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Theoretical & Experimental Studies on Vibration & Damping of Fibre-Reinforced Cantilever Laminates

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ABSTRACT

In this paper, vibration and damping analyses of glass fibre-reinforced laminated composite cantilever beams and plates are studied using C^1 finite element using shear deformation theory and also through experiments. The formulation in the theoretical model includes in-plane and rotary inertia terms. The governing equations for the complex eigenvalue problem based on complex elastic moduli are formulated. The solutions are obtained using QR algorithm. Parametric study is carried out to highlight the effects of lay-up and ply-angle of the laminates. A limited number of experimental investigations on cantilever laminates are conducted for obtaining the natural frequencies, damping factor and frequency responses. The comparison between the theoretical and the experimental results shows good agreement.

1. INTRODUCTION

The activity and the effort in the field of fibre-reinforced composites are on the increase for the laminated construction, and this is mainly due to their high specific strength and stiffness. The static and the dynamic analyses of the fibre-reinforced composite material plates are generally carried out by replacing a laminated plate by a homogenous orthotropic material plate. This simplification may not lead to the true representation of the deformation in the layers because of neglecting shear deformation due to high ratio of the in-plane Young's modulus to the transverse shear modulus for most of the composite materials. Hence, analysis based on shear deformation theory is essential for predicting accurate behaviour. Further, the damping in composite materials plays an important role in controlling the resonant response of aerospace structures and thus in prolonging their service life under repeated loading or impact. Fibre-reinforced composites, in general, have higher damping than metals. However, their values depend on fibre and resin types, fibre orientation, and stacking sequence. Research on the damping analysis of composites is not so extensive as that of undamped free vibration analysis. Experimental and analytical efforts based on refined theories and reliable instruments to characterise the actual dynamic characteristics of composite laminates are essential for the designers/engineers in optimising the structural design.

Considerable research has been carried out on the vibration and damping of laminated beams, such as constrained layer/sandwich layer and it has been reviewed by Nakra[1-3] on the topic dealing with vibration control with viscoelastic

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material. Rao[4] has extensively dealt with the dynamics of plates made of isotropic and composite materials. Some research has also been carried out on optimum design of viscoelastic damping layer treatment for beams and plates. The notable contributions are the studies by Yildiz and Stevens[5], Hajela and Lin[6], and Marcelin[7], *et al.* Dynamic response due to forced vibrations of a beam with constrained damping layer treatment has been studied by Mead and Markus[8], and Roy and Ganesan[9]. In all these studies, complex modulus, which consists of a real part representing elastic stiffness and an imaginary part representing dissipation, has been widely used to model the behaviour of linear viscoelastic materials under harmonic vibration. Furthermore, all these studies are based on the classical theory. The investigation using shear deformation theory is considered by Moser and Lumassegger[10], He and Rao[11], and Rikards[12]. Imaino and Harrison[13] pointed out that the classical theory is not suitable to predict the system loss factors when the viscoelastic layer has a considerably lower modulus compared to the base and constraining layer moduli or core layer in the sandwich case. Therefore, it may be necessary to use a sandwich beam theory which satisfies interface stress and displacement continuity with vanishing shear stress on the top and bottom surfaces of the beam and is based on higher-order deformation theory.

Gibson and Plunkett[14], and Gibson[15] reviewed experimental and analytical efforts to characterise the damping properties of fibre-reinforced materials. The important contributions are cited here. The analysis of vibration and damping of fibre-reinforced composite plate has been carried out by Alam and Asnani[16], Malhotra, Ganesan and Veluswami[17], and Koo and Lee[18]. Alam and Asnani employed solution in the form of series summation, and the finite element procedure was adopted by Malhotra, Ganesan and Veluswami[17], and Koo and Lee[18]. Shiau, *et al.*[19] recently investigated the dynamic response and stability characteristics of rotating composite blades with frictional damping.

In the present study, vibration and damping analyses of glass fibre-reinforced laminated composite beams and plates are studied employing finite element based on shear deformation theory, as outlined by Beakou and Touratier[20]. The formulation in the theoretical model includes in-plane and rotary inertia terms. The governing equations are solved using standard eigenvalue approach. Numerical investigations, considering cantilever laminated beams and plates, are carried out to bring out the influences of different parameters. Some experimental studies on the laminates are also conducted for obtaining the natural frequencies, damping factor and frequency responses. A good correlation between numerical and experimental results is established.

2. FORMULATION

A laminated composite plate is considered with the coordinates x, y along the in-plane directions and z along the thickness direction, respectively. Using formulation based on shear flexible theory, the displacements in k^{th} layer $u^{(k)}$, $v^{(k)}$ and $w^{(k)}$ at a point (x, y, z) from the median surface are expressed as functions of mid-plane displacement u, v, w and independent rotation θ_x and θ_y of normal in xz and yz planes, respectively, as

$$\begin{aligned} u^{(k)}(x,y,z,t) &= u(x,y,t) - z\partial w/\partial x + [f_1(z) + g_1^{(k)}(z)] \\ &\quad \{\partial w/\partial x + \theta_x\} + g_2^{(k)}(z)\{\partial w/\partial y + \theta_y\} \\ v^{(k)}(x,y,z,t) &= v(x,y,t) - z\partial w/\partial y + g_3^{(k)}(z)[\partial w/\partial x \\ &\quad + \theta_x] + [f_2(z) + g_4^{(k)}(z)]\{\partial w/\partial y + \theta_y\} \\ w^{(k)}(x,y,z,t) &= w(x,y,t) \end{aligned} \quad (1)$$

where t is the time. The functions involved in Eqn (1) for defining the kinematics are as follows:

$$\begin{aligned} f_1(z) &= h/\pi \sin(\pi z/h) - h/\pi\, b_{55}\cos(\pi z/h) \\ f_2(z) &= h/\pi \sin(\pi z/h) - h/\pi\, b_{44}\cos(\pi z/h) \\ g_i^{(k)}(z) &= a_i^{(k)} z + d_i^{(k)},\ i = 1,2,3,4;\ k = 1,2,3,\ldots N \end{aligned} \quad (2)$$

where N is the number of layers of the multilayered structure, h is the total thickness of the laminate, π is equal to 3.141592, and b_{44}, b_{55}, $a_i^{(k)}$, $d_i^{(k)}$ are coefficients to be determined from contact conditions for displacements and stresses between the layers and from the boundary conditions on the top and bottom surfaces of the plate. The details of the derivations of these coefficients are presented by Beakou and Touratier[20].

The linear strains in terms of mid-plane deformation can be written as

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon^o \\ \chi \\ \omega \\ \gamma^o \end{Bmatrix} \tag{3}$$

The mid-plane strains $\{\varepsilon^o\}$, bending strains (due to lower and higher-order terms involved in defining the kinematics, Eqn (1)), $\{\chi\}$, $\{\omega\}$ and shear strains $\{\gamma^o\}$ in Eqn (3) are written as

$$\{\varepsilon^o\} = \begin{Bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial u/\partial y + \partial v/\partial x \end{Bmatrix}; \quad \{\chi\} = -\begin{Bmatrix} \partial^2 w/\partial x^2 \\ \partial^2 w/\partial y^2 \\ 2\partial^2 w/\partial x\partial y \end{Bmatrix}$$

$$\{\omega\} = \begin{Bmatrix} \partial\gamma_1^0/\partial x \\ \partial\gamma_2^0/\partial y \\ \partial\gamma_1^0/\partial y \\ \partial\gamma_2^0/\partial x \end{Bmatrix}, \quad \{\gamma^o\} = \begin{Bmatrix} \gamma_1^0 \\ \gamma_2^0 \end{Bmatrix} = \begin{Bmatrix} \partial w/\partial x + \theta_x \\ \partial w/\partial y + \theta_y \end{Bmatrix} \tag{4}$$

If $\{N\}$ represents the membrane stress resultants (N_{xx}, N_{yy}, N_{xy}) and $\{M\}$, $\{\widetilde{M}\}$, represent the bending stress resultants due to lower and higher-order terms involved in defining the kinematics [(M_{xx}, M_{yy}, M_{xy}), ($\widetilde{M}_{xx}, \widetilde{M}_{yy}, \widetilde{M}_{xy}$)], one can relate these to membrane strains $\{\varepsilon^o\}$ and bending strains [$\{\chi\}$, $\{\omega\}$] through the constitutive relations as

$$\begin{aligned} \{N\} &= [A]\{\varepsilon^o\} + [B]\{\chi\} + \{E\}\{\phi\} \\ \{M\} &= [B]^T\{\varepsilon^o\} + [D]\{\chi\} + [\widetilde{B}]\{\omega\} \\ \{\widetilde{M}\} &= [E]^T\{\varepsilon^o\} + [B]^T\{\chi\} + [\widetilde{D}]\{\omega\} \end{aligned} \tag{5}$$

Similarly, the transverse shear stress resultants $\{Q\}$ representing the quantities (Q_{xz}, Q_{yz}) are related to the transverse strains $\{\gamma^o\}$ through the constitutive relation as

$$\{Q\} = [\widetilde{A}]\{\gamma^o\} \tag{6}$$

The different matrices involved in Eqns (5) and (6) are defined by Beakou and Touratier[20].

For a composite laminate of layer thickness h_k (k = 1,2,3..), and the ply-angle ϕ_k (k = 1,2,3..), the necessary expressions for computing the stiffness coefficients, available in the literature [21] are used. For the damping analysis, the complex moduli of an orthotropic material are defined, according to the elastic-viscoelastic correspondence principle, as follows:

$$\begin{aligned} &E_1^* = E_1^R + iE_1^I,\ E_2^* = E_2^R + iE_2^I,\ E_3^* = E_3^R + iE_3^I, \\ &G_{12}^* = G_{12}^R + iG_{12}^I,\ G_{23}^* = G_{23}^R + iG_{23}^I,\ G_{13}^* = G_{13}^R + iG_{13}^I \end{aligned} \tag{7}$$

Here, E^* and G^* are Young's modulus and shear modulus, respectively. The subscript 1 denotes longitudinal direction, whereas subscripts 2 and 3 refer to the transverse directions, wrt the fibres. The superscripts R and I denote the real and imaginary parts of the complex moduli. The material loss factors η_1, η_2, η_3 under tension-compression and η_{12}, η_{23}, η_{13} under shear are defined as

$$\begin{aligned} &\eta_1 = E_1^I/E_1^R,\ \eta_2 = E_2^I/E_2^R,\ \eta_3 = E_3^I/E_3^R, \\ &\eta_{12} = G_{12}^I/G_{12}^R,\ \eta_{23} = G_{23}^I/G_{23}^R,\ \eta_{13} = G_{13}^I/G_{13}^R \end{aligned} \tag{8}$$

The total potential energy functional U consisting of energy stored in the plate is given by:

$$U(\delta) = \frac{1}{2}\int \left([N]^T\{\varepsilon^o\} + [M]^T\{\chi\} + [\widetilde{M}]^T\{\omega\} + [Q]^T\{\gamma^o\}\right)dA + \int \{f\}w^T\,dA \tag{9}$$

where δ is the vector of the degrees-of-freedom (DOFs) associated to the displacement field in a finite element discretisation and f is the force acting on the structure.

The kinetic energy of the plate is written as

$$T(\delta) = \frac{1}{2}\int \left(\int_{-h/2}^{h/2} \rho_i [\dot{u}^{(k)}\ \dot{v}^{(k)}\ \dot{w}^{(k)}]^T [\dot{u}^{(k)}\ \dot{v}^{(k)}\ \dot{w}^{(k)}]\,dz\right)dA \tag{10}$$

where the dot over the variable denotes the partial derivative wrt time and ρ is the mass density.

Substituting Eqns (9) and (10) in Lagrange's equation of motion, one obtains the governing equation for the vibration of the beam structure as

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = \{F\} \qquad (11)$$

where $[M]$ is the consistent mass matrix, $[K]$ the structural stiffness matrix of the laminate which is a complex matrix, and $\{F\}$ is the load vector.

The eigenvalues for the damped structure can be determined from Eqn (11) by letting $\{F\}$ equal to zero for the free vibrations.

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = \{0\} \qquad (12)$$

The complex eigenvalues of the form $\lambda^* = (\lambda^R + i\lambda^I) = (\omega^*)^2$, where $\omega^* = (\dot{\omega}^R + i\,\omega^I)$ are obtained for Eqn (12) employing QR algorithm. The resonance frequencies ω and the system loss factors η are calculated from the eigenvalues as

$$\omega = \omega^R = (\lambda^R)^{1/2},\ \eta = \lambda^I/\lambda^R \qquad (13)$$

Once the natural/resonance frequencies are obtained, the structure is excited around the natural frequencies so as to obtain the response of the structure from the equation.

$$\left[[K] - \omega_F^2\,[M]\right]\{\delta\} = \{F \qquad (14)$$

where ω_F is the harmonic forcing frequency.

3. ELEMENT DESCRIPTION

The eight-noded element used here is based on Hermite cubic function for transverse displacement (w) according to the C^1 continuity requirement, Serendipity quadratic function for the in-plane displacements u, v and rotations θ_x, θ_y. Further, the element needs eight nodal DOFs (u, v, w, $\partial w/\partial x$, $\partial w/\partial y$, $\partial^2 w/\partial x\partial y$, θ_x, θ_y) for all corner nodes and four DOFs (u, v, θ_x, θ_y) for the mid-node of all four sides. The element is developed based on new kinematics as given in Eqn (1) which accounts for interlayer continuity for displacements and transverse shear stresses of the laminate. The element behaves very well for both thick and thin situations. It has no spurious mode and is represented by correct rigid body modes.

4. RESULTS & DISCUSSION

There is no need of using shear correction factor here, as the transverse strain is represented by cosine function, which is of higher order in nature. Based on progressive mesh refinement, 16 elements idealisation and 8 × 8 grid size are found to be adequate to model the laminated beams and plates, respectively, for the flexural/bending

Table 1. Comparison of flexural frequencies and loss factors of simply-supported sandwich beams

h_1/h_2	G Core N/M[2]	Mode N	Frequency 'ω' (rad/s) Present	Frequency 'ω' (rad/s) Analytical[23]	Loss factor 'η' Present	Loss factor 'η' Analytical[23]
			865	878	0.4954	0.50
	5.50 E 06	2	2560	2458	0.3212	0.34
		3	5275	4927	0.1927	0.20
			1646	1643	0.1220	0.11
	.00 E 08	2	5244	5456	0.3320	0.31
		3	9672	9877	0.4629	0.45
			1101	1106	0.3204	0.32
	2.50 E 08	2	3623	3481	0.1922	0.20
		3	7705	7300	0.1096	0.11
			1592	1581	0.1134	0.10
7	2.50 E 07	2	5266	5357	0.2714	0.26
		3	10304	10187	0.3207	0.32

Table 2. Natural frequencies and loss factors for cantilever cross-ply laminated plates (90° /0° /90°.....)

No. of layers	Frequencies (ω)(Hz)/Loss factors (η)							
	Mode 1		Mode 2		Mode 3		Mode 4	
	ω_1	$\eta_1 \times 10^{-3}$	ω_2	$\eta_2 \times 10^{-3}$	ω_3	$\eta_3 \times 10^{-3}$	ω_4	$\eta_4 \times 10^{-3}$
2	80.68	5.4910	187.09	9.4329	501.00	5.4859	713.77	6.41567
4	91.34	3.5161	197.22	8.7144	566.49	3.5499	747.76	6.60910
6	93.61	3.2274	199.80	8.5973	580.46	3.2650	761.20	6.38490
8	94.52	3.1307	200.95	8.5573	586.06	3.1702	766.86	6.30900
10	94.99	3.0868	201.60	8.5389	588.99	3.1269	769.91	6.27430

damping analysis. Thus, the present formulation can be verified numerically by comparing the results based on different models, which are used for studying the thin and thick laminates. Before proceeding for the detailed analysis, flexural frequencies and the damping/loss factors obtained for sandwich beam are compared in Table 1 and they are found to be in good agreement with the available analytical/numerical solutions[18,23]. The materials considered here are[22]:

GFRP (Glass/DX-210): E_1^R = 37.78 GPa, E_2^R = 10.90 GPa, E_3^R = 10.90 GPa, G_{12}^R = 4.91 GPa, G_{23}^R = 4.91 GPa, G_{13}^R = 4.91 GPa, ν_{12} = 0.30, η_1 = 13.8465 × 10^{-4}, $\eta_2 = \eta_3 = \eta_{12} = \eta_{23} = \eta_{13}$ = 0.208, ρ = 1870 kg/m^3

where ν_{12}, ρ are Poisson's ratio and mass density, respectively.

The geometry of the cantilever laminates assumed here are given as

Beam: Length (a) = 300 mm; breadth (b) = 37 mm; thickness (h) = 03 mm

Plate: Length (a) = 200 mm; breadth (b) = 180 mm; thickness (h) = 06 mm

Table 3. Influence of ply-angle of 11-layered cantilever laminated plate (θ°/- θ°/ θ°/θ°....)

Ply-angle (θ) (deg)	Frequencies (ω)(Hz)/Loss factors (η)(×10^{-001})							
	Mode 1		Mode 2		Mode 3		Mode 4	
	ω_1	η_1	ω_2	η_2	ω_3	η_3	ω_4	η_4
90	80.68	2.08	197.35	2.08	546.73	2.08	738.21	2.08
45	80.39	2.08	256.91	2.08	485.94	2.08	787.27	2.08
30	97.77	2.08	253.35	2.08	583.71	2.08	724.82	2.08
15	112.90	2.08	231.09	2.08	627.47	2.08	715.26	2.08

4.1 Numerical Results

Free vibration analysis of cross-ply cantilever laminated plate is carried out and the results are shown in Table 2 varying the number of layers in the laminate. It is seen from Table 2 that the effect of number of layers is to increase the frequency values and to reduce the damping/loss factor of the structures. Further study is made considering a laminate with 11 layers to highlight the influence of ply-angle and the results are tabulated in Table 3. It can be concluded from Table 3 that the ply-angle can significantly alter the frequency values, and this is due to the directional stiffness provided by the anisotropic properties in the multilayered laminate.

Table 4(a). Comparison of theoretical and experimental results for free vibrations of 11-layered cross-ply cantilever plate (90°/ 0°/90°.....)

Frequencies (ω)	Theoretical (Hz)	Experimental (Hz)
ω_1	88.0	88.0
ω_2	197.3	184.0
ω_3	546.3	–
ω_4	728.5	–

Table 4(b). Comparison of theoretical and experimental results for free vibrations of 11-layered cross-ply cantilever beam (90°/ 0°/90°.....)

Frequencies (ω)	Theoretical (Hz)	Experimental (Hz)
ω_1	17.22	17.0
ω_2	107.85	96.0
ω_3	236.63	

4.2 General Experimental Setup

The experimental setup is accomplished by setting a bench–vice on a rigid mounting table to simulate the boundary condition of one side clamped and other sides free. The vice is mounted rigidly onto the mounting table with the help of a steel channel. The equipment required for the tests include a dynamic signal analyser, two sets of tunable bandpass filters, two sets of signal conditioning amplifiers, a frequency control meter and power amplifier, an electromagnetic shaker, one accelerometer and force transducer, and an impact hammer with necessary standard cables. The vice and the layout of the instruments are planned

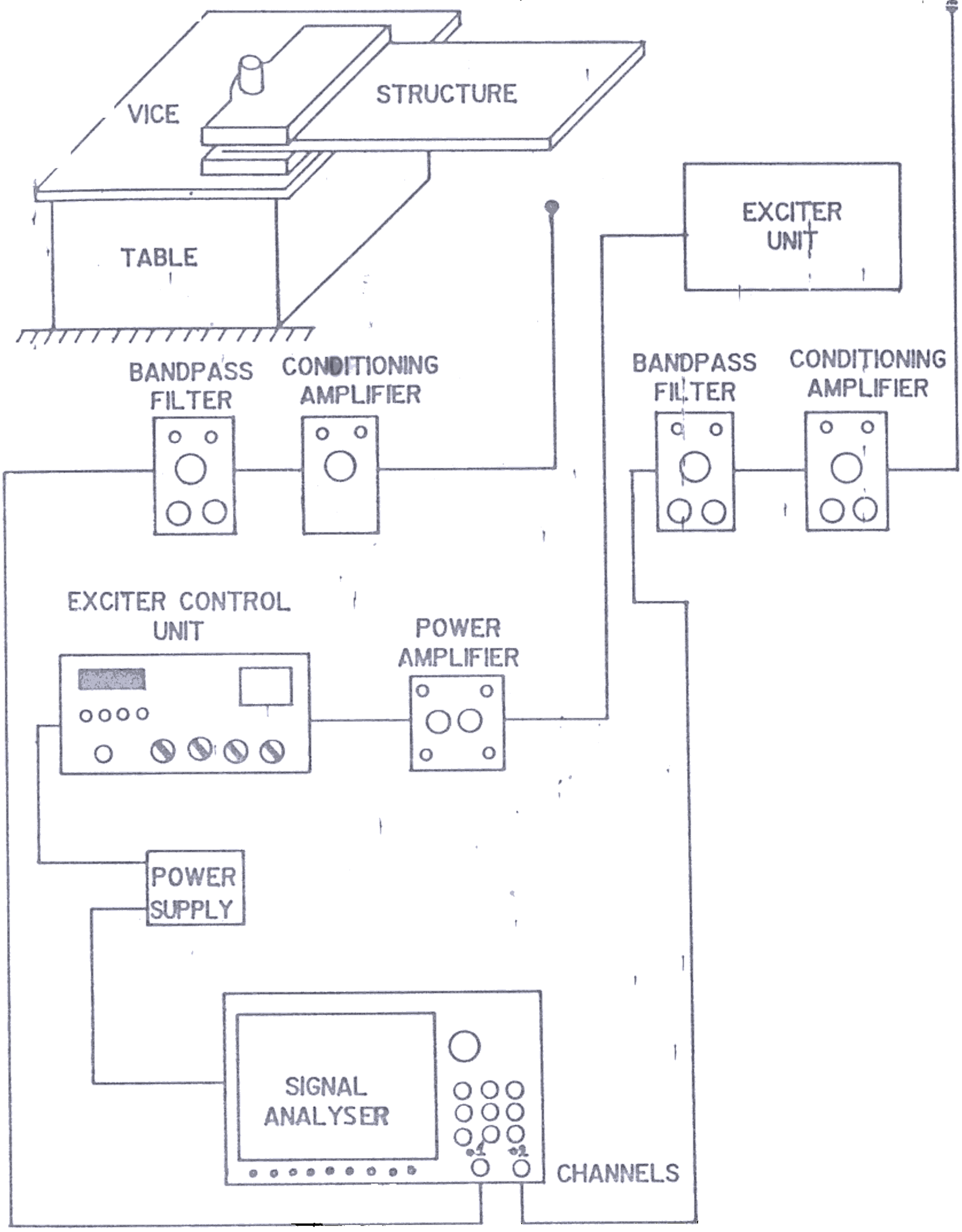

Figure 1. General schematic experimental setup

so as to enable easy recording of measurements as shown in the schematic diagram (Fig. 1). Based on the calibration charts, settings of various instruments are initialised before proceeding the experiment. A photograph showing the actual experimental layout containing the amplifiers, filters and the analyser and the setup ready for vibration tests is shown in Fig. 2.

4.3 Experimental Setup for Laminated Plate/Beam

Here, an instrumented hammer is readied by assembling all components of the hammer. The other required instruments like conditioning amplifier, filters, cables, accelerometers, etc. are placed in position. All the measuring instruments are connected using standard cables. The analyser used is Hewlett Packard dynamic signal analyser. The measuring accelerometer is placed on the top of the mounted composite laminates at the centre. With the help of instrumented hammer, small impact is given to the beam/plate on the top surface and then the measured time response characteristics are fed to the dynamic signal analyser for obtaining the natural frequencies. The consistency of the results is ensured by repeating the experimental procedure, i.e., by striking the laminate at a number of points marked on it. The mode shape of the laminate is then obtained by impacting (same amount of force) at different marked points on the laminate using the impact hammer and then recording the frequency response behaviour at the centre of the laminate. Finally, the frequency response due to the forced excitation, by means of placing an electromagnetic shaker below the laminate at centre of the free end of the beam/plate, is obtained. Further, for forced response measurement, the shaker is kept in contact with the laminate at the free end.

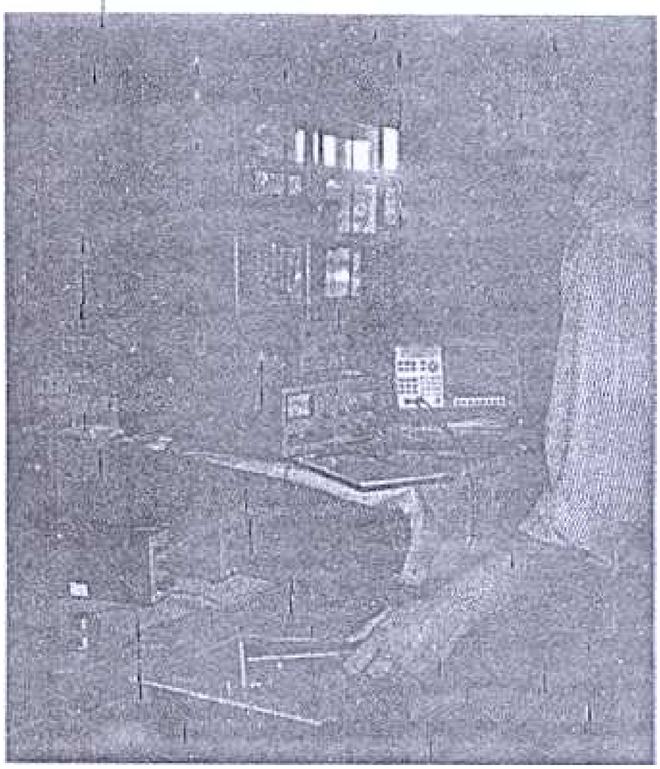

Figure 2. Actual experimental layout

4.4 Experimental Results & their Comparisons

The cross-ply laminates (GFRP) with 11 layers (90°/0°/90°/......), as test specimens, are made. The natural frequencies obtained for the laminates (beam and plates) from the experiment are compared in Table 4 with those of theoretical model presented. It can be observed from this table that the results are in good agreement. The little discrepancy in the results may be attributed to the possible variation in the material properties (assumed for numerical study and that of actual laminates used for experiment), and to a lesser extent in simulating the boundary conditions in experiment.

The frequency responses are studied for laminated plate through experiment and theoretical investigations. A force of 1.6421 N is applied at the centre of the free end of plate and the response is measured at the centre of the laminate and depicted in Fig. 3. Furthermore, the response of the laminated plate is recorded for the range of frequencies, around fundamental frequency. Using half-power point method, the values of the damping factors are calculated from the experimental response shown in Fig. 3. Introducing the damping factor obtained from experimental response, the frequency response characteristics is evaluated

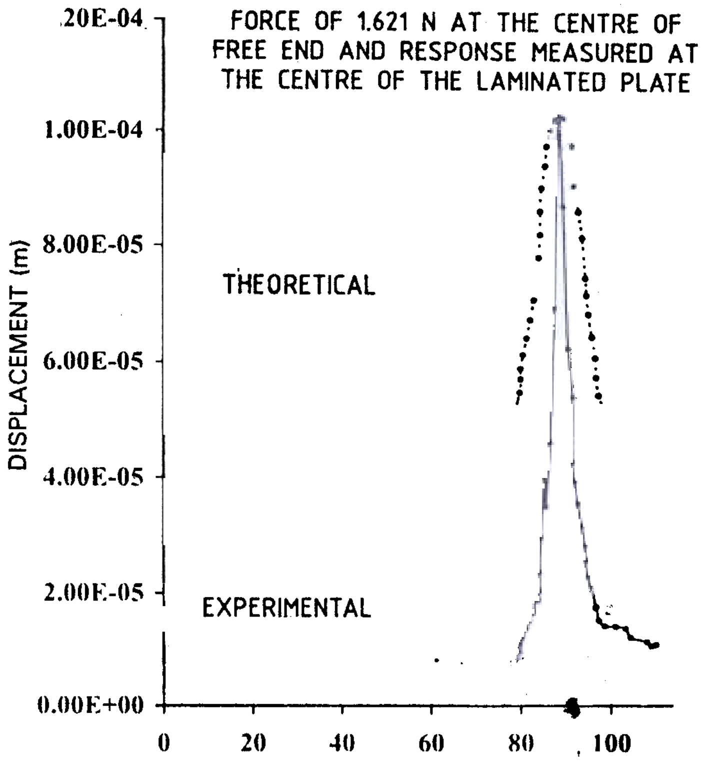

Figure 3. Frequency response of laminated cantilever 11-layered plate.

theoretically using Eqn (14) and is presented along with experimental response in Fig. 3. The behaviours predicted (by experimental and theoretical work), are qualitatively similar and the displacement at resonance agrees very well. However, the variation in the results may possibly be attributed to the difficulty in ensuring the simulation of the exact boundary conditions (the shaker, which is used for excitation, is physically in contact with the laminate at the free end).

The frequency response of laminated beam, consisting of 11-layered cross-ply one, obtained from experiment is demonstrated in Fig. 4. The first peak in the response, which is predominant, corresponds to the fundamental frequency (17 Hz) and second peak is near the second natural mode (96 Hz) of the beam.

5. CONCLUSIONS

The vibration and damping analyses of laminated cross-ply beams and plates are studied

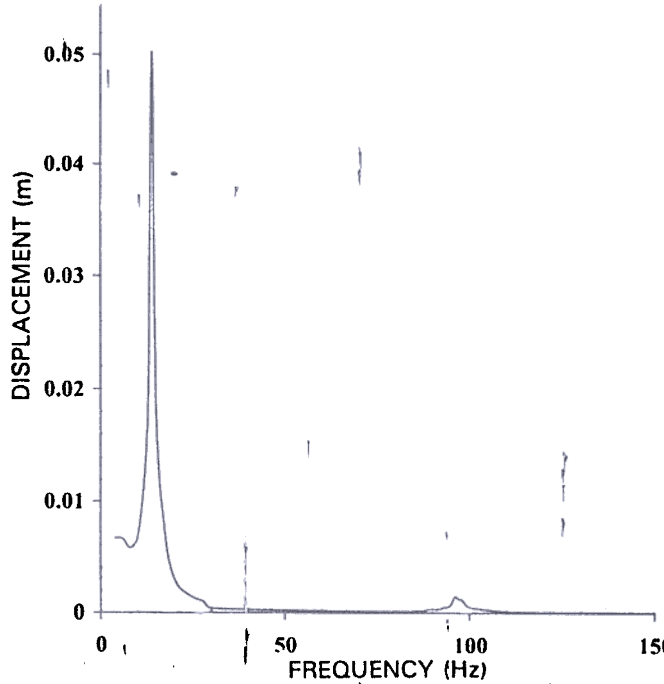

Figure 4. Frequency response of laminated cantilever 11-layered beam.

through theoretical and experimental studies of models. The theoretical study is made using finite element procedure utilising shear deformation theory. Some experimental studies are also carried out. The following conclusions can be made from this investigation:

- The ply-angle in a layered laminate affects the behaviour of the laminate by increasing the free vibration frequencies as the angle is decreased. The change in ply-angle does not affect the loss factor for the modes studied.
- The overall response behaviour predicted by theory and that obtained by experimental investigations are qualitatively similar.
- A good correlation is observed between theoretical and experimental values for free vibration frequencies and displacements for the first mode of vibration for the laminates.
- The discrepancies in results for certain cases of plates and beams could be attributed to variations in material properties and difficulties in simulating precise and exact boundary conditions.

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