances for blueprint dimensions and stock removal simultaneously. Fainguelunt[12], *et al.* have developed microcomputer-based software for tolerancing. This software takes into account several categories of tolerances, such as setting, positioning, and machining. First of all, the process plan is checked wrt its technical feasibility and then the software optimises the distribution of tolerances so as to completely use the tolerance range allowed by the designer to produce the most economical manufacturing. Several alternative process plans can be compared quickly. The program has small database for setting dimension tolerance. However, it lacks geometrical tolerance consideration.

Kaushal[13], *et al.* suggested a low-cost solution to the problem of tolerance assignment. An overview of the methodology is shown in Fig. 2. The AutoCAD drawings are manipulated to extract features using Auto LISP programming language. Feature inferencing is then done utilising both the expert system and the database software. A tolerance database is created to store handbook data. With user interaction, tolerances based on dimensions are regenerated by referring to this tolerance database. The CAD drawing is then updated with these tolerances. The application to rotational parts with linear and circular dimensions has been demonstrated.

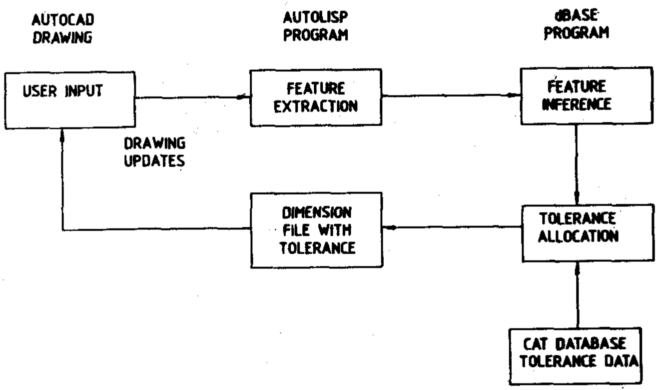


Figure 2. Overview of feature-based tolerancing

The traditional tolerance chart analysis has a limitation that it can be built only after all the initial engineering decisions have been taken. Or in other words, traditional tolerance chart analysis can only reactively check existing tolerance stack-up rather than pro-actively minimising tolerance stack-up by selecting an appropriate setup plan. Huang[14], *et al.* have attempted this problem by representing the design specification of a part into a graph. The problem of identifying the optimal setup plan is transformed into a graph search problem. A setup planning algorithm for rotational parts was developed simultaneously.

Ngoi[15,16], *et al.* have demonstrated the use of black box strategy which offers several advantages over other traditional approaches. These are:

(a) It does not require a graphical representation. In place of graphic aids, codes and IDs are used to describe data relationship.

(b) It does not use matrices or arrays. All variables and reference data are tabulated as structures so that they may be stored as file records.

(c) It does not require intermediate calculations and all unknown variables are solved simultaneously, and hence balancing dimensions are not required.

(d) There is no need for an elaborate tracing algorithm.

Hoffman[17] has discussed the problem of part design analysis and tolerance allocation using linear programming. After the tolerance specification has been input to the system, the designer can simply declare the type of tolerance function, the variability of which interest to him and point of its arguments. The system can then find the answer without the user performing any manipulation on algebraic equations. Chase and Greenwood[18] have proposed a new method called unified model of tolerance analysis based on process mean shift. It includes worst case and root mean square as extreme cases and with significant improvement in the model any thing can be simulated between these extreme conditions.

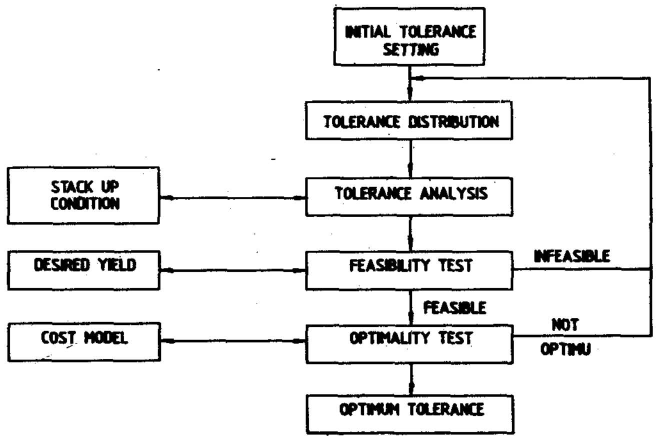


Figure 3. Basic scheme of tolerance synthesis

Lee and Woo[19] have developed a new procedure for tolerance synthesis by distributing tolerances so as to satisfy the stack-up conditions. The algorithm of tolerance synthesis is shown in Fig. 3 in which the cost minimisation is used as global criteria. A probabilistic approach has been used for synthesis and analysis. Branch and bound algorithm has been used for optimum selection of tolerances using production cost under the constraints of tolerance stack-up condition. The probabilistic optimisation problem for tolerance synthesis is simplified into deterministic nonlinear programming problem. Contributions of other researchers in this field are reported[20-25].

3. COST-TOLERANCE MODEL

The specification of tolerancing on the dimension of the manufactured parts has a significant impact on the final production cost. Tight tolerance can result in excessive process cost, while loose tolerance may cost less but lead to increased wastage and assembly problems. Wu and Elmaraghy[26] have reported that the proper selection of design tolerance is the key element to increase in productivity, control on product quality and cut down in the manufacturing cost. Various researchers have developed models for cost tolerance. Wu and Elmaraghy have discussed eight tolerance analysis models, namely, Sutherland function, reciprocal function, reciprocal square function, exponential function and Michral-Siddall functions. Dong[27], *et al.* have made tolerance synthesis in terms of cost tolerance relation model, and a strategy for optimisation has been suggested. Based on modelling error analysis, six new cost tolerance models have been developed[26, 27]. These models consist of two major classes:

(a) Hybrid models
- Combined reciprocal power and exponential function
- Combined linear and exponential function B-spline

(b) Polynomial models
- Cubic polynomial
- Fourth-order polynomial
- Fifth-order polynomial

These models have been evaluated using empirical production cost tolerance data of typical production process, including hole machining, turning, milling, grinding and casting. The new models have shown significantly less inherent modelling error than the model suggested by Wu and Elmaraghy. Dong[27], *et al.* have also reported that hybrid model formulation of tolerance synthesis can provide additional modelling error deduction.

Chase[28], *et al.* have performed tolerance analysis from the design point of view, and have reported that tolerance allocation is an optimisation problem in which it is required to optimise cost tolerance function. They have advocated that in order to make advanced tolerance analysis and optimisation methods available for tolerance allocation by designers, quality control technique must be used to determine process capability and track cost.

Speckhart[29], has presented an analytical method for finding the optimum set of dimension tolerance for a mechanical device that minimises manufacturing cost and also meets the imposed constraints, such as physical requirements that critical parts must fit relative to each other with a given precision. He has used nonlinear cost functions subjected to nonlinear constraints.

Michael and Siddall[30] have developed a nonlinear optimisation model which integrates the relationship between the design and production activities to cope with the problem of optimally allocating tolerances in manufacturing process. An important distinction between design and manufacturing scrap has been explained and the use of cell technique is suggested to estimate system scrap. Using graph approach, Irani[11], *et al.* have presented an integrated tolerance cost model which takes into account, an alternative machine selection method within the desired tolerance range, variation in manufacturing on tolerance value, variation in manufacturing cost and optimisation of interdependent tolerance stack-up. Tang[10], *et al.* have chosen a cost function as an objective function to be minimised, the value of which varies with design variables, i.e., process tolerance and stock removal at individual cuts.

4. TOLERANCE FOR QUALITY CONTROL

Quality engineering uses design to improve the product quality by reducing the effect of variability. Variability of output can be reduced by two actions: (i) parameter design which is supposed to adjust the nominal value so that output is less sensitive to the causes of variability, and (ii) tolerance design which is bound to reduce the tolerance to control variability.

In general, there is no manufacturing cost associated with parameter design's changing of nominal values of product parameters. However, reduction in product tolerance leads to higher manufacturing cost. Hence, it is always better to carry out parameter design before tolerance design for economic justification.

Earlier work in the area of optimal tolerance design for quality improvement was carried out by Taguchi[31], *et al.* Kapur[32] and Fathi[33]. They addressed the problem of tolerance design by considering Taguchi's famous quality loss function. Fathi[33] suggested that total expected loss can be a good criterion to measure diminution of quality caused by deviation of each critical dimension from its target value. Using Dong's[27] fifth-order polynomial function for cost tolerance relationship, Fathi has presented a methodology where total tolerance cost, which is a function of total expected cost, is minimised to obtain optimal tolerance. Taguchi[31], *et al.* have used the quality loss function in determining the tolerance allocation for multiple components of a final assembled product. They have determined the expressions for optimal tolerance and expected total quality loss through classical optimisation method.

Jeang[34], has made a good contribution in the field of tolerancing for quality improvement. He has reported that there are mainly two types of cost which occurs in a product life cycle: (i) manufacturing cost, occurs before the product is sold, and (ii) quality loss, which occurs after the sale. He also proposed a set of models which determine optimal tolerances considering both manufacturing cost and scrap/rework costs. These models include:

(a) Single component product with no inspection
(b) Single component product with inspection
(c) Multiple component product with no constraints
(d) Multiple component product with design constraints

He has illustrated use of these models with examples and compared results with Taguchi's model. Jeang[35], in another work, has discussed a set of models which determines optimal tolerances and minimises combined manufacturing and quality related costs. These models include following cases:

(a) Nominal–the best
(b) Smaller–the better
(c) Larger–the better, and
(d) Asymmetric loss function

He has suggested the use of process capability index to build the functional relationship between the product variability and the product tolerance. Feng and Kusiak[36] have suggested the use of stochastic integer programming approach for selection of optimal tolerance. They have used quality loss function for formulation of objective function.

5. DISCUSSION

A brief survey of various aspects of tolerancing techniques has been presented. It is found that tolerancing technique developed by Wade is very popular. However, this technique has gained importance only after the computer explosion, because development of tolerance chart manually is very cumbersome and time consuming. A large number of software have been developed for tolerancing charting which include various approaches, such as random gauss, graph theoretic, matrix-tree, window, black box and feature extraction. A close look at various models reveal that black box approach suggested by Ngoi, *et al.* is a promising approach which utilises chain identification code, and all files are link-listed in computer memory. The feature extraction approach suggested by Kaushal[13], *et al.* is another important technique which can be implemented in low-end CAD software, such as AutoCAD. According to a survey, most software are developed for mainframe computer and there are not enough software available on PC platform except the one developed by Fainguelunt[12], *et al.* and Kaushal[13]. These software address the problem of tolerance allocation for two-dimensional drawing and lacks three-dimensional modelling and geometric tolerancing. These software tools also lack from the point of view of statistical tolerance analysis and synthesis. Since manufacturing process is stochastic in nature, the statistical method of tolerance could provide more realistic and cost-effective tolerances on the parts.

The efforts of Wu and Elmaraghy[26] and Don[27], *et al.* in developing cost-tolerance models for different manufacturing processes are really commendable. The use of cost-tolerance model for optimisation of interdependent tolerance stack-up, using graph theoretic approach, suggested by Irani[11], *et al.* is another pioneer work in this field. The current need of the industry is to produce quality parts at low cost with limited-process capability machine tools. Controlling tolerance either at design or manufacturing level can fulfil this requirement. The mathematical models suggested by Jeang[34,35] for optimal allocation of tolerance for improving the product quality, can address this problem because these models include the cost of manufacturing as well as tolerancing. Survey reveals some efforts made to use artificial intelligence techniques[37,38] to address this problem. However, the use of neural network in computer-aided tolerancing has yet to emerge.

6. CONCLUSION

In this paper, a survey of various tolerancing techniques is presented. Some of the promising techniques from the point of view of cost-effectiveness and quality improvement have been discussed.

REFERENCES

1. Marks, C. J. Tolerance chart control production machining. *American Machinist*, **97**(5), 1953, 114-16.
2. Anderson, J.F. Assembly tolerance chart saves time and money. *Tool Engineer*, **37**(6),1956, 95-97.
3. Gadzala, J.L. Dimensional control in precision manufacturing. McGraw Hill, New York, 1959.
4. Wade, O.R. Tolerance control in design and manufacturing. Industrial Press, New York, 1967.
5. Wade, O.R. Tolerance control–tool and manufacturing engineering handbook, Vol. 1. Society of Manufacturing Engineers, 1983.
6. Farmer, L.E. *et al.* Tolerancing technology computer-based analysis. *CIRP Annals*, 1986, **35**(1), 7-10.
7. Kim, S.H. *et al.* A pseudo–Boolean approach to determining least cost tolerance. *Int. J. Prod. Res.*, 1988, **26**(1), 157-67.
8. Michael, W. & Siddal, J.N. The optimisation problem with optimal tolerance assignment and full acceptance. *ASME J. Engg. Ind.*, 1981, **103**, 842-45.
9. Ahluwalia, R. S. & Karolin, A.V. CATC–A computer-aided tolerance control system. *J. Manuf. Syst.* 1986, **3**, 153-60.
10. Tang, X. & Davies, B.J. Computer-aided dimensional planning. *Int. J. Prod. Res.*, 1988, **26**, 283-97.
11. Irani, S.A. *et al.* Tolerance chart optimisation. *Int. J. Prod. Res.*, 1989, **27**(9), 1531-552.
12. Fainguelunt, D. *et al.* Computer-aided tolerancing and dimensioning in process planning. *CIRP Annals*, 1986, **35**(1), 381-86.
13. Kaushal, P. *et al.* Computer-aided tolerance assignment procedure for design dimensioning. *Int. J. Prod. Res.* 1992, **30**(3), 599-610.
14. Huang, S.H. *et al.* Tolerance analysis for setup planning: A graph theoretical approach. *Int. J. Prod. Res.*, 1997, **35**(4), 1107-124.
15. Ngoi, B.K.A. *et al.* Computer-aided tolerancing charting black box approach. *Int. J. Prod. Res.*, 1995, **33**(4), 1117-134.
16. Ngoi, B.K.A. *et al.* A window approach for tolerancing charting. *Int. J. Prod. Res.*, 1996, **34**(4), 1093-1107.

17. Hoffman, P. Analysis of tolerances and process inaccuracies in discrete part manufacturing. *Comput. Aided Des.*, 1982, **14**(2), 83-88.

18. Chase, K.W. *et al.* A new tolerance analysis method for designers and manufactures. *ASME J. Engg. Ind.,* 1987, **109**, pp. 112-116.

19. Lee, W. & Woo, T.C. Tolerance: Their analysis and synthesis. *ASME J. Engg. Ind.,* 1990, **112**, 113-19.

20. Zhang, H.C. *et al.* Operational dimensioning and tolerancing in *CAPP, CIRP Annals*, 1991, **40**(1), 419-22.

21. Zhang, H.C. *et al.* Tolerancing technique: The state of art. *Int. J. Prod. Res.*, 1992, **30**(9), 2111-135.

22. Stantsikopoulos, M. M. A cost tolerance analytical approach for design and manufacturing. *Int. J. Adv. Manuf. Technol.,* 1990, **5**(2), 126-34.

23. Parkinson, D.B. Tolerancing of component in CAD. *Comput. Aided Des.,* 1984, **16**(1), 25-31.

24. Ji, P. Linear programming model for tolerance assignment in tolerance chart. *Int. J. Prod. Res.*, 1993, **31**(3), 739-51.

25 Ji, P. A tree approach for tolerance charting. *Int. J. Prod. Res.*, 1993, **31**(5), 1023-33.

26. Wu, Z. & Elmaraghy. Evaluation of cost tolerances algorithms for design tolerance analysis and synthesis. *Manufacturing Review*, 1988, **1**(3), 168-78.

27. Dong, Z. *et al.* New production cost tolerance models for tolerance synthesis. *ASME J. Engg. Ind.*, 1994, **116**, 119-206.

28. Chase, K.W. *et al.* Least cost tolerance allocation for mechanical assemblies with automated process selection. *Manufacturing Review*, 1990, **3**(1), 44-59.

29. Speckhart, F.H. Calculation of tolerance based on a minimum cost approach. *ASME J. Engg. Ind.*, 1972, 447-53.

30. Michael, W. *et al.* The optimal tolerance assignment with less than full acceptance. *ASME J. Mech. Des.*, 1982, **104**, 855-60.

31. Taguchi, G. Introduction to quality engineering. White Plain, New York, 1989.

32. Kapur, K.C. An approach for development of specifications for quality improvement. *Quality Engineering*, 1989, **1**(1), 63-77.

33. Fathi, Y. *et al.* Alternative manufacturing sequence and tolerance build up: A point of view and case study. *Int. J. Prod. Res.*, 1997, **35**(1), 123-36.

34. Jeang, A. Optimal tolerance design for product life. *Int. J. Prod. Res.*, 1996, **34**(8), 2187-209.

35. Jeang, A. An approach of tolerance design for quality improvement and cost reduction. *Int. J. Prod. Res.*, 1997, **35**(5), 1193-211.

36. Feng, C. X. & Kusiak, A. Robust tolerance design with integer programming approach. *ASME J. Manuf. Sci. Engg.*, 1997, **119**, 603-10.

37. Jankiram, D. *et al.* Tolerancing of parts using an expert system. *Int. J. Adv. Manuf. Syst.*, 1989, **4**, 157-67.

38. Manivannan, S. *et al.* A knowledge-based system for the specification of manufacturing tolerances. *J. Manuf. Syst.*, 1989, **81**(2), 153-60.