# Input Excitation Techniques for Aerodynamic Derivatives-Estimation of Highly Augmented Fighter Aircraft

## N. Shantha Kumar

#### National Aerospace Laboratories, Bangalore - 560 037.

### ABSTRACT

This paper presents the results of an investigation related to the estimation of lateral-directional aerodynamic derivatives of highly augmented and advanced fighter aircraft from the flight like response data. Different types of pilot inputs are used to generate aircraft response data in the engineer-in-loop flight simulator to determine which input excitation might provide the most accurate estimates of aircraft stability and control derivatives. Also, MATLAB/ SIMULINK-based simulation platform is used to generate aircraft response with single-surface excitation to evaluate the usefulness of the method for stability and control derivatives estimate the derivatives from the aircraft simulation response data. The results indicate that accuracy of the estimated derivatives improve with persistence excitation and single-surface excitation.

m

Mass of aircraft (kg)

### NOMENCLATURE

$a_{y}$	Lateral acceleration (g)	М	Mach number		
b	Wing span (m)	р	Roll rate (rad/s)		
$C_D$	Drag coefficient	q	Pitch rate (rad/s)		
$F_T$	Thrust (N)	$\overline{\mathbf{q}}$	Dynamic pressure (N/m <sup>2</sup> )		
g	Acceleration due to gravity (m/s <sup>2</sup> )	r	Yaw rate (rad/s)		
H	Altitude (m)	<b>S</b> /	Wing reference area (m <sup>2</sup> )		
$I_{xx}$	Moment of inertia about X-axis (kg-m <sup>2</sup> )	V	True air speed (m/s)		
I <sub>xz</sub>	Cross moment of inertia about X and Z-axis (kg-m <sup>2</sup> )	X <sub>β</sub>	Position of $\beta$ sensor in X-direction (body axis) from centre of gravity (m)		
$I_{yy}$	Moment of inertia about Y-axis (kg-m <sup>2</sup> )	$Z_{\beta}$	Position of $\beta$ sensor in Z-direction (body axis) from centre of gravity (m)		
Izz	Moment of inertia about Z-axis (kg-m <sup>2</sup> )	. Р ·			

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X <sub>ay</sub>	Position of $a_y$ sensor in X-direction (body axis) from centre of gravity (m)	$C_{n_0}$	Yawing moment coefficient at zero angle of attack				
Y <sub>ay</sub>	Positive of $a_y$ sensor in Y-direction (body axis) from centre of gravity (m)	C <sub>n</sub> β	Change in yawing moment coefficient due to change in $\beta/(rad)$				
$Z_{ay}$	Position of $a_y$ sensor in Z-direction (body axis) from centre of gravity (m)	$C_{n_p}$	Change in yawing moment coefficient due to change in $p/(rad/s)$				
α	Angle of attack (rad)	$C_{n_r}$	Change in yawing moment coefficient due				
β	Angle of side slip (rad)		to change in r/(rad/s)				
δ <sub>a</sub>	Aileron deflection (rad)	$C_{n_{\mathfrak{S}a}}$	Change in yawing moment coefficient due to change in $\delta a/(rad)$				
$\delta_r$	Rudder deflection (rad)	C	• · · ·				
φ	Roll angle (rad)	$C_{n_{\mathrm{dr}}}$	Change in yawing moment coefficient due to change in $\delta r/(rad)$				
θ	Pitch angle (rad)	$C_{y_0}$	Side force coefficient at zero angle of attack				
$\sigma_T$	Nozzle deflection (rad)	$C_{y_{B}}$	Change in side force coefficient due to				
Θ	Vector of true values (linear model) of		change in $\beta/(rad)$				
_	derivatives	$C_{y_p}$	Change in side force $p/(c)$ coefficient due to change in $p/(rad/s)$				
Ô	Vector of estimated values of derivatives						
$C_{l_0}$	Rolling moment coefficient at zero angle of attack	C <sub>yr</sub>	Change in side force coefficient due to change in $r/(rad/s)$				
С <sub>4</sub>	Change in rolling moment coefficient due to change in $\beta/(rad)$	$C_{y_{\delta a}}$	Change in side force coefficient due to change in $\delta a/(rad)$				
$C_{l_p}$	Change in rolling moment coefficient due to change in $p/(rad/s)$	$C_{y_{\delta r}}$	Change in side force coefficient due to				
$C_{l_r}$	Change in rolling moment coefficient due to change in <i>r</i> /(rad/s)	Subsci	change in δr/(rad)				
$C_{l_{\delta a}}$	Change in rolling moment coefficient due to $\frac{1}{2} \frac{1}{2} $	m	Measured				
C	change in $\delta a/(rad)$	0	Value at trim condition				
$C_{l_{\mathrm{ör}}}$	Change in rolling moment coefficient due to change in $\delta r/(rad)$	Ŭ	Dot on the variable indicates time derivative.				

# 1. INTRODUCTION

The estimation of stability and control derivatives is an important tool for flight test engineers in determining the aerodynamic characteristics of new and untested aircraft. Flight-determined stability and control derivatives are also useful in updating the flight simulator model and in improving the flight control laws and handling qualities. The system identification techniques are successfully applied in estimating the aircraft stability and control derivatives

In case of highly augmented fly-by-wire fighter aircraft, estimation of aerodynamic parameters of

from flight test data<sup>1-3</sup>. In these methods, the aircraft

system under investigation is assumed to be modelled

by a set of dynamic equations containing the unknown

parameters. The system is excited by a suitable input

and the input and system response are measured. The

values of the unknown parameters are then inferred

based on the requirement that the model response to

the given input match the actual system response.

basic unstable aircraft poses several problems<sup>4</sup>. Due to the feedback action, the stability augmentation system constantly controls the aircraft response, and the measured responses may not exhibit the required modes for estimation of unknown parameters. Also, feedback action introduces correlation among the input and output variables which could cause inaccurate and biased estimates. The test aircraft under investigation, though laterally stable, is provided with lateral-directional control system with schedule gains to achieve desired handling qualities.

Because of feedback loop in lateral-directional control and due to aileron-rudder interconnection, the directional response of the aircraft to standard pilot inputs (i.e., system identification inputs) is tightly controlled and often does not exhibit required modes and information for estimating some important derivatives. Also, there is a strong correlation between input and response signals which causes more uncertainty in estimation, particularly at high angle of attack.

This paper addresses these issues and presents techniques to overcome the problems from parameter estimation point of view. For the present study, the lateral-directional response data is generated in engineer-in-the-loop simulator (ELS), which is a dedicated flight simulator for flight control law design. Also, the MATLAB/SIMULINK-based simulation platform is used to generate the aircraft response with single-surface excitation (SSE), as this provision is not available in ELS.

### 2. ESTIMATION TECHNIQUE

The stability and control derivatives, represented as unknown parameters in aircraft dynamical equations, are estimated by output error method (OEM) based on maximum likelihood estimation (MLE) technique<sup>1</sup>. In this method, a probability that the aircraft model response time history attains values near to the measured aircraft response time history, is defined in terms of possible estimate of unknown parameters, and the MLE are defined as those that maximise this probability. MLE as many desirable statistical characteristics for example, yields asymptotically-unbiased, consistent and efficient estimates<sup>5</sup>. MLE also provides a measure of reliability of each estimate based on the information obtained from each dynamic maneouver, called Cramer-Rao bound. In the presence of measurement noise, Cramer-Rao bound is analogous to the standard deviation (SD) and provides an estimate of the uncertainty interval. A comparison of SD of like parameters for different input techniques indicates which technique is providing accurate estimates of stability and control derivatives.

# 3. LATERAL-DIRECTIONAL EQUATIONS OF MOTION

The following fourth-order model is used to fit the simulated lateral-directional response data and to estimate relevant aerodynamic derivatives:

### 3.1 State Equations

$$\dot{\beta} = \frac{\overline{q}_{S}}{mV_{0}} \left[ \begin{cases} C_{y_{0}} + C_{y_{\beta}} \beta + C_{y_{p}} p \frac{b}{2V_{0}} \\ + C_{y_{r}} r \frac{b}{2V_{0}} + C_{y_{8_{a}}} \delta_{a} + C_{y_{8_{r}}} \delta_{r} \end{cases} \cos \beta \\ + C_{D} \sin \beta \right] \\ + p \sin \alpha_{0} - r \cos \alpha_{0} + \frac{F_{T}}{mV_{0}} \cos (\alpha_{0} + \sigma_{T}) \sin \beta \\ + \frac{g}{V_{0}} [\sin \phi \cos \theta_{0} \cos \beta + \sin \beta \cos \alpha_{0} \sin \theta_{0} \\ - \sin \alpha_{0} \cos \phi \cos \theta_{0} \sin \beta] \end{cases}$$

$$\dot{p} = \frac{\bar{q}sb}{I_{xx}I_{zz} - I_{xz}^2} \begin{bmatrix} I_{zz} \\ I_{zz} \\ I_{zz} \end{bmatrix} \begin{bmatrix} C_{I_0} + C_{I_\beta}\beta + C_{I_p}p \frac{b}{2V_0} \\ + C_{I_r}r \frac{b}{2V_0} + C_{I_{\delta_a}}\delta_a + C_{I_{\delta_r}}\delta_r \end{bmatrix} + \\ \begin{bmatrix} C_{n_0} + C_{n_\beta}\beta + C_{n_p}p \frac{b}{2V_0} \\ + C_{n_r}r \frac{b}{2V_0} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r \end{bmatrix} \\ - q_m r \frac{I_{xz}^2 - I_{yy}I_{zz} + I_{zz}^2}{I_{xx}I_{zz} - I_{xz}^2} + p q_m \frac{I_{xz}(I_{xx} - I_{yy} + I_{zz})}{I_{xx}I_{zz} - I_{xz}^2} \end{bmatrix}$$

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(2)

$$\dot{r} = \frac{\overline{q}sb}{I_{xx}I_{zz} - I_{xz}^{2}} \begin{bmatrix} I_{xx} \begin{cases} C_{n_{0}} + C_{n_{\beta}} \beta + C_{n_{p}} p \frac{b}{2V_{0}} \\ + C_{n_{r}} r \frac{b}{2V_{0}} + C_{n_{\delta_{a}}} \delta_{a} + C_{n_{\delta_{r}}} \delta_{r} \\ + I_{xz} \end{cases} \begin{bmatrix} C_{I_{0}} + C_{I_{\beta}} \beta + C_{I_{p}} p \frac{b}{2V_{0}} \\ + C_{I_{r}} r \frac{b}{2V_{0}} + C_{I_{\delta_{a}}} \delta_{a} + C_{I_{\delta_{r}}} \delta_{r} \end{bmatrix} \\ - q_{m} r \frac{I_{xz} (I_{xx} - I_{yy} + I_{zz})}{I_{xx}I_{zz} - I_{xz}^{2}} + p q_{m} \frac{I_{xz}^{2} - I_{xx}I_{yy} + I_{xx}^{2}}{I_{xx}I_{zz} - I_{xz}^{2}} \\ \dot{\phi} = p + q_{m} \sin \phi \tan \theta_{0} + r \cos \phi \tan \theta_{0}$$
(1)

### **3.2 Measurement Equations**

$$\beta_{m} = \beta - \frac{Z_{\beta}}{V_{0}}p - \frac{X_{\beta}}{V_{0}}r$$

$$p_{m} = p$$

$$r_{m} = r$$

$$\phi_{m} = \phi$$

$$a_{y_{m}} = \frac{1}{g} \begin{bmatrix} \frac{\overline{qs}}{m} \begin{cases} C_{y_{0}} + C_{y_{\beta}}\beta + C_{y_{p}}p \frac{b}{2V_{0}} \\ + C_{y_{r}}r \frac{b}{2V_{0}} + C_{y_{\delta_{a}}}\delta_{a} + C_{y_{\delta_{r}}}\delta_{r} \\ + (pq_{m} + \dot{r}) X_{ay} - (r^{2} + p^{2}) Y_{ay} \\ + (rq_{m} - \dot{p}) Z_{ay} \end{bmatrix}$$

$$p_{m} = \dot{p}$$

$$r_{m} = \dot{r}$$

In the above model, all the derivatives except  $C_{y_0}$ ,  $C_{I_0}$ ,  $C_{n_0}$  are estimated. These derivatives are ignored as they are very small and insignificant.

# 4. ESTIMATION WITH STANDARD PILOT INPUT

The lateral-directional aerodynamic derivatives are generally estimated by exciting lateral-directional modes through Dutchroll maneouver. Initially, the aircraft is excited in ELS with standard pilot input consisting of 10 mm doublet-to-roll stick followed by 10 mm doubled-to-rudder pedal Fig. 1(a). The aircraft response is recorded with sensor noise at three flight condition, viz.,

(a) M = 1.0, H = 8000 m,  $\alpha_0 = 2.46^{\circ}$ 

(b) M = 0.6, H = 8000 m,  $\alpha_0 = 6.59^\circ$ 

(c) M = 0.4, H = 8000 m,  $\alpha_0 = 12.45^\circ$ 

Using OEM algorithm, most of the aerodynamic derivatives are estimated accurately at all the three flight conditions from simulated response data. As angle of attack increases,  $C_{n_{\rm B}}$  and  $C_{n_{\rm r}}$  appear to be difficult to estimate accurately. This is because, with under carriage up, the aileron and rudder interconnection gain in flight control system, increases with angle of attack causing small deflection of rudder surface for the standard pilot input to the rudder pedal at higher angle of attack. This small rudder deflection causes small variation in sideslip angle  $\beta$  and yaw rate r indicating low information in these signals. This insufficient information in  $\beta$  and r results in inaccurate estimate of important derivatives like  $C_{n_{\rm B}}$  and  $C_{n_{\rm r}}$ . Also because of feedback gains, there is a strong correlation among response signals and control surface signals due to which estimated derivatives show large uncertainty levels. To overcome this deficiency, the pilot inputs are modified exclusively for parameter identification experiment to increase the information content in aircraft response signals and to reduce the response signal correlation.

# 5. PERSISTENT EXCITATION WITH INCREASED AMPLITUDE

One way to get sufficient excitation and information in  $\beta$  and *r* responses is to increase the amplitude and bandwidth of pilot input signal to rudder pedal. The bandwidth of input signal can be increased<sup>6</sup> with 3 2 1 1 type input. Since parameter estimation is based on small perturbation analysis, the amplitude of pilot input is to be chosen to restrict the aircraft response within the linear range. Keeping in mind the above restriction, the following types of pilot inputs are considered to study the effect of pilot input on parameter estimation:

- Type 1 10 mm doublet input to roll stick followed by 10 mm doublet input to rudder pedal (standard input)
- *Type* 2 10 mm doublet input to roll stick followed by 30 mm doublet input to rudder pedal.
- Type 3 10 mm doublet input to roll stick followed by three repeated doublet of 30 mm amplitude to rudder pedal.
- *Type* 4 10 mm doublet input to roll stick followed by 3 2 1 1 input of 30 mm amplitude to rudder pedal.

These input signals are plotted against time (Fig. 1). The aircraft response to these input signals

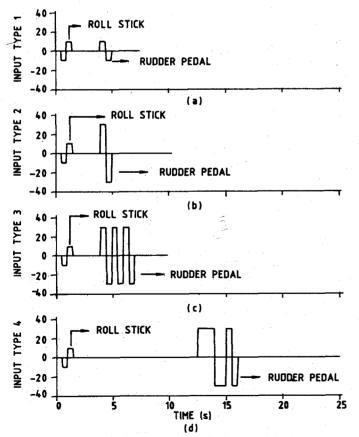


Figure 1. Types of pilot inputs, (a) similar doublet inputs to roll stick and rudder, (b) doublet input with increased amplitude to rudder, (c) repeated doublets input with increased amplitude to rudder, (d) and 3 2 1 1 input with increased amplitude to rudder.

are recorded with sensor noise at each of the abovementioned flight condition, and unknown aerodynamic derivatives are estimated along with SD. The SD of some important estimated derivatives shown in Fig. 2 indicates that estimation accuracy improves with persistent excitation.

The simulation program in ELS has a facility to generate linear model values<sup>7</sup> of derivative at a given flight condition which represents true values of derivatives. From these known true values of derivatives, the percentage value of parameter estimation error norm (PEEN) given by

$$\left(\frac{\sum |\Theta - \hat{\Theta}|}{\sum |\Theta|}\right) * 100 \tag{3}$$

is computed to show how close are the estimated parameters to their true values. Less PEEN means estimated parameters are closer to true values. PEEN is plotted against input signal type at each flight condition (Fig. 3). The results indicate that persistent excitation, viz., repeated doublet and 3 2 1 1 input signals gives consistently better estimates.

### 6. SINGLE-SURFACE EXCITATION

Single-surface excitation (SSE), wherein an additional input signal is applied directly to the control surfaces bypassing the control system, is recommended as one of the techniques to reduce the signal correlation. SSE also helps in accurate estimation of control surface effectiveness of individual control surfaces. Elsewhere, SSE technique<sup>8</sup> has been applied in real flight and has successfully demonstrated the improvement in parameter estimation.

In flight test, SSE can be implemented by using flight control computers to generate separate input signal and fed directly to control surface actuators. Alternative way is to use external devices like flutter test box<sup>8</sup> to excite control surfaces. Since in ELS, no such provision is provided, alternative simulation platform is developed in MATLAB/SIMULINK to implement SSE. With considerable effort, this linear DEF SCI J, VOL 49, NO 3, JULY 1999

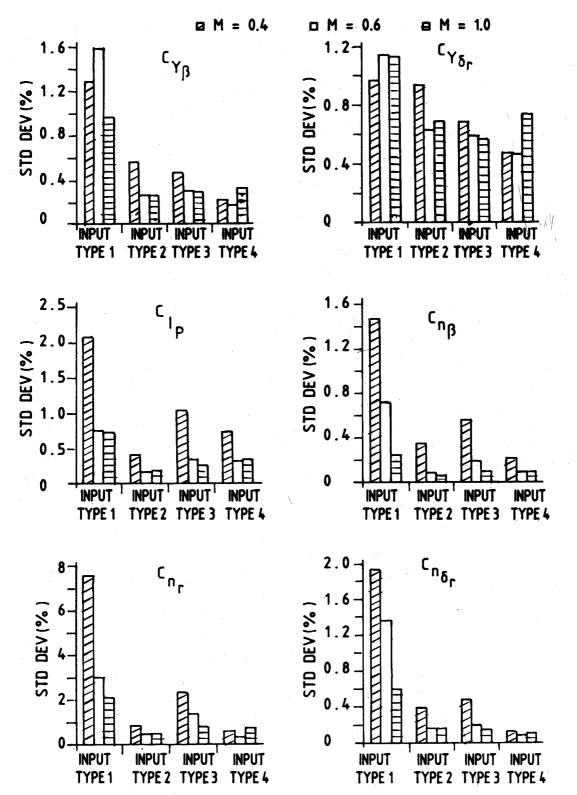
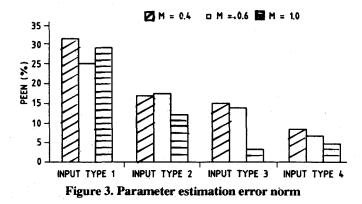


Figure 2. Standard deviation of estimated parameters

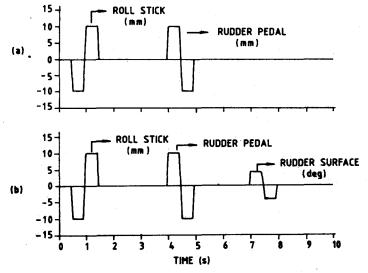
simulation platform has been built from the given control law details and validated by comparing the

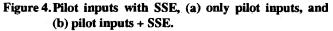
responses with those generated in ELS at several flight conditions. This linear simulation platform is able to



generate lateral-directional responses for roll stick and rudder pedal inputs.

Using this simulation platform, lateraldirectional response is generated by giving standard doublet input shown in Fig. 4(a) to roll stick and rudder pedal, at all the above-mentioned flight conditions given in Section 4. The response is then generated with the same input followed by SSE given directly to rudder surface as shown in Fig. 4(b). These simulated response signals are analysed for collinearity<sup>9</sup> and parameter estimation. Since correlation coefficient between two signals can point to a possible correlation problem, the matrix of correlation coefficients is computed (Table 1). The highlighted numbers indicate strong correlation amongst the response variables. The results clearly show that the correlation between response signals is





considerably reduced with SSE. Also, the aerodynamic derivatives estimated from simulated response signals show lower SD with SSE indicating more accurate estimates. Figure 5 shows the comparison of SD of estimates with and without SSE for some important derivatives.

### 7. CONCLUSION

Different types of pilot inputs are used to generate aircraft response data in ELS to determine which input might provide the most accurate estimates of aircraft stability and control derivatives. Also, MATLAB/ SIMULINK-based simulation platform is used to generate aircraft response with single-surface excitation to evaluate the usefulness of the method for stability and control derivatives estimation. MLE based on OEM technique is used to estimate the derivatives from the aircraft response data. The estimation results indicate that:

- At larger angle of attack, standard pilot input is not adequate to excite directional motion sufficiently and related derivatives are difficult to estimate accurately. A strong correlation exists among response signals due to which there is large uncertainty in estimates.
- Repeated doublet and wider bandwidth input signal like 3 2 1 1 with increased amplitude to rudder pedal causes more excitation in both lateral and directional motion. This results in more information in the response data and estimates improve with lower uncertainty levels.
- SSE reduces correlation among response signals which results in estimates with lesser uncertainty levels. Also, it gives best estimates for the control effectiveness parameters due to direct uncorrelated excitation of corresponding control surfaces. However, there might be some practical difficulty in implementing SSE in actual flight test, particularly when aircraft is new and untested.

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	Only pilot inputs					Pilot inputs + SSE						
	β	p	r	δa	δ,		β	р	r	δα	δ,	
β	1.000	-0.9995	-0.9989	0.5597	0.4175	β	1.0000	-0.7874	-0.6117	-0.2808	-0.5705	
P		1.0000	0.9990	-0.5628	-0.4215	P		1.0000	0.9067	-0.3027	-0.0118	
r			1.0000	-0.5953	-0.4571	r			1.0000	-0.5792	-0.2929	
δa				1.0000	0.9862	δ <sub>a</sub>				-1.0000	0.9212	
δ <sub>r</sub>					1.0000	δ <sub>r</sub>					1.0000	

### Table 1. Matrix of correlation coefficients of response signals

(a) M = 0.4, H = 8000 m,  $\alpha_0 = 12.45^\circ$ 

Pilot inputs + SSE								
	β	р	r	δα	δ,			
β	1.0000	-0.9391	-0.7778	0.5528	-0.6551			
Р		1.0000	0.8204	-0.6900	0.4330			
r			1.0000	-0.8758	0.1454			
δα				1.0000	-0.0185			
δŗ					1.0000			

		Only pil	lot inputs		
	β	р	r	δα	δr
β	1.000	-0.9972	-0.9966	0.9240	0.5956
р		1.0000	0.9995	-0.9294	0.5486
r			1.0000	-0.9362	0.5317
δα				1.0000	-0.319
δ <sub>r</sub>	e en	· · · · · · · · · · · · · · · · · · ·			1.0000

(b) M = 0.6, H = 8000 m,  $\alpha_0 = 6.59^\circ$ 

β

1.000

Only pilot inputs			Pilot inputs + SSE							
p	r	δα	δ,	· · · · · · · · · · · · · · · · · · ·	β	р	r	δα	δ,	
0.9989	0.9989	0.9592	-0.9522	β	1.0000	0.9862	0.3255	0.3506	-0.4928	
1.0000	0.9812	-0.9641	0.9575	Р		1.0000	-0.4432	0.4158	0.5270	
	1.0000	-0.9813	0.9660	r			1.0000	-0.8936	-0.3929	
		1.0000	-0.9963	δa				1.0000	0.1206	
			1.0000	δ <sub>r</sub>					1.0000	

(c) M = 1.0, H = 8000 m,  $\alpha_0 = 2.46^\circ$ 

β

р

r

δa

δŗ

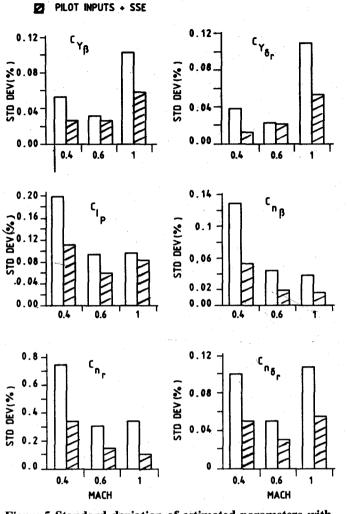


Figure 5. Standard deviation of estimated parameters with and without SSE.

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ONLY PILOT INPUTS

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#### Contributor



Mr N Shantha Kumar obtained his MTech (Aerospace Engineering) from Indian Institute of Technology (IIT) Bombay, in 1987. Presently, he is working as Scientist at the Flight Mechanics and Control Division of the National Aerospace Laboratories (NAL), Bangalore. His areas of work include: modelling and parameter estimation of aerospace vehicles and sensor failure detection and management. He worked as Guest Scientist at the Institute of Flight Mechanics of German Aerospace Research Establishment and Institute of Flight Mechanics, Braunschweig, Germany, for six months during 1993. He is a member of the Aeronautical Society of India.