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REVIEW PAPER

Advances in Shannon Sampling Theory

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ABSTRACT

In this paper, the authors have provided a brief review of the recent advances in the Shannon sampling theory. In particular, they discuss the work related to 1-D signal reconstruction involving the samples taken below the Nyquist rate using nonlinear/time-variant systems. The extensions of the sampling theorems to the fractional Fourier and Linear canonical transform domains are also discussed.

Keywords: Fourier transform, fractional Fourier transform, linear canonical transform, sampling theorem, signal reconstruction, Nyquist rate

1. INTRODUCTION

Sampling expansions usually deal with the issue of perfect signal reconstruction from the samples taken in time domain using some kind of interpolation or filtering technique provided that the sampling rate satisfies some well defined criteria[1-12]. Intimately connected with the sampling rate required for perfect reconstruction of signals is the dimensionality and support/bandwidth of the signal in the frequency domain[1-12]. The sampling waveform used for sampling operation, which is generally assumed to be an ideal impulse-train or ideal pulse train for natural sampling[2], also matters most. It must, however, be kept in mind that the continuous-time ideal impulse function, also known as the Dirac delta function, is an idealization and it is not possible to generate it in real life by any physical circuit or system[1-12]. The nonuniform sampling of signals and sampling of periodic or random signals are other important issues which have been widely investigated in the literature[1,5].

Most of the sampling theorems deal with signals which are band limited or band pass in the conventional Fourier domain (CFD). The most widely-known Shannon sampling theorem deals with the issue of signal reconstruction using the samples of a band limited signal $f(t)$ satisfying the Nyquist condition[1-9]. Further, it is well known that a signal cannot be recovered from its samples by performing any linear time-invariant (LTI) filtering operation if it is under sampled. However, perfect signal reconstruction through nonlinear or time-variant filtering is not ruled out either. It may be mentioned here that the real life signals are time-limited, and hence, in theory, cannot be band limited/band pass in the CFD[1-5]. It is also known that the sampling of band pass signals is more involved than the sampling of band limited signals[2,10].

Therefore the challenging issues involved in the sampling theory are:

- Reduction in sampling rate below the Nyquist rate
- Signal reconstruction using nonlinear or time variant systems
- Signal reconstruction using samples in domains other than the usual time or frequency domains.

Recently time and frequency domains have been generalized to a continuum of domains called as fractional Fourier transform (FRFT) and linear canonical transform (LCT) domains[13-16]. The FRFT is characterized by the angle parameter α and the FRFT domain with angle α may be seen as the axis at an angle α from the time domain axis[6-8]. These FRFT domains are shown in Fig. 1 for the ease of discussion reproduced from[13].

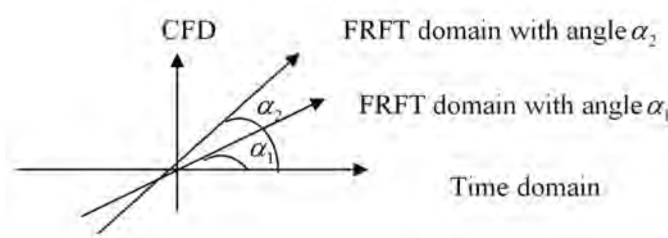

Figure 1. The FRFT domains.

From the Fig.1, it is natural to investigate the signal reconstruction issues in the FRFT/LCT domains other than the time or CFD. In particular, having a signal reconstruction formula in one FRFT/LCT domain using the samples in the same or other domain would be desirable. Similarly, signal reconstruction in some FRFT/LCT domain using samples taken in multiple domains will also crop up in mind. Of course the challenging issues discussed earlier continue to attract the attention of the researchers in the FRFT/LCT domains also. Moreover, it has been shown that the FRFT/LCT domains

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are specially suited for processing of chirp signals commonly encountered in radars/sonars[6-8], as chirp signals can be band limited in some FRFT/LCT domains other than the CFD. It is worth mentioning here that the area of nonuniform sampling is also equally connected here and much remains to be explored here in the FRFT/LCT domains. The sampling of 2-D and higher dimensional signals also need to be investigated in the LCT domains although several 1-D sampling theorems have been derived in the literature involving the FRFT and LCT domains and can be seen in[13-28] and the references therein.

In this paper, authors have provided a brief discussion of the recent advances in the Shannon sampling theory related to 1-D signal reconstruction involving the samples taken below the Nyquist rate using nonlinear/time-variant systems. The extensions of the sampling theorems to the FRFT and LCT domains are also discussed.

2. SAMPLING SCHEMES

The classical Shannon sampling theorem for signals band limited in the CFD gives us the minimum sampling rate (often called the Nyquist rate) required to reconstruct the signal uniquely from its samples[1-12].

If the sampling rate is below the Nyquist rate (twice the maximum frequency of the signal) or the signal is not band limited in the CFD, aliasing takes place in the CFD and it is not possible to recover the original signal from the under sampled signal by performing the filtering with an ideal low-pass or using any other linear time-invariant system[1-12]. It provides us the following reconstruction formula for a low-pass filter signal *f(t)* band limited to a frequency *B* Hz as[1-12]

$$f(t)=\sum_{n=-\infty}^{\infty} f(nT)\frac{\sin\left[2\pi B(t-nT)\right]}{2\pi B(t-nT)}$$

where *T* is the sampling interval. Sampling at half the Nyquist rate is possible if we sample the analytical signal corresponding to a signal as discussed in[5]. It may however, be noted that the analytical signal, which is obtained using the Hilbert transform of the signal, becomes a complex signal even if the original signal happens to be a real signal[5].

Recently, an interesting and innovative sampling scheme for a signal using an ideal impulse-train is proposed by Sharma[26], *et al.* The scheme allows signal reconstruction using samples taken at a rate which is less than the Nyquist rate. In fact this reduction in sampling rate is possible because of the use of the linear time-variant system in place of the LTI system in the conventional sampling scheme. The following paragraph is reproduced from[26] for the ease of reference. This scheme decomposes a low-pass signal *f(t)* band limited to a frequency *B* Hz as[26]

$$f(t)=f_1(t)\,f_2(t)\,, \tag{1}$$

where the signals $f_1(t)$ and $f_2(t)$ are assumed to be band limited to B_1 and B_2 Hz respectively. It may be obtained from the theory of conventional Fourier transform that[2,4]

$$B=B_1+B_2\,. \tag{2}$$

The signal $f_1(t)$ can be expressed as[26]

$$f_1(t)=f(t)\,/\,f_2(t)\,. \tag{3}$$

The signal $f_2(t)$ in Eqn (1) can be taken as any known but otherwise arbitrary signal band limited to B_2 Hz. As the signal $f_1(t)$ is band limited to B_2 Hz, it is possible to reconstruct the signal $f_1(t)$ using samples obtained using the ideal sampling waveform shown in Fig. 2, at the Nyquist rate of $2B_1$ samples/second only[2].

This requires the samples of *f(t)* and $f_2(t)$ at a rate of $2B_1$ samples/second only. Once the signal $f_1(t)$ is constructed using the ideal sinc interpolation[4] technique (By passing the samples of $f_1(t)$ through an ideal low pass filter with cut-off frequency B_1 Hz) as given by[4]

$$f_1(t)=\sum_{n=-\infty}^{\infty}\left(f(nT)\,/\,f_2(nT)\right)\frac{\sin\left[2\pi B_1(t-nT)\right]}{2\pi B_1(t-nT)}\,, \tag{4}$$

we can obtain the signal *f(t)* using Eqn. (1). The whole scheme is reproduced here from[26] and is shown in Fig. 3.

It must also be mentioned that the samples must satisfy the condition $f_2(nT)\neq 0$, a condition which is not difficult to be satisfied in practice[26]. One may further consider the problem of reconstruction of $f_1(t)$ in terms of decomposition similar to (1) at lower rate. So the whole process can be written as[26]

$$f(t)=\left[\left\{\left(f_1(t)\,f_2(t)\right)f_3(t)\right\}\ldots f_N(t)\right], \tag{5}$$

where signals $f_1(t)\;f_2(t)\;f_3(t)\;f_N(t)$ are band limited signals with bandwidth $B_1\;B_2,\;\ldots,B_N$ respectively satisfying $B=\sum_{i=0}^{N}B_i$.

Another scheme applicable only for natural sampling[2] is given by Sharma & Joshi[24]. The natural sampling is known to employ pulses of finite duration and is often used in practice, as the ideal impulse-train sampling is unrealizable. It is further

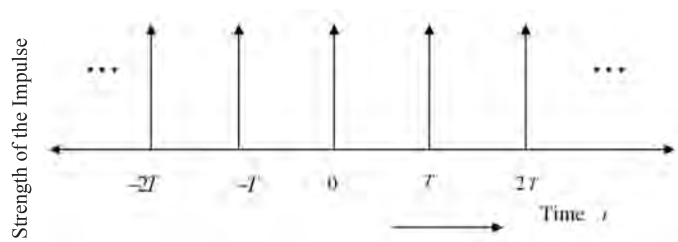

Figure 2. Ideal impulse-train sampling waveform $\delta_T(t)$.

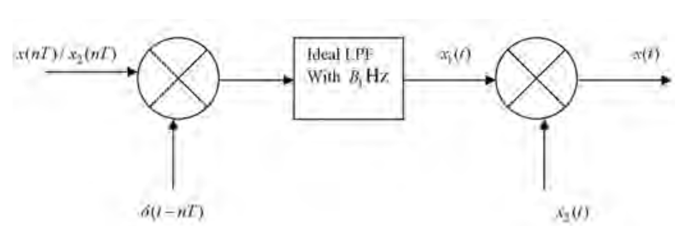

Figure 3. Reconstruction scheme using a time-variant system[26].

replaced by flat-top sampling for the ease of circuit realization[2]. Using a slightly modified version of the conventional natural sampling waveform with a specific width of the pulses taken, it is shown in[24] that the sampling rate in the classical Shannon sampling theorem can be further reduced by a factor of two. In the Sharma-Joshi scheme the sampling is performed using the waveform *s(t)* shown in Fig. 4, where δ denotes the width of the pulses and T stands for the periodic time of the sampling waveform.

The basic philosophy behind the Sharma-Joshi sampling scheme is to eliminate all the even harmonics and one specific odd harmonic of the sampling frequency from the spectrum of the sampled signal[24]. This is achieved by taking the alternate positive and negative pulses in the sampling waveform and by properly selecting the width of the pulses[24]. This increases the gap between two adjacent replicas in the spectrum of the sampled signal around the particular odd harmonic that has been eliminated by proper selection of the width of the pulses in the sampling waveform[24]. This increased gap in the spectrum of the sampled signal can be exploited either for increasing the maximum frequency of the signal to be sampled for a given sampling rate, or for reducing the sampling rate by a factor of two for a fixed maximum frequency of the signal[24].

For the sampling waveform *s(t)* shown in Fig. 4, the Fourier series can be written as[24]

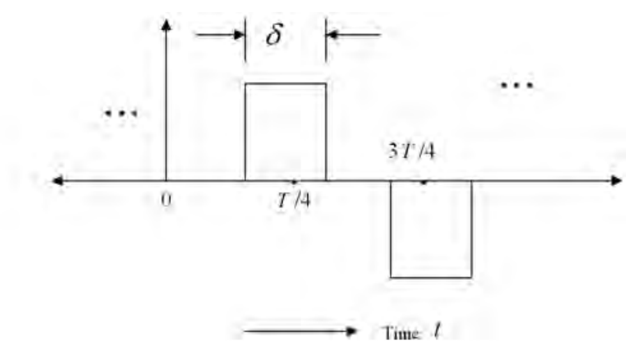

Figure 4. Sampling waveform *s(t)*.

$$s(t) = \sum_{n=1,\,3,\,5,\ldots} \frac{4}{n\pi} \sin\frac{n\omega_0\delta}{2} \sin n\omega_0 t \tag{6}$$

where $\omega_0 = 2\pi/T = 2\pi f_0$, and T is the periodic time of the sampling waveform. Here due to the alternation symmetry in the sampling waveform along the time-axis, all the even harmonics ($n = 2,\ 4,\ 6,\ldots$) are absent. Now accounting the positive and negative pulses in one cycle of the sampling waveform, the sampling rate f_s can be written as[24]

$$f_s = 2f_0 \tag{7}$$

Using Eqn (6) in Eqn (7), we obtain[24]

$$s(t) = \sum_{n=1,\,3,\,5,\ldots} \frac{4}{n\pi} \sin\frac{n f_s\delta}{4} \sin n\frac{f_s}{2} t \tag{8}$$

For a specific choice of δ, any one of the odd harmonic can further be eliminated from the spectrum of the sampled signal[24]. Clearly the higher the order of the harmonic to be eliminated, the lower will be the value of δ. The shape of the spectrum will remain intact (no aliasing) around this eliminated odd harmonic, and the original signal can be recovered from the sampled signal by band pass filtering and appropriate frequency translations[24].

The signal has been recovered from the undersampled signal, in the Sharma-Joshi scheme, by performing the filtering and frequency translation operation, which is basically a signal reconstruction from using a linear time-variant (LTV) system[24]. It can be easily seen that the required sampling rate f_s for exact signal reconstruction is independent of the order of the odd harmonic eliminated from Eqn (8) and must satisfy the condition given below:

$$f_s \geq B, \tag{9}$$

where B is the maximum frequency of the signal $x(t)$.

3. SAMPLING IN THE FRFT AND LCT DOMAINS

The research related to the FRFT/LCT has opened up new activities relating to the signal reconstruction in one FRFT/LCT domain using the samples in the same or other domain, or using partial samples in multiple domains[17-27].

Band limited signals and their sampling theorems in the FRFT domains have been investigated by Xia[17]. Xia has presented sampling theorems for the class of signals band limited in the FRFT domains. For instance, a signal *f(t)* band limited in the FRFT domains, i.e., $F_\alpha(u) = 0$, for $|u| > B_\alpha$, admits the following sampling expansion[17]:

$$f(t) = e^{-j(\cot\alpha)t^2/2} \sum_{n=-\infty}^{\infty} f(nT) e^{j(\cot\alpha)(nT)^2/2} \operatorname{sinc}\left[\frac{B_\alpha(t-nT)}{\pi\sin\alpha}\right] \tag{10}$$

where the sampling points are nT with $T = \pi\sin\alpha / B_\alpha$, $n \in \mathbb{Z}$, and $\operatorname{sinc}(x) = \sin(\pi x)/\pi x$.

Candan[19], *et al.* have also derived sampling and series expansion theorems using the FRFT and other signal transforms. Stern[21] has derived few sampling relations and corollaries for the signals band limited in the LCT domains. We reproduce here the sampling theorem for linear canonical transformed signals for ready reference from[21]. Let *f(t)* be a function with a compact support in some LCT domain with parameter matrix $M = [a,\ b;\ c,\ d]$ such that $F_M(u) = 0$ for $|u| > B_M$, where B_M is a positive real number. The function *f(t)* can be exactly reconstructed from its sampled version at points $t_n = nT$, $n \in \mathbb{Z}$ denoted as $f_T(t)$, using the signal reconstruction formula given below[21]

$$f(t) = T\Re_M^{-1}\left[rect\left(\frac{u}{2B_M}\right)\left[\Re_M f_T\right](u)\right](t) \tag{11}$$

provided the sampling interval T satisfies the inequality:

$$T \leq \pi|b| / B_M. \tag{12}$$

here $\Re_M$ and $\Re_M^{-1}$ denote the forward and inverse LCT operators with parameter matrix $M = [a,\ b;\ c,\ d]$ and determinant $ad - bc = 1$. By interchanging the role of the LCT domain with time domain and replacing $[a,\ b;\ c, d] \to [d,\ -b,\ -c,\ a]$ a dual form of the theorem of Eqn (12) can also be obtained[21-22]. Li [22], *et al.* have provided an alternative proof and a more explicit version of Eqn (11) as given in the following sampling theorem. Let signal *f(t)* be band limited to B_M in the LCT domain with parameter matrix $M = [a,\ b;\ c,\ d]$ and $b > 0$. Then the following sampling expansion for *f(t)* holds[22]:

$$f(t) = e^{-j(a/2b)t^2} \sum_{n=-\infty}^{\infty} f(nT) e^{j(a/2b)t^2} \operatorname{sinc}\left[\frac{B_M(t-nT)}{\pi b}\right] \tag{13}$$

where $T = \pi b / B_M$ is the sampling period and $1/T = B_M / \pi b$ is called the Nyquist rate of sampling theorem associated with the LCT.

In the sampling theorems discussed in this section so far, the signal in a particular FRFT/LCT domain is reconstructed from the samples of the signal in the same FRFT/LCT domain.

Many other sampling theorems involving the signal and its derivative, signal and its generalized Hilbert transform (GHT) have been investigated[18]. Zayed[18], *et al.* have derived several sampling expansions to reconstruct the FRFT of a time limited or band limited signals in the CFD using samples of the signal and its conventional Hilbert transform (CHT), each at half the Nyquist rate. For a signal that is conventionally band limited to *B* the main sampling relation reads as follows[18]:

$$f(t) = \sum_{n=-\infty}^{\infty} \left\{ f(nT)\cos[\beta(t-nT)] - \tilde{f}(nT)\sin[\beta(t-nT)] \right\} \frac{\sin[\beta(t-nT)]}{\beta(t-nT)} \tag{14}$$

where $\beta = B/(2\sin\alpha)$, $T = 2\pi\sin\alpha / B$, $n \in \mathbb{Z}$, and $\tilde{f}(\cdot)$ denotes the CHT of the signal $f(\cdot)$.

Cetin[20], *et al.* have also considered the signal reconstruction problem from the partial FRFT domains information of the signal using an iterative algorithm based on the method of projections onto convex sets (POCS). Of course the charming issues discussed in the introduction section continue to attract the attention of the researchers in these FRFT/LCT domains. The main reason which allow the reduction in sampling rate in the FRFT/LCT domains can be attributed to two facts:

- The signal *f(t)* is band limited in the CFD as well as some LCT domain but the bandwidth/support in the LCT domains can be less as compared to support in CFD. This is based on the fact that if *f(t)* and $F(\omega)$ are Fourier transform pair

  $$f(t) \xleftrightarrow{CFT} F(\omega),$$

  then the LCT of the signal $f(t)\exp(-jat^2/2b)$ will have compact support determined from $F(u_M/b)$, i.e., the support of the LCT of the signal will be *b* times the bandwidth of the signal in the CFD, where *b* is the parameter of the LCT matrix[21,22]. Clearly for values of the parameters $b < 1$, the support of LCT of a signal will be less than the support in CFD.
- The signal is not band limited in CFD but is band limited in some LCT domain.

The signal reconstruction in the above two different cases can either use samples of the transformed version of the original signal or the original signal itself.

As opposed to some of the early works in this direction[17,21,22], Sharma & Joshi[24] discuss a more general class of signals and also discuss the band pass sampling in the LCT domains. We reproduce the following theorems[21,24]. Let a signal *f(t)* whose FRFT with angle α, i.e., $F_\alpha(u)$ is band limited in the LCT domain with parameter matrix *M* to B_M be sampled at a rate u_s, then it admits the following reconstruction formula:

$$f(t) = \frac{1}{T}\mathcal{F}_\alpha^{-1}\Re_M^{-1}\left\{ rect\left(\frac{u}{2B_M}\right)\left(\Re_M F_\alpha^s\right)(u)\right\} \tag{15}$$

where $F_a^s(u)$ denotes the FRFT with angle α of the sampled signal, $\mathcal{F}_\alpha^{-1}$ and $\Re_M^{-1}$ denote the inverse FRFT and inverse LCT operators respectively, and

$$rect(u/2\Omega_M) = \begin{cases} 1 & \text{for } -\Omega_M \le u \le \Omega_M, \\ 0 & \text{otherwise.} \end{cases}$$

The sampling rate for perfect signal reconstruction must also satisfy the condition given below:

$$u_s(a\sin\alpha + b\cos\alpha) \ge 2B_M. \tag{16}$$

Clearly the sampling rate in Eqn (16) is dependent on the parameters *a* and *b* of the LCT and it can be less than the Nyquist rate for some specific values of the parameters. The band pass counterpart of the above theorem is as follows[24]:

Let a signal *f(t)* whose FRFT with angle α, i.e., $F_\alpha(u)$ is band pass from B_{ML} to B_{MH} in the LCT domain with parameter matrix *M* be sampled at a rate u_s, then it admits the following reconstruction formula:

$$f(t) = \frac{1}{T}\mathcal{F}_\alpha^{-1}\Re_M^{-1}\left\{ rect\left(\frac{u-B_{MC}}{B_{MH}-B_{ML}}\right)\left(\Re_M F_\alpha^s\right)(u)\right\} + \frac{1}{T}\mathcal{F}_\alpha^{-1}\Re_M^{-1}\left\{ rect\left(\frac{u+B_{MC}}{B_{MH}-B_{ML}}\right)\left(\Re_M F_\alpha^s\right)(u)\right\} \tag{17}$$

where $B_{MC} = (B_{MH} + B_{ML})/2$, and $u_s = 2\pi/T$. The required sampling rate u_s must satisfy the inequalities given below 24):

$$(a\sin\alpha + b\cos\alpha)u_s \le 2B_{Mw}\left(\frac{k-1}{N-1}\right), \text{ and}$$

$$(a\sin\alpha + b\cos\alpha)u_s \ge 2B_{Mw}\left(\frac{k}{N}\right), \tag{18}$$

where $B_{Mw} = B_{MH} - B_{ML}$ and $k = B_{MH}/B_{Mw}$, $k \ge N$ and *N* is a natural number.

The sampling expansion given by Papoulis[8] has also been generalized for FRFT and LCT domains[8,23,28]. The signal reconstruction using partial samples in multiple but related LCT domains has also been discussed[27]. It is worth mentioning here that the area of nonuniform sampling and sampling of periodic signals[25] is also equally connected here and much remains to be explored here in the FRFT/LCT domains. The sampling of 2-D and higher dimensional signals also need to be investigated in the LCT domains.

4. CONCLUSIONS

In this paper, the authors have reviewed the recent advances in the Shannon sampling theory of 1-D signals involving the samples taken below the Nyquist rate using nonlinear/ time-variant systems. The extensions of the sampling theorems to the fractional Fourier and Linear canonical transform domains are discussed. The sampling theory for 2D signals remains to be explored further.

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