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Review Paper

Global Contour and Region Based Shape Analysis and Similarity Measures

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ABSTRACT

More and more images have been generated in digital form around the world. There is a growing interest in finding images in large collections or from remote databases. In order to find an image, the image has to be described or represented by certain features. Shape is an important visual feature of an image. Searching for images using shape features has attracted much attention. There are many shape representation and description techniques in the literature. Object classification often operates by making decisions based on the values of several shape properties measured from an image of the object. Shape analysis is a useful tool for recognition of an object. This paper treats various aspects that are needed to solve shape matching problems: choosing the precise problem of global contour and region based shape analysis, selecting the properties of the similarity measure that are needed for the problem and choosing the specific similarity measure to compute the similarity.

Keywords: Shape analysis, object classification, shape measure, pattern recognition, computer vision, circular fit, elliptic fit, rectangular fit

1. INTRODUCTION

An important task in image analysis is the discrimination of objects based on their appearance. Various properties of appearance such as texture, color and shape can be measured. Shape is a powerful tool for describing objects and differentiating between them, and has been extensively applied in many areas of computer vision[1,2]. In fact, the issue of shape extends far beyond computer vision into areas such as the graphic arts[3], physics, chemistry and biology[4,5].

There is no universal definition of what shape is. The word 'figure' is used for shape. Here we consider shape as something geometrical. We will use the term shape for a geometrical pattern, consisting of a set of points, curves, surfaces, solids etc. This is commonly done, although 'shape' is sometimes used for a geometrical pattern modulo some transformation group, in particular similarity transformations. Shape analysis deals with transforming a shape, and measuring the resemblance with another one, using some similarity measure. So, shape similarity measures are an essential module in shape matching. Although the term similarity is often used, dissimilarity corresponds to the notion of distance: small distance means small dissimilarity, and large similarity. Shape similarity measures are used in building extraction and oil tank detection from remote sensing images[2,6,7,8].

Various shape features are often evaluated by how accurately they allow one to retrieve similar shapes from a designated database. However, it is not sufficient to evaluate a representation technique only by the effectiveness of the features employed. This is because the evaluation ignores other important characteristics of a shape representation technique. For example, in the new multimedia application content-based image retrieval (CBIR), efficiency is envisaged as equally important as effectiveness due to the online retrieval demand. In fact, MPEG-7 has set several principles to measure a shape descriptor, that is, good retrieval accuracy, compact features, general application, low computation complexity, robust retrieval performance and hierarchical coarse to fine representation[9].

The algorithm to compute the similarity depends on the precise measure, required properties and particular matching problems for the application at hand. In this paper, global contour and region based shape analysis is described. The classification of matching problems and similarity measure properties are explained, and various similarity measures are presented.

2. SHAPE ANALYSIS

Shape analysis methods analyze the objects in a scene. In this section, we concentrate on shape representation and description aspects of shape analysis. Shape representation methods result in a non-numeric representation of the original shape so that the important characteristics of the shape are preserved. The word important typically has different meanings for different applications. Shape representation generally looks for effective and perceptually important shape features based on either shape boundary information or boundary plus interior content. Various features have been designed, including shape signature, signature histogram, shape invariants, moments, curvature, shape context, shape matrix, spectral features etc. Shape description refers to the methods that result in a numeric

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descriptor of the shape and is a step subsequent to shape representation. A shape description method generates a shape descriptor vector (feature vector) from a given shape. The goal of description is to uniquely characterize the shape using its shape descriptor vector. The required properties of a shape description scheme are invariance to translation, scale, and rotation. This is required because these three transformations, by definition, do not change the shape of the object.

The problem of the shape analysis has been published by many authors and a great amount of research papers are available in the literature. A number of review papers[10-12], as well as books[13-20] have been written on the subject of shape analysis.

2.1 Classifications

Shape analysis methods can be classified according to many different criteria. Pavlidis[10] has proposed the following classifications. The first classification is based on the use of shape boundary points as opposed to the interior of the shape. The two resulting classes of algorithms are known as boundary (external) and global (or internal), respectively. Examples of the former class are algorithms which parse the shape boundary[15-24] and various Fourier transforms of the boundary[24-29]. Examples of global methods include the medial axis transform (MAT) proposed by Blum and described in[15,18,20,30-34], moment based approaches[37-47], and methods of shape decomposition into other primitive shapes[48-50]. Another classification of shape analysis algorithms can be made on the basis of whether the result of the analysis is numeric or non-numeric. For example, the MAT produces another image and is therefore called a space-domain technique. On the other hand, scalar transform techniques produce numbers as results. Examples of later methods include various Fourier[24-29] and moment-based[37-47] procedures for shape analysis.

A third classification of shape analysis methods can be made on the basis of information preservation. Methods which allow for the accurate reconstruction of a shape from its descriptor are called information preserving methods, as opposed to methods only capable of partial reconstruction which are called information non-preserving techniques. An example of an information non-preserving method is area to perimeter square ratio. Many significantly different shapes can have the same area to perimeter square ratio, and therefore it is not possible to reconstruct the original shape knowing only its area to perimeter square ratio. Many simple shape descriptors suffer from the same problem.

2.1.1 Contour-based Shape Representation and Description Techniques

Contour shape techniques only exploit shape boundary information. There are generally two types of very different approaches for contour shape modeling: continuous approach (Global) and discrete approach (Structural). Continuous approaches do not divide shape into sub-parts; usually a feature vector derived from the integral boundary is used to describe the shape. The measure of shape similarity is usually a metric distance between the acquired feature vectors. Discrete approaches break the shape boundary into segments, called primitives using a particular criterion. The final representation is usually a string or a graph (or tree) the similarity measure is done by string matching or graph matching. In the present paper we discuss about the continuous approach. One can go through Zhang and Lu paper[12] for discrete approach.

2.1.1.1 Continuous Approach

Continuous contour shape representation techniques usually compute a multi-dimensional numeric feature vector from the shape boundary information. The matching between shapes is a straightforward process, which is usually conducted by using a metric distance, such as Euclidean distance or city block distance. Point (or point feature) based matching is also used in particular applications.

2.1.1.1.1 Simple Shape Descriptors

Common simple global descriptors are area, rectangularity, circularity (*perimeter*^{2}/*area*), eccentricity (length of major axis/length of minor axis) and major axis orientation[51]. These simple global descriptors usually can only discriminate shapes with large differences; therefore, they are usually used as 1lters to eliminate false hits or combined with other shape descriptors to discriminate shapes. They are not suitable to be standalone shape descriptors. Other simple global contour shape descriptors have been proposed by Peura and Iivarinen[52]. These descriptors include convexity, ratio of principle axis, circular variance and elliptic variance. Young, Walker, and Bowie[51] proposed an interesting concept of bending energy. According to this approach, a shape can be represented by its bending energy defined by $E = \frac{1}{P}\int_0^P |K(p)|^2\, dp$ where $K(p)$ is the curvature function, p is the arc length parameter, and P is the total curve length. To actually compute the bending energy above equation was not used directly, but instead the Fourier transform of the boundary was computed first. Using Fourier coefficients and Parseval's relation, the bending energy was computed in a more efficient way. In addition, the authors proved that the circle was the shape having the minimum average bending energy.

Object classification often operates by making decisions based on the values of several shape properties measured from an image of the object. In computer vision, circularity is extensively used as a shape measure[53,54]. Haralick[53] proposed a measure for circularity where the center of the fitting circle is assumed to coincide with the centroid of the object border. But there is little work on the measurement of the other shapes. Proffitt[51] and Peura and Livarinen[56] have both described ellipticity measures. Although one rectangularity measure[50] is reasonably well known it is not as widespread as circularity. More recent work by Rosin described several new approaches to rectangularity measurement[54]. Another recent work by Rosin[1] proposed several algorithms for calculating ellipticity, rectangularity and triangularity shape descriptors.

(a) Circularity

- Geometry based (C_G)

  Circularity measure is defined as P^2/A where P and A are the perimeter and area of the object.

- Statistical based (C_S)

Haralick[53] proposed a measure for circularity where the center of the fitting circle is assumed to coincide with the centroid of the object border. So $\left(\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \right)$ be the centroid where $(x_i, y_i), i = 1,2,...,n$ are the border points of the object.

Let $\mu_R = \frac{1}{n}\sum_{i=1}^{n}\left\|(x_i - y_i) - (\bar{x}, \bar{y})\right\|$ be the mean redial distance and $\sigma_R = \left(\frac{1}{n}\sum_{i=1}^{n}\left[\left\|(x_i - y_i) - (\bar{x}, \bar{y})\right\| - \mu_R\right]^2 \right)^{1/2}$ be the standard deviation of radial distance. The circularity measure is $circularity = \frac{\mu_R}{\sigma_R}$.

- Statistical Structural Descriptor (C_{SD})

Zhang and Wenyin[67] proposed a novel shape descriptor based on the histogram matrix of pixel-level structural. The computation of the statistical structural descriptor can be briefly summarized into 3 steps:

1. The centroid of a shape is computed based on the Distance Transform,
2. Two structural attributes, the length ratios and angles of each point on the contour, are calculated by taking the centroid as a fixed reference point.
3. Statistics are conducted on the two attributes to generate the structural feature histogram matrix (*SFHM*).

This shape descriptor can measure circularity, smoothness, and symmetry of shapes, and be used to recognize shapes. Here brief description has been made about the measure of circularity using those three steps.

Distance transform based centroid: By exploring the properties of distance transform (DT), Zhang and Wenyin[67] have presented a DT based centroid. Compared with the centroid obtained by averaging all points in a shape, DT based centroid is closer to the centroid given by the human visual system and more robust to noise. Assume $I(x,y)$ is a point in a shape, $D(x,y)$ is the corresponding point after DT, whose value $|D(x,y)|$ indicates the distance to the closest boundary point from $I(x,y)$. The DT based centroid is computed by:

$$C_{DT} = \frac{\sum_n \left[(x,y) \times |D(x,y)| \right]}{\sum_n |D(x,y)|}$$

where n is the number points in the shape.

Structural feature histogram matrix: Based on the obtained centroid, we start to compute the proposed shape descriptor. Assume that $C(x,y)$ is the centroid of a shape, P_i and P_j are two different points on the contour $\{P_1, P_2, \ldots, P_n\}$, $|P_iC|$ and $|P_jC|$ denote the lengths of the two vectors P_iC and P_jC, θ_{ij} is the angle between them, as shown in Fig. 1, two structural attributes d_{ij} and θ_{ij} are defined as follows:

$$d_{ij} = \min\left(\frac{|P_iC|}{|P_jC|}, \frac{|P_jC|}{|P_iC|} \right), d_{ij} \in [0,1] \text{ and } \theta_{ij} = \angle P_iCP_j, \theta_{ij} \in [0,\pi].$$

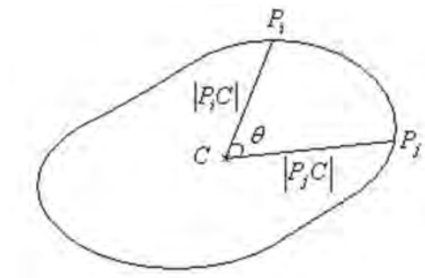

Figure 1. Two structural attributes of SFHM.

Clearly, they are invariant to scaling and rotation.

It's true that any pair of points (P_i, P_j) out of the n points on the contour has a corresponding pair (d_{ij}, θ_{ij}). Hence, there are $n(n-1)/2$ pairs of (d_{ij}, θ_{ij}), $i \in [1, n-1]$, $j \in [i+1, n]$ for a shape. The set $(D, \Lambda) = \{(d_{ij}, \theta_{ij}) \mid i \in [1, n-1];\ j \in [i+1, n]\}$ are utilized to describe the shape. First, we transform the shape into a feature space with a new coordinate system, in which θ denotes the X axis, d denotes the Y axis, and every element in the set (D, Λ) is a point in the feature space. Second, since $d_{ij} \in [0,1]$ and $\theta_{ij} \in [0,\pi]$, [0,1] divided into M equal bins, and $[0,\pi]$ into N equal bins. As a result, the feature space is divided into $M \times N$ blocks.

Lastly, Zhang and Wenyin[67] describe the formula to calculate the percentage of the contour points in the block at the u^{th} row and the v^{th} column by:

$$q(u,v) = \frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} h(d_{ij}, \theta_{ij}, x_{u-1}, x_u, y_{v-1}, y_v) \quad ;$$

$u \in [0, M-1]$ and $v \in [0, N-1]$.

where

$$h(d_{ij}, \theta_{ij}, x_{u-1}, x_u, y_{v-1}, y_v) = \begin{cases} 1 \text{ if } d_{ij} \in [x_{u-1}, x_u), \theta_{ij} \in [y_{v-1}, y_v) \\ 0 \quad \text{otherwise} \end{cases}$$

Hence, there is $\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} q(u,v) = 1$.

Finally, a $M\times N$ histogram matrix $\boldsymbol{Q}$ structural feature histogram matrix (SFHM) is constructed based on all $q(u,v)$, $u \in [0, M-1]$, $v \in [0, N-1]$.

Now the circularity (*Cir*) of a shape can compute by using structural feature histogram matrix[67]. Assume the size of Q of a shape is $M\times N$ with the distance d=1 at 1st row and d=0 at M^{th} row.

- Assign the elements in i^{th} row of Q a weight i so that a bigger d has a smaller weight and a smaller d has a bigger weight.
- Let *Cir* equal to the weighted summation of all elements in Q. The pseudo code is:

```
Cir = 0;
FOR i =1:M
Cir=Cir+(summation of all element on ith row)*i;
END
```

It's true that all elements in the *SFHM* of circles are at the first row where d=1 and the summation of all elements is 1. Hence, circle has smallest circularity, *Cir* = 1. A shape with *Cir* value closer to 1 is more similar to a circle.

(b) Ellipticity

- Moment Invariants (E_1)

The first approach is based on moment invariants. Since any ellipse can be obtained by applying an affine transform to a circle, we use the simplest affine moment invariant[58] (based on the central moments μ_{pq}) of the circle to characterize ellipses

$$I_1 = \frac{\mu_{20}\mu_{02} - \mu_{11}^2}{\mu_{00}^4}$$

while all perfect circles will produce identical values of I_1, other shapes can also produce the same value. To help discriminate shape more precisely it would be possible to also incorporate higher-order invariants. However, the disadvantage is that higher-order moments are less reliable. In contrast, I_1 only requires relatively low powers of second-order moments, and is thus more practical. The moments for the unit-radius circle are

$$m_{pq} = \int_{-1}^{1} \int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} x^p y^q dydx$$

allowing us to calculate the value of the invariant as

$$I_1 = \frac{1}{16\pi^2}.$$

The measure of ellipticity is then taken as

$$E_1 = \begin{cases} 16\pi^2 I_1 & \text{if } I_1 \le \dfrac{1}{16\pi^2} \\ \dfrac{1}{16\pi^2 I_1} & \text{otherwise} \end{cases}$$

which ranges over [0, 1], peaking at 1 for a perfect ellipse.

- Elliptic Variance (E_V)

Peura and Iivarinen[56] described an 'elliptic variance' which they used to measure ellipticity. The mean μ and covariance $\boldsymbol{C}$ of the N data points p_i are calculated. The distances of points from the mean are weighted by the covariance matrix

$$r_i = \sqrt{(p_i - \mu)^T \mathbf{C}^{-1} (p_i - \mu)}$$

and the mean distance calculated as

$$\mu_r = \frac{1}{N}\sum_i r_i.$$

Then the normalized *elliptic* variance in distance is

$$\Sigma = \frac{1}{N\mu_r^2}\sum_i (r_i - \mu_r)^2,$$

which for uniformity with the other measures we modify to

$$E_V = \frac{1}{1+\Sigma}.$$

- Euclidean Ellipticity (E_E)

A more direct and potentially more reliable approach is to robustly fit an ellipse to the region's boundaries and measure the Euclidean distances between the ellipse and the region's boundaries. Rosin[61] and Roth and Levine[62] proposed a least median of squares approaches to feature fitting. Minimal subsets (i.e. five points) of the data are selected at random and used them to generate the ellipses. The ellipse with the lowest median error over the full data set is retained as the best fit. The true point error, which is the distance from the point along the line normal to the ellipse, involves solving a quadratic equation. The orthogonal conic distance approximation method is more accurate and easy to calculate and this is used as an alternative of other Rosin's paper[60]. Using this fit the outliers are robustly detected and rejected, and the estimate is 'polished' by applying a least-square fit. If necessary the data is refitting using Fitzgibbon et al.'s ellipse-specific algorithm[59], which is guaranteed to return an ellipse. The result of the above process is to robustly and accurately determine the summed errors of the ellipse fit $E = \sum_{i=1}^{N} d_i$ where d_i's are the conic distance approximations. The normalized ellipse measure, which is scale invariant, is defined as

$$E_E = \frac{1}{1 + \frac{1}{N\sqrt{A}} E}$$

where A is the original region's area.

- DFT (E_F)

Proffitt[55] proposed an approach for measuring ellipticity and circularity based on the discrete Fourier transform (DFT). An ellipse is fitted to the shape by centering it on the region's centroid. The ellipse is then scaled such that its mean square of the lengths of the lines from the centroid to the boundary points matches the region's. Ellipticity is calculated (via the DFT) as the distance between corresponding points on the ellipse and the region.

- Moment Matching (E_M)

Voss and S¨uße proposed a method for fitting geometric primitives based on moments[63]. First, by applying an affine transformation the data is normalized into a canonical frame (which for an ellipse they take as the unit circle). Then by applying the inverse transformation to the primitive (i.e. the circle) produces the fitted ellipse. An ellipticity measure can be calculated as the root mean square of the differences between the normalized moments of the data (m'_{ij}) and the moments of the canonical primitive (m_{ij}), where only the moments not used to determine the normalization are included:

$$E_M = \frac{1}{1 + \sum_{i+j\le 4} \left(m'_{ij} - m_{ij}\right)^2}.$$

(c) Rectangularity

- MBR (R_B)

The standard approach to measuring rectangularity is to use the ratio of the area of the region to the area of its minimum bounding rectangle (MBR). The MBR of a convex polygon can be calculated in linear time by Toussaint's[64] 'rotating calliper' method. Since the convex hull of a simple polygon can be found in linear time the overall algorithm remains linear.

- Rectangular Discrepancy (R'_D)

The rectangularity measure MBR is very much sensitivity to noise and to overcome the problem Rosin[57] described an alternative in which a rectangle is fitted to the region based on its moments. Rectangularity is then measured as the normalised discrepancies between the areas of the rectangle and the region. Let R be the area difference between the rectangle and the region, D be the area difference between the region and the rectangle, and finally, B be the rectangle's area, then

$$R_D = 1 - \frac{R + D}{B}.$$

It was found that a weakness of the moment-based approach was that the orientation estimates were unreliable for compact regions. A significant improvement could be made by considering both the original orientation estimate with and without a 45° offset[57]. The maximum of the two is retained as the final rectangularity measure R'_D.

- Moment Matching (R_M)

Again Voss and S¨uße is applied fitting method[65]. The normalisation procedure is performed first by translates and rotates the data at the origin and aligned with the axes. Then the scaling is normalised to provide unit area. This leaves the rectangle's aspect ratio, which is calculated by a one-dimensional optimisation using the higher-order moments (up to fourth order). The minimum RMS error is used to calculate the rectangularity

$$R_M = \frac{1}{1 + \sum_{i+j \le 4} \left(m'_{ij} - m_{ij}\right)^2}$$

(d) Triangularity

- Moment invariants (T_1)

Flusser and Suk[58] proposed the same approach as previous in ellipses to characterise triangles by moment invariants. They have considered any triangle as a simple right-angled triangle aligned with the axes after an affine transformation. The moments are

$$m_{pq} = \int_0^x \int_0^1 x^p y^q \, dy dx,$$

which results in $I_1 = \frac{1}{108}$.

Thus the triangularity measure is

$$T_1 = \begin{cases} 108 I_1 & \text{if } I_1 \le \frac{1}{108} \\ \frac{1}{108 I_1} & \text{otherwise} \end{cases}$$

- Polygonal triangle approximation (T_A)

In polygonal triangle approximation approach[66], first we have fitted a geometric triangle model to the data, and measure the error between the model and the data. Fitting is performed by finding a polygonal approximation of the boundary. Dynamic programming is used to find the optimal three-line polygon approximation minimizing $E = \sum_i d_i$ the summed L_1 error, where d_i is the shortest Euclidean distance from $\boldsymbol{p}_i$ to the triangle[66]. The final triangularity measure is

$$T_A = \frac{1}{1 + \frac{1}{N\sqrt{A}} E},$$

where A is the area of the region.

- Minimum bounding triangle (T_B)

Another alternative approach to fitting a triangle is to use the region's minimum bounding triangle (MBT). O'Rourke[69], *et al.* describe an optimal $O(n)$ algorithm to determine the MBT. Analogous to the standard approach to measuring rectangularity, triangularity is calculated as the ratio of the area of the region to the area of its minimum bounding triangle.

- Moment Matching (T_M)

Voss and S¨uße's fitting method[65] is applied to fit a triangle, where the canonical model is an equilateral triangle with $m_{10} = m_{01} = 0$. As with their ellipse fitting all the parameters are recovered without additional optimisation and triangularity is measured as

$$T_M = \frac{1}{1 + \sum_{i+j \le 4} \left(m'_{ij} - m_{ij}\right)^2}.$$

(e) Unique Relative Measure for Shape Descriptors

Different definitions of measures are available for different object shapes detection. Even feature based approaches although potentially being based on local features requires the presence of most of the object to compute the statistic of the features. It applies to all shape descriptors presented in the special issue of pattern recognition on shape similarity by Latecki[69], *et al*. as well as to the shape descriptors presented in Belongie[70], *et al*. and Grigorescu and Petkov[71]. The subject of shape perception remains a fertile area of research. The power of such primitive shapes for scientific analysis is that they can be applied to a vast range of tasks involving not only man-made objects but also natural forms.

Nayak and Stojmenovic[72] proposed a variety of schemes that compute global shape measures, which can be categorized as techniques based on minimum bounding rectangles, other bounding primitives, fitted shape models, geometric moments, and Fourier descriptors are described. Farin[73] compared a variety of triangle shape measures using concepts such as smoothness and convexity. Stojmenovic and Nayak[74] propose several measures, all of which are based on existing linearity measures that have been adapted to measure circularity. In order to make use of these linearity measures, they transferred the Cartesian coordinates of the input set into polar coordinates. The linearity of the polar coordinate set corresponds to the circularity of the original input set given a suitable centre. They separately considered the circularity of ordered and unordered point sets. The circularity of unordered data is determined directly from the linearity measure, whereas the circularity of ordered data is derived by multiplying the unordered data circularity measure by a monotonicity factor. A shape descriptor based on the histogram matrix of pixel-level structural features is presented in Zhang and Wenyin's paper[75]. In their paper first, length ratios and angles between the centroid and contour points of a shape are calculated as two structural attributes. Then, the attributes are combined to construct a new histogram matrix in the feature space statistically. This shape descriptor can measure circularity, smoothness, and symmetry of shapes, and be used to recognize shapes. Stojmenovic and Nayak[76] proposed a method of measuring the accuracy of ellipse fits against the original point set. The evaluation of fits proceeds by their ellipticity measure which transforms the point data into polar representation where the radius is equal to the sum of distances from the point to both foci, and the polar angle is equal to the one the original point makes with the centre relative to the *x*-axis. The linearity of the polar representation will correspond to the quality of the ellipse fit for the original data. They also proposed an ellipticity measure based on the average ratio of distances to the ellipse and to its centre.

Already we have discussed many shape descriptors on continuous approach. There is no single definition is available in the literature which can be used for finding the shape of an object. As for example, for finding rectangular or circular or elliptical shape object one has to define different measure for different shape. A new definition of measure for different geometrical shapes detection is presented by Chaudhuri[6], *et al*. The definition is a unique (single) measure by which different shape of objects can be identify on the basis of their degree of fitness parameter. In our paper[6], first, we have fitted a polygon/curve optimally on the object then compute the degree of fitness, which is a ratio of matching area and the non-matching area due to object and polygon/curve both. Next, we are identified the object of a particular geometrical shape (rectangle/circle/ellipse) on the basis of minimal value of degree of fitness for different fitted polygon/curve.

- Best fitting Methods of 2-D Parametric Polygons/curves

Computing geometrical features is an important intermediate level vision task that has many applications. This step serves as a gateway to high-level matching and understanding of elements in the image. Among many other 2-D parametric polygons/curves fitting of rectangle/ square, circle, and ellipse plays important roles in real life applications. In general, approaches for locating parametric polygons/curves can be divided into two steps[7,8,77]. The first step is to detect the boundary of the object. The second step is to estimate its parameters based on the boundary points[2, 7,8,77]. The parameters estimation procedures are discussed briefly.

- Best-fitted Rectangle

Chaudhuri[2,7], *et al*. proposed for computing the best-fitted rectangle for closed regions. The coordinates of the vertices of the best-fitted rectangle are computed using a bisection method of the upper estimated rectangle (UPER) vertices and the under estimated rectangle (UNER) vertices based on difference area minimization between object and best-fitted rectangle areas. The coordinates of the vertices of UPER and UNER are computed directly using closed-form solutions based on the border points of the object. The approaches for UPER and UNER are based on simple coordinate geometry and least square fitting approach. Using a least square approach the directions of major and minor axes of the object, which gives the orientation of the object are extracted. The four vertices of UPER are computed by pair-wise solving the four straight lines[7]. Also the four vertices of UNER are computed by pair-wise solving the four straight lines, which are formed by least square fitting approach[2]. Finally, the four vertices of the best-fitted rectangle are computed by bisection method of the UPER and UNER vertices in a iterative way based on the constraint of area unchanged of the fitted rectangles between the last and previous iterations, which is same as the difference between the area of the object and the area of the best-fitted rectangle is minimum[2].

- Elliptic and Circular fit

Fitting circles and ellipses of an object is a problem that arises in many application areas. Ellipse or circle computation[8] begins with finding the boundary points[7,8,77] for each blob. The border points of a perfect elliptical or circular object will satisfy the equation of the ellipse or circle and in such situation the error due to fitting will be zero. But if the border point of an object does not lie on the fitted ellipse or circle then it will generate an error. Here, errors function for all border points of the object is defined and then estimate the other parameters of ellipse or circle by minimizing the error[8].

- Unique Relative Measure

There are five steps in this approach. First fit a polygon/curve (rectangle/circle/ellipse) on the object. Fig. 2(a) shows an image (object) and its fitted circle is shown in Fig. 2(b). Next step is to find the matching area. Fig. 2(c) shows the matching area (dark black region) of the object and the fitted circle. Then find the non-matching area due to fitted circle and it is shown in Fig. 2(d) (dark regions inside the circle upper and lower portions). Next, find the non-matching area due to object and shown in Fig. 2(e) (little bit bright gray regions inside the object left and right sides). The total non-matching area is the sum of both non-matching areas due to object and curve. Then

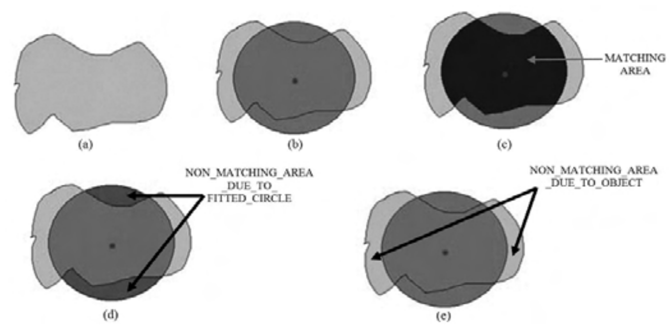

Figure 2. Example of an object (a) Object (b) Fitted circle (c) Match area of the object and fitted circle (d) Non-match area with respect to circle (e) Non-match area with respect to object.

compute the degree of fitness as the ratio of non-matching area and matching area. Mathematical formulation of the proposed definition is as follows.

Let A be a region and B be the fitted polygon/curve (rectangle/circle/ellipse). Without loss of generality we assume that the area of a region means the number of points of the object. Similarly, the area of the polygon/curve (rectangle/circle/ellipse) is the number of points inside the polygon/curve including the boundary points. The matching region of A and B is $A \cap B$. The non-matching region due to object is $A-(A \cap B)$ and the non-matching region due to fitted curve is $B-(A \cap B)$. Let $\# A$ be the area of region A and let M_A^B is called the measure of degree of fitness of object A with respect to fitted polygon/curve B. So the measure of degree of fitness M_A^B is defined as

$$M_A^B = \frac{\text{Total non-matching area}}{\text{Matching area}} = \frac{\#[B-(A \cap B)] + \#[A-(A \cap B)]}{\#(A \cap B)}$$

"+" means union, so from the set theory $[B-(A \cap B)] \cup [A-(A \cap B)] = (A \cup B)-(A \cap B)$, so M_A^B can be written as $M_A^B = \frac{\#[(A \cup B)-(A \cap B)]}{\#(A \cap B)}$

The above measure of degree of fitness M_A^B depends on fitted polygon/curve (rectangle/circle/ellipse). Since fitted polygons/curves are invariant under translation, rotation and scaling, so the measure of fitness is also invariant under translation, rotation and scaling. Also the proposed measure of fitness is dimensionless and always finite. Perfectly fitted curve of an object has zero value for degree of fitness. The image with 8 different shape objects is shown in Fig. 3(a). The degree of fitness of different shape objects of Fig. 3(a) with respect to rectangle, circle and ellipse are shown in Table 1. It is reflected from Table 1 that the objects 2 and 4 are detected as rectangle and since all sides of the fitted rectangle are equal for object 2, so object 2 is a square. The objects 4, 7 and 8 are detected as ellipse and object 5 is detected as circle. Since the values of degree of fitness for objects 1 and 3 with respect to all curves (rectangle, circle and ellipse) are higher and greater than tolerance level ($T < 0.3$), so they are not detected as any of the curves, though the minimum values of the degree of fitness for these objects are for ellipticity. Chaudhury[2,6-8], *et al.* used this unique relative measure for building extraction and oil tank detection from remote sensing images.

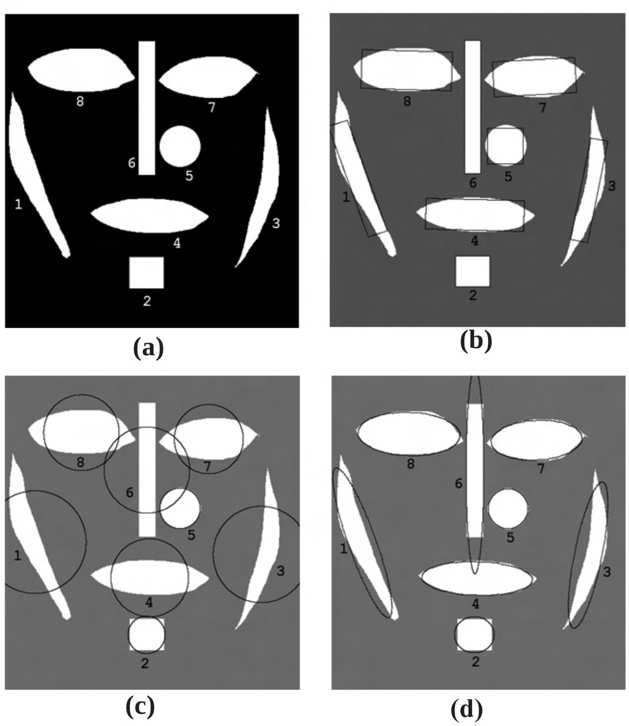

Figure 3. (a) Different shapes binary image (b) Fitted rectangle (c) Fitted circle and (d) Fitted ellipse

Table 1. Degree of fitness of different objects in Fig. 3(a)

Objects of Fig. 3 (a)	Degree of Fitness		
	Rectangularity	Circularity	Ellipticity
Object 1	1.378815	3.835594	0.615690
Object 2	0.000000	0.207373	0.219076
Object 3	1.288053	3.766476	0.897333
Object 4	0.366713	1.229934	0.049481
Object 5	0.274922	0.0318950	0.032764
Object 6	0.000000	3.661320	0.677681
Object 7	0.442777	0.696997	0.074589
Object 8	0.338803	0.747632	0.064988

(f) Discussion

Global contour shape techniques take the whole shape contour as the shape representation. The matching between shapes can either be in space domain or in feature domain. For shape description, the techniques are accurate and efficient. On the one hand, shape should be described as accurately as possible and a shape description should be as compact as possible to simplify indexing and recovery. Efficient offline feature extraction is also desirable. Simple global shape descriptors are compact; however, they are very inaccurate shape descriptors. They need to be combined with other shape descriptors to create practical shape descriptors. Fourier descriptor is simple to implement, and involves less computation by either using fast Fourier transform (FFT) or using truncated Fourier transform computation. The resulting descriptor is also compact and the matching is very simple. Fourier descriptor (FD) is simpler to compute and more robust compared to curvature scale space. Boundary moment descriptor is similar to Fourier descriptor, and is easy to acquire. However, unlike Fourier descriptor, only the few lower order moment descriptors have physical interpretation.

2.1.2 Region-based Shape Representation and Description Techniques

In contour base methods only use boundary information; but in region based techniques, all the pixels within a shape region are taken into account to obtain the shape representation. Common region based methods use moment descriptors to describe shapes. Other region based methods include grid method, shape matrix, convex hull and media axis. Similar to contour based methods, region based shape methods can also be divided into global and structural methods, depending on whether they separate shapes into sub parts or not.

2.1.2.1 Global Methods

The concept of moment in mathematics evolved from the concept of moment in physics. It is an integrated theory system. For both contour and region of a shape, one can use moment's theory to analyze the object.

(a) Region Moments

Among the region-based descriptors, moments are very popular. These include invariant moments, Zernike moments, Radial Chebyshev moments, etc. The general form of a moment function m_{pq} of order $(p + q)$ of a shape region can be given as:

$$m_{pq} = \sum_x \sum_y \psi_{pq} f(x, y),\ p, q = 0, 1, 2, ...$$

where ψ_{pq} is known as the moment weighting kernel or the basis set; $f(x, y)$ is the shape region.

(b) Geometric moment invariants

Hu published the first significant paper on the use of image moment invariants for two-dimensional pattern recognition applications[33]. Geometric moments, are the simplest of the moment functions with basis $\psi_{pq} = x^p y^q$ while complete, is not orthogonal. His approach is based on the theory of algebraic forms:

$$m_{pq} = \sum_x \sum_y x^p y^q f(x, y),\ p, q = 0, 1, 2, ...$$

The geometric central moments, which are invariant to translation, are defined as:

$$\psi_{pq} = \sum_x \sum_x (x - \bar{x})^q (y - \bar{y})^q f(x, y),\ p, q = 0, 1, 2, ...$$

where $\bar{x} = \frac{m_{10}}{m_{00}}$ and $\bar{y} = \frac{m_{01}}{m_{00}}$.

A set of 7 Geometric moments are given by Hu[37], which are useful shape recognition.

Geometrical moments are computationally simple. Moreover, they are invariant to rotation, scaling and translation. Since the values of the acquired moment invariants are usually very small, a normalization process, such as zscore normalization[38], is needed in the implementation. The main problem with geometric moments is that only a few invariants derived from lower order moments is not sufficient to accurately describe shape. Higher order invariants are difficult to derive. However, they have several drawbacks[78]:

- Information redundancy: since the basis is not orthogonal, these moments suffer from a high degree of information redundancy.
- Noise sensitivity: higher-order moments are very sensitive to noise.
- Large variation in the dynamic range of values: since the basis involves powers of p and q, the moments computed have large variation in the dynamic range of values for different orders. This may cause numerical instability when the image size is large.

(c) Algebraic Moment Invariants

The algebraic moment invariants are computed from the first m central moments and are given as the eigen values of predefined matrices, $M_{[j,k]}$, whose elements are scaled factors of the central moments[42]. The algebraic moment invariants can be constructed up to arbitrary order and are invariant to affine transformations. However, algebraic moment invariants performed either very well or very poorly on the objects with different configuration of outlines.

(d) Zernike Moments *(ZM)*

Zernike Moments (ZM) are orthogonal moments[78]. The complex Zernike moments are derived from orthogonal Zernike polynomials:

$$V_{mn}(x, y) = V_{nm}(r\cos\theta, \sin\theta) = R_{nm}(r)\exp(jm\theta)$$

where $R_{nm}(r)$ is the orthogonal radial polynomial:

$$R_{nm}(r) = \sum_{s=0}^{(n-|m|)/2} (-1)^s \frac{(n-s)!}{s!\left(\frac{n-2s+|m|}{2}\right)!\left(\frac{n-2s-|m|}{2}\right)!} r^{n-2s}$$

$n = 0,1,2,...$, $0 \le |m| \le n$ and $n - |m|$ is even.

Zernike polynomials are a complete set of complex valued functions orthogonal over the unit disk, i.e. $x^2 + y^2 \le 1$. The Zernike moment of order n with repetition m of shape region $f(x, y)$ is given by:

$$Z_{nm} = \frac{n+1}{\pi} \sum_r \sum_\theta f(r\cos\theta, r\sin\theta).R_{nm}(r).\exp(jm\theta),\ r \le 1$$

Zernike moments (ZM) have the following advantages[79]:

- Rotation invariance: the magnitudes of Zernike moments are invariant to rotation.
- Robustness: they are robust to noise and minor variations in shape.
- Expressiveness: since the basis is orthogonal, they have minimum information redundancy.

However, the computation of ZM (in general, continuous orthogonal moments) poses several problems:

- Coordinate space normalization: the image coordinate space must be transformed to the domain where the orthogonal polynomial is defined (unit circle for the Zernike polynomial).
- Numerical approximation of continuous integrals: the continuous integrals must be approximated by discrete summations. This approximation not only leads to numerical errors in the computed moments, but also severely affects the analytical properties such as rotational invariance and orthogonality.
- Computational complexity: computational complexity of the radial Zernike polynomial increases as the order becomes large.

(e) Radial Chebyshev Moments *(RCM)*

The radial Chebyshev moment of order p and repetition q is defined as[80]:

$$S_{pq} = \frac{1}{2\pi\rho(p, m)} \sum_{r=0}^{m-1} \sum_{\theta=0}^{2\pi} t_p(r).\exp(-jq\theta).f(r, \theta)$$

where $t_p(r)$ is the scaled orthogonal Chebyshev polynomials for an image of size $N \times N$:

$$t_0(x) = 1,\ t_1(x) = \frac{(2x - N + 1)}{N},\ldots,$$

$$t_p(x) = \frac{(2p-1)t_1(x)t_{p-1}(x) - (p-1)\left\{1 - \frac{(p-1)^2}{N^2}\right\}t_{p-2}(x)}{p}; \quad p > 1$$

$\rho(p, N)$ is the squared-norm:

$$\rho(p,m) = \frac{N\left(1-\frac{1}{N^2}\right)\left(1-\frac{2^2}{N^2}\right)\cdots\left(1-\frac{p^2}{N^2}\right)}{2p+1}; \quad p = 0,1,\ldots,N-1$$

and $m = (N/2)+1$

The mapping between (r, θ) and image coordinates (x, y) is given by:

$$x = \frac{rN}{2(m-1)}\cos\theta + \frac{N}{2}, \; y = \frac{rN}{2(m-1)}\sin\theta + \frac{N}{2}$$

Compared to Chebyshev moments, radial Chebyshev moments possess rotational invariance property.

(f) Generic Fourier Descriptor

Although Zernike moment descriptor has a robust performance but it has several drawbacks:

- The kernel of Zernike moments is complex to compute and before computing the moment features the shape has to be normalized into a unit.
- The radial features and circular features captured by Zernike moments are not consistent, one is in spatial domain and the other is in spectral domain. It does not allow multi-resolution analysis of a shape in radial direction.
- The circular spectral features are not captured evenly at each order, this can result in loss of significant features which are useful for shape description.

To overcome these drawbacks, a generic Fourier descriptor (GFD) has been proposed by Zhang and Lu[81]. The GFD is acquired by applying a 2-D Fourier transform on a polar-raster sampled shape image:

$$PF_2(\rho,\varphi) = \sum_r \sum_i f(r,\theta_i)\exp\left[j2\pi\left(\frac{r}{R}\rho + \frac{2\pi i}{T}\varphi\right)\right]$$

where $0 \le r < R$ and $\theta_i = i(2\pi/T)$, $(0 \le i < T)$; $0 \le \rho < R,\; 0 \le \varphi < T$. R and T are the radial frequency resolution and angular frequency resolution respectively. The normalized coefficients are the GFD. The similarity between two shapes is measured by the city block distance between their GFDs.

GFD is simpler to compute then Zernike moments computation. Also the features are pure spectral features and have better retrieval performance due to multi-resolution analysis in both radial and circular directions of the shape. With an enhanced process, GFD can achieve retrieval performance on perspectively transformed shapes as high as it achieves on similarity transformed shapes[82]. Zhang and Lu have also shown that GFD outperforms contour shape descriptors such as CSS, FD and region-based shape descriptors such as Zernike moments, geometric moments and grid method[83].

(g) Discussion

Global region based methods treat the shape region as a whole and make effective use of all the pixel information within the region. These methods measure pixel distribution of the shape region, which are less likely affected by noise and variations. As a result, they usually can cope well with shape of significant defection, which poses a problem for contour-based methods. Particularly popular region methods are moment methods. Moment methods extract statistical distribution of region pixels. The lower order moments or moment invariants carry physical meanings associated with region pixel distribution. However, because of their global nature, the disadvantage of moment-based methods is that it is difficult to correlate high order moments with a shape's salient features. Besides the previous moments, there are other moments for shape representation, for example, homocentric polar-radius moment[84], orthogonal Fourier-Mellin moments (OFMMs)[85], pseudo-Zernike Moments[86], etc. The study shows that the moment-based shape descriptors are usually concise, robust and easy to compute. It is also invariant to scaling, rotation and translation of the object. Present paper discusses about the global approach. One can go through Zhang and Lu[12] for structural approach.

3. CLASSIFICATION OF MATCHING PROBLEMS AND SIMILARITY MEASURE PROPERTIES

This section discusses about shape matching problems and properties. Shape matching is studied in various forms. Given two patterns and a dissimilarity measure, it can define the problem as follows:

- Computation problem: compute the dissimilarity between the two patterns.
- Decision problem: for a given threshold, decide whether the dissimilarity is smaller than the threshold. Also for a given threshold, decide whether there exists a transformation such that the dissimilarity between the transformed pattern and the other pattern is smaller than the threshold.
- Optimization problem: find the transformation that minimizes the dissimilarity between the transformed pattern and the other pattern.

Sometimes the time complexities to solve these problems are rather high, so that it makes sense to devise approximation algorithms:

- Approximate optimization problem: find a transformation that gives dissimilarity between the two patterns that is within a constant multiplicative factor from the minimum dissimilarity.

These problems play an important role in the following categories of applications.

Shape retrieval: search for all shapes in a typically large database of shapes that are similar to a query shape. Usually all shapes within a given distance from the query are determined (decision problem), or the first few shapes that have the smallest distance (computation problem). If the database is large, it may be infeasible to compute the similarity between the query and every database shape. An indexing structure can help to exclude large parts of the database from consideration at an early stage of the search, often using some form of triangle inequality property.

Shape recognition and classification: determine whether a given shape matches a model sufficiently close (decision problem), or which of k class representatives is most similar (k computation problems).

Shape alignment and registration: transform one shape so that it best matches another shape (optimization problem), in whole or in part.

Shape approximation and simplification: construct a shape of fewer elements (points, segments, triangles, etc.), that is still similar to the original. There are many heuristics for approximating polygonal curves[66] and polyhedral surfaces[87]. Optimal methods construct an approximation with the fewest elements given a maximal dissimilarity, or with the smallest dissimilarity given the maximal number of elements. (Checking the former dissimilarity is a decision problem, the latter is a computation problem.)

- Properties

It can be desirable that a similarity measures has such properties. Whether or not specific properties are wanted will depend on the application at hand, sometimes a property will be useful, sometimes it will be undesirable. Some combinations of properties are contradictory, so that no distance function can be found satisfying them. A shape dissimilarity measure, or distance function, on a collection of shapes S is a function d: $S \times S \to R$. In the properties listed below, it is understood that they must hold for all shapes A, B, or C in S.

- Metric Properties

(i) Non-negativity: $d(A,B) \geq 0 \ \forall A,B \in S$
(ii) Identity: $d(A,A) = 0 \ \forall A \in S$
(iii) Uniqueness: $d(A,B) = 0$ implies $A = B$
(iv) Symmetry: $d(A,B) = d(B,A) \ \forall A,B \in S$ and
(v) Triangular inequality:
$d(A,B) + d(A,C) \geq d(B,C) \ \forall A,B,C \in S$.

A distance function satisfying identity, uniqueness, and triangular inequality is called a metric. If a function satisfies only identity and triangular inequality, then it is called a semi-metric. Symmetry follows from the triangular inequality and identity and symmetry is not always wanted. Indeed, human perception does not always find that shape A is equally similar to B, as B is to A. In particular, a variant A of prototype B is often found more similar to B than vice versa[88].

- Continuity Properties

It is often desirable that a similarity function has some continuity properties. The following four properties are about robustness, a form of continuity. Such properties are for example useful to be robust against the effects of discretization.

Perturbation robustness: for each $\varepsilon > 0$ there is an open set F of deformations sufficiently close to the identity, such that $d(f(A),A) < \varepsilon \ \forall f \in F$. For example, it can be desirable that a distance function is robust against small affine distortion.

Crack robustness: for each ε>0, and each 'crack' x in bd(A), the boundary of A, an open neighbourhood U of x exists such that for all B, $A - U = B - U$ implies $d(A,B) < \varepsilon$.

Blur robustness for each $\varepsilon > 0$, an open neighbourhood U of $bd(A)$ exists, such that $d(A,B) < \varepsilon$ for all B satisfying $B - U = A - U$ and $bd(A) \subseteq bd(B)$.

Noise and occlusion robustness: for each $x \in \Re^2 - A$, and each $\varepsilon > 0$, an open neighbourhood U of x exists such that for all B, $B - U = A - U$ implies $d(A,B) < \varepsilon$.

- Invariance

A distance function d is invariant under a chosen group of transformations G if for all $g \in G$, $d(g(A), g(B)) = d(A,B)$. For object recognition, it is often desirable that the similarity measure is invariant under affine transformations, since this is a good approximation of weak perspective projections of points lying in or close to a plane. However, it depends on the application whether a large invariance group is wanted. For example, Thompson[89] showed that the outlines of two hatchet-fishes of different genus, Argyropelecus olfersi and Sternoptyx diaphana, can be transformed into each other by shear and scaling, see Fig. 4. So, the two fishes will be found to match the same model if the matching is invariant under affine transformations.

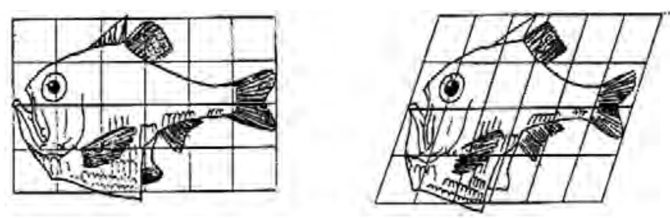

Figure 4. Two hatchet-fishes of different genus: Argyropelecus olfersi and Sternoptyx diaphana. From Thompson[88] paper.

4. SIMILARITY MEASURES

Measure of similarity is an important concept for shape analysis problem. On this issue distance is a very important measure used widely in applied science problems such as pattern recognition and image processing. It is desirable that the distance is a metric. In this section it discusses various distances, which are used in shape analysis problem.

4.1 Discrete Metric

Finding an affine invariant metric for patterns is not so difficult. Indeed, a metric that is invariant not only for affine transformations, but for general homeomorphisms is the discrete metric:

$$d(A,B) = \begin{cases} 0 \text{ if } A = B \\ 1 \text{ otherwise} \end{cases}$$

However, this metric lacks useful properties. For example, if a pattern A is only slightly distorted to form a pattern A', the discrete distance $d(A,A')$ is already maximal.

Under the discrete metric, computing the smallest $d(A,B)$ over all transformations in a transformation group G is equivalent to deciding whether there is some transformation g in G such that $g(A)$ equals B. This is known as exact congruence matching. For sets of n points in $\Re^k$, the algorithms with the best known time complexity run in $O(n \log n)$ time if G is translations, scaling, or homotheties (translation plus scaling), and $O\left(n^{\lceil k/3 \rceil} \log n\right)$ time for rotations, isometries, and similarities[90].

4.2 L_p Distance, Minkowski Distance

Many similarity measures on shapes are based on the L_p distance between two points. Three special cases of the L_p (Minkowski metric), namely City-block distance (d_C), Euclidean distance (d_E) and Chessboard distance (d_M) are popular. For two n dimensional points $\mathbf{X} = (x_1, x_2, ..., x_n)$ and $\mathbf{Y} = (y_1, y_2, ..., y_n)$, the L_P metric is defined as

$$d_p(\mathbf{X}, \mathbf{Y}) = \left\{ \sum_{i=1}^{n} |x_i - y_i|^p \right\}^{1/p}$$

where d_C, d_E and d_M correspond to $p = 1,2$ and ∞, respectively.

4.3 CMC Distance (d_{CMC})[91]

Since the conventional data space is Euclidean, it is natural to use Euclidean distance in such a space. But computation of Euclidean distance is expensive in a high dimensional space, especially when such computation is to be performed on a large amount of data. An example is the processing of multi-spectral/hyper-spectral imagery which contains more than a million pixels per image frame and many such frames are to be processed. In statistical pattern recognition methods also, the distance is to be computed iteratively on a large amount of data. In order to reduce the computation, City-block and Chessboard distances are often used in image processing and related problems. While these distances are computationally more efficient, they derivate markedly from the Euclidean frame work and cause the accuracy of the final result to suffer, because City-block distance is upper estimate and Chessboard distance is under estimate with respect to Euclidean distance.

Chaudhuri, Murthy and Chaudhuri (CMC) metric[91], which is close to Euclidean distance and yet requires significantly less computational effort. Given two n-dimensional points, $X = (x_1, x_2, ..., x_n)$ and $Y = (y_1, y_2, ..., y_n)$, let $|x_i - y_i|$ be the maximum for $i = i_0$; the CMC distance (d_{CMC}) is defined as

$$d_{CMC}(X, Y) = \left|X_{i_0} - Y_{i_0}\right| + \frac{1}{n - \left\lfloor \frac{n-2}{2} \right\rfloor} \sum_{\substack{i=1 \\ i \neq i_0}}^{n} |x_i - y_i|$$

where $\lfloor a \rfloor$ is the floor of "a", i.e. the largest integer $\leq a$.

Rhodes proved in his paper[98], it is natural to expect a suitable linear combination of d_C and d_M to give an approximation to d_E better than either of them. Note that Rosenfeld and Pfaltz[99] obtained a 2-dimensional approximation by combining d_C and d_M nonlinearly as $D(x) = \max\left\{(d_C(x) + 1)/3, d_M(x)\right\}$.

4.4 Hausdorff Distance

In many applications, for example stereo matching, not all points from A need to have a corresponding point in B, due to occlusion and noise. Typically, the two point sets are of different size, so that no one-to-one correspondence exists between all points. In that case, a dissimilarity measure that is often used is the Hausdorff distance. The Hausdorff distance is defined not only for finite point sets, it is defined on nonempty closed bounded subsets of any metric space. Hausdorff distance is a classical correspondence-based shape matching method, it has often been used to locate objects in an image and measure similarity between shapes[92-97]. Given two shapes represented by two set of points: $A = (a_1, a_2, ..., a_n)$ and $B = (b_1, b_2, ..., b_n)$ the Hausdorff distance between two sets A and B is defined as

$$H(A, B) = \max\left\{h(A, B), h(B, A)\right\};$$

where $h(A, B) = \max_{a \in A} \min_{b \in B} \|a - b\|$ and $\|.\|$ is the underlying norm on the points of A and B, usually Euclidean distance. However, this distance measure is too sensitive to noise or outlier. A single point in A that is far from anything in B will cause *h(A,B)* to be large. Therefore, a modified Hausdorff distance is introduced by Rucklidge[96]:

$$h^f(A, B) = f^{th}_{a \in A} \min_{b \in B} \|a - b\| \text{ where } f^{th}_{x \in X} g(x) \text{ denotes } f^{th}$$

quantile value of $g(x)$ over set X, for some value of f between 0 and 1. For example, the f^{th} quantile value is the maximum and the $1/2^{th}$ quantile value is the median. In practice[92], f is usually set to be ½. The advantage of shape matching using Hausdorff distance is that shape can be matched partially. However, the Hausdorff distance is not translation, scale and rotation invariant. In order to match a model shape with a shape in the image, the model shape has to be overlapped on the image in different positions, different orientations and different scales. As the result, the matching is prohibitively expensive.

4.5 Bottleneck Distance

Let A and B be two point sets of size n, and $d(a,b)$ a distance between two points. The bottleneck distance $F(A,B)$ is the minimum over all 1-1 correspondences f between A and B of the maximum distance $d(a, f(a))$. For the distance $d(a,b)$ between two points, an L_p distance could be chosen. An alternative is to compute an approximation $\overline{F}$ to the real bottleneck distance F. An approximate matching between A and B with $\overline{F}$ the furthest matched pair, such that $F < \overline{F} < (1 + \varepsilon)F$, can be computed with a less complex algorithm[100].

The decision problem for translations, deciding whether there exists a translation ℓ such that $F(A + \ell, B) < \varepsilon$ can also be solved, but takes considerably more time[100]. Because of the high degree in the computational complexity, it is interesting to look at approximations with a factor ε: $F(A + \ell, B) < (1 + \varepsilon)F(A + \ell^*, T)$, where ℓ^* is the optimal translation. Variations on the bottleneck distance are the minimum weight distance, the most uniform distance, and the minimum deviation distance.

4.6 Area of Symmetric Difference, Template Metric

For two compact sets A and B, the area of symmetric difference, also called template metric, is defined as $area((A - B) \cup (B - A))$. Unlike the area of overlap, this measure is a metric. Translating convex polygons so that their centroids coincide also gives an approximate solution for the symmetric difference, which is at most 11/3 of the optimal solution under translations[3]. This also holds for a set of transformations F other than translations, if the following holds: the centroid of A, $c(A)$, is equivariant under the transformations, i.e. $c(f(A)) = f(c(A))$ for all f in F, and F is closed under composition with translation.

5. CONCLUSION

Extracting a shape feature in accordance with human perception is not an easy task. Due to the fact that human vision and perception are an extraordinary complicated system, it is a utopia to hope that the machine vision has super excellent performance with small complexity. In addition, choosing appropriate features for a shape recognition system must consider what kinds of features are suitable for the task. There exists no general feature which would work best for every kind of images.

The last few decades have resulted in an enormous amount of work related to shape analysis. The course of development has been influenced by achievements from other related research disciplines as well as by image analysis applications. A selection of the most characteristic methods has been discussed briefly. The main goal of this review was to attempt to cover the diversity of methods for shape description and provide a set of references that the reader can use for further research.

In this paper, existing shape representation and description techniques have been reviewed. Generally, there are two classes of approaches in shape representation and description: contour-based versus region-based. Under each class, the methods can be divided into structural and global methods. The different methods can be further distinguished between methods working in space domain and methods working in transform domain. We have discussed several approaches of global contour and region based shape description methods. Contour-based approaches are more popular than region-based approaches in literature. This is because human beings are thought to discriminate shapes mainly by their contour features. Another reason is because in many of the shape applications, the shape contour is the only interest, whilst the shape interior content is not important. However, there are several limitations with contour-based methods. First, contour shape descriptors are generally sensitive to noise and variations because they only use a small part of shape information, that is, contour information. Second, in many cases, the shape contour is not available. Third, in some applications, shape content is more important than the contour features. These limitations can be overcome by using region-based methods. The findings in the survey are in favour of region-based methods. Region-base methods are more robust as they use all the shape information available; they can be applied to general applications; and they generally provide more accurate retrieval. In addition, region-based methods can cope well with shape defection, which is a common problem for contour-based shape representation techniques. Although region-based methods make use of all the shape information, it is not necessarily more complex than contour-based methods, as some promising methods such as the moment methods and GFD are simple to implement.

The article has also discussed a number of shape similarity properties. It is a challenging research task to construct similarity measure with a chosen set of properties. The difficulty is that the distance measure must be suitable for partial matching. The dissimilarity must be small when two shapes contain similar regions, and the measure should not penalize for regions that do not match. Also, the number of local minima of the distance can be large.

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