

Atmospheric Reentry Dispersion Correction Ascent Phase Guidance for a Generic Reentry Vehicle

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ABSTRACT

Launch vehicle explicit guidance mechanism depends on the estimation of the desired burnout conditions and driving the vehicle to achieve these conditions. The accuracy of the vehicle at the target point depends on how tightly these conditions are achieved and what is the strategy used to define the trajectory. It has been observed in the literature that most of the guidance mechanisms during reentry use vacuum guidance equations that is during reentry the atmospheric effects are not considered. In order to achieve minimum miss distance at the target point the atmospheric effects are to be considered during the guided phase and appropriate corrections should be executed, otherwise depending on the reentry flight path angle and ballistic coefficient the errors can be as high as tens of nautical miles. In this paper, the authors develop a novel approach to these vacuum guided launch vehicle problems. The paper elaborates how to calculate a priori the reentry dispersion during the ascent phase guidance and provide guidance corrections such that the terminal conditions are achieved with higher accuracy.

Keywords: Reentry dispersion, launch vehicle injection point accuracy, atmospheric effects, reentry vehicle

1. INTRODUCTION

Launch vehicles consist of single or multiple stages based on the requirement to place payload at the specified location (ground/space) with higher accuracy. In order to achieve the mission objective, guidance algorithm lays the path by considering the mission and trajectory constraints such as system capabilities, control bounds, structural limitation, etc.

Broadly launch vehicle trajectory consists of three phases: boost phase (powered flight), ballistic flight (free flight) and reentry flight¹. During boost phase, vehicle follows a pre-programmed pitch program till vehicle attains sufficient desirable conditions (altitude, velocity, dynamic pressure), once vehicle attains desirable conditions, closed loop guidance take over for higher stages. During closed loop guidance, taking the current and target (earth rotation rate compensated) inputs from navigation the guidance algorithm steers the vehicle such that at the end of powered phase, the vehicle is placed on the desired trajectory (burnout condition). Once the vehicle is placed on the desired trajectory the vehicle follows the ballistic flight path, where it experiences only the gravitational force up to reentry point. During the reentry phase the vehicle experiences not only the gravitational force but also aerodynamic force and this phase continuous till the point of impact.

Several guidance schemes are available in the literature to place the vehicle on the desired trajectory at burnout. Not all, but most of them rely on the required velocity concept, where it is assumed that at each point on the powered flight, there exists a velocity vector which, if achieved gives desired impact

point during the free flight¹. In order to find the desired velocity vector, hit equation² needs to be solved. But the drawback of the solution which we got from this technique that, the guidance equation did not consider the reentry aerodynamic forces into account while formulating the trajectory. And from the literature it is clear that the error due to this consideration is as high as 10 nautical miles. The major effects of atmosphere during reentry are :

- The time required to reach the target point will increase, leads to impact point displacement
- Short fall of the desired downrange in the trajectory plane.

Rapid analytical techniques for determining the reentry range and time-of-flight are developed by Moe³ and Blum⁴, Bate and Johnson⁵ according to which the error for a typical ICBM trajectory remain well below 2 kilometres⁶. Changsheng⁷, *et al.* uses optimization techniques to define the optimal lateral force required to guide the vehicle towards the desired target.

The procedure described in the current paper is based on running a parallel 6 degrees of freedom (6-DOF) simulation from the estimated burnout point⁸ to the impact point. The error at the impact is the difference between the desired and parallel simulation achieved parameters (Latitude and Longitude). The error thus obtained is given as an augmentation to the desired parameters in the actual flight, by virtue of which the vehicle will be steered to a pseudo target. Because of the aerodynamic forces during reentry, the vehicle attains the desired terminal conditions (target). The above procedure is iterated during the ascent guided phase, such that the error at the impact will be well within the desired tolerance bounds.

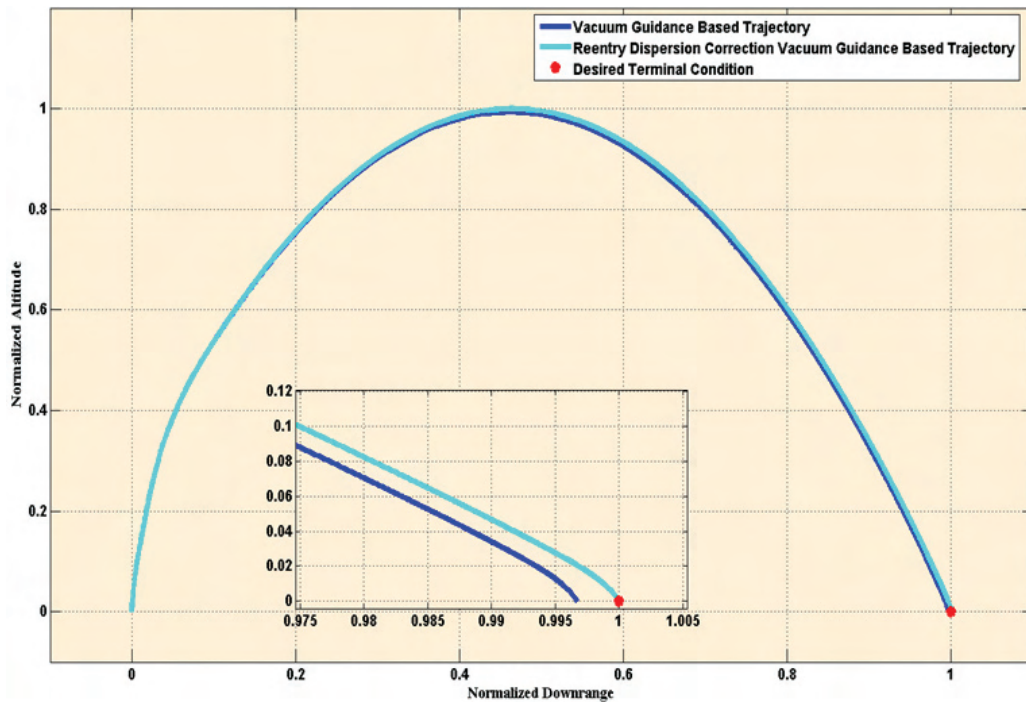


Figure 1. Typical ballistic trajectory with and without reentry dispersion correction.

2. DESIGN METHODOLOGY

The basic principle of the current method is shown in Fig. 1. The trajectory shown in the blue line is the trajectory obtained by using the explicit guidance, required velocity vector based guidance mechanism, where the hit equation is the basis. It is clear from this that if we go with this guidance mechanism the impact point is not achieved with the desired accuracy, since the hit equation doesn't include the reentry atmospheric effects⁸. In order to improve the terminal accuracy of the system the current work proposes an iterative atmospheric reentry dispersion correction ascent phase guidance (ARDCAG), where the estimated terminal error at the impact becomes an internal part of the ascent phase guidance. The ARDCAG design procedure is as follows:

- From burnout conditions (estimated/achieved)⁹, simulate the 6-DOF up to the impact point.
- Calculate the error at the impact, i.e. (desired – simulation achieved) latitude and longitude.
- Augment the ascent phase guidance desired coordinates with the above error values.
- With the above augmented coordinates initiate the guidance.
- Repeat the steps 1-5 till the error at impact lies below the desired tolerance bounds.

The typical guidance flow chart is shown in Fig. 2 and the generic guidance problem is shown in Fig. 3.

3. MATHEMATICAL MODELLING OF THE LAUNCH VEHICLE

A Non-linear 6-DOF mathematical model with 3 forces and 3 moments is considered for the current work^{10,14}.

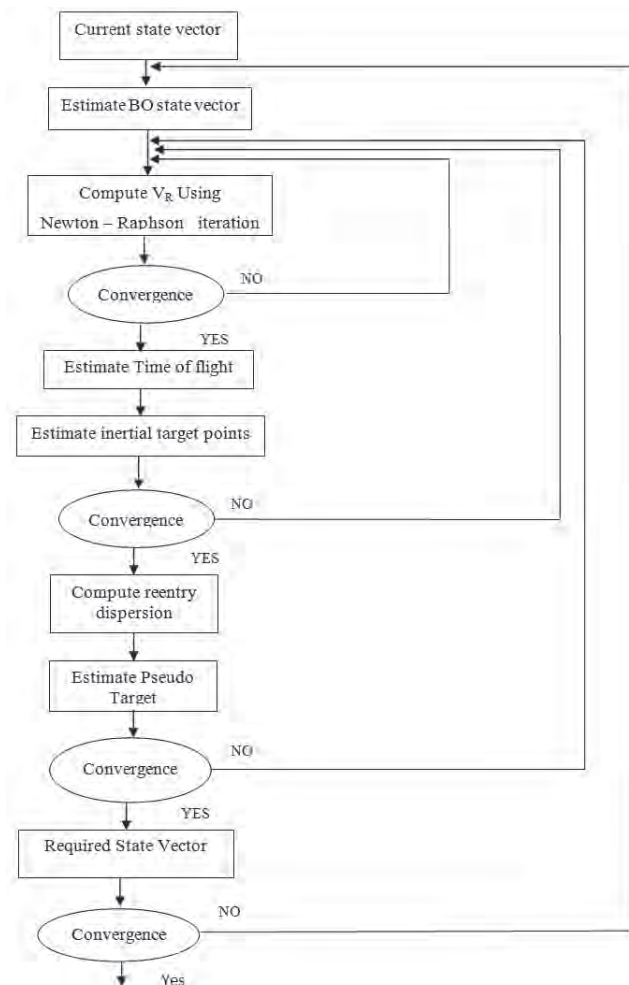


Figure 2. ARDCAG design procedure.

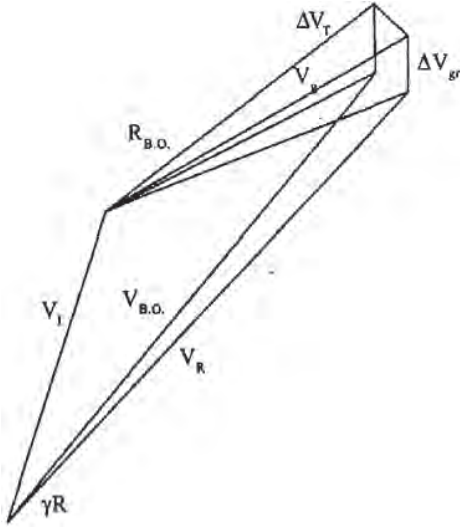


Figure 3. Geometrical representation of guidance problem.

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{(T_{fx} - D_{fx})}{m} \right) - g_x \\ \left(\frac{(T_{fy} - A_{fy})}{m} \right) - g_y \\ \left(\frac{(T_{fz} - A_{fz})}{m} \right) - g_z \\ M_x / I_x \\ (M_y + (I_z - I_x)pr) / I_y \\ (M_z + (I_x - I_y)pq) / I_z \end{bmatrix} \quad (1)$$

where u, v, w and p, q, r are translation and rotational components, T_{fx}, T_{fy}, T_{fz} and A_{fx}, A_{fy}, A_{fz} are thrust and aerodynamic force components, D_{fz} is the drag force, and m is the mass of the vehicle g_x, g_y, g_z are the gravitational components M_x, M_y, M_z are moments, I_x, I_y, I_z are moments of inertia Eqn (1), can be written in a concise form as

$$\dot{X} = f(X, U) \quad (2)$$

where $X = (u, v, w, p, q, r)$ and $U = (T_x, T_y, T_z)$.

Here the moments include both control (reaction control) and aero (reentry) components, where control moments used to control the attitude of the vehicle. The 6-DOF consists of all necessary input models like atmospheric¹¹, wind¹², gravitational force⁸, earth obsoleteness and rotational rate¹³, navigation, aero, nonlinear auto pilot, actuator so as to make the simulation closed to the real time environment.

The launch vehicle considered for the current study consists of two stages, propelled by solid propellant motors controlled by flex nozzle control system. Closed loop guidance initiated after the launch vehicle completes a pre-programmed attitude turn phase. Once closed loop guidance starts, based on the current state vector the guidance system estimates the desired burnout state vector (position, velocity, flight path angle) required to place the vehicle on the desired trajectory^{8,9}. With these desired burnout state vector, a background ARDCAG algorithm is initiated iteratively till the 6-DOF achieved impact latitude and longitude coincides with the desired one's within tolerance bounds.

4. SIMULATION RESULTS

To validate the ARDCAG algorithm, various combinations of burnout state vectors are considered for a given target coordinates as shown in Table 1.

In the current formulation the flight path angle is defined with respect to the local vertical and the burnout velocity is

Table 1. Combinations of burnout state vectors

Case	Burnout state vector (flight path angle, velocity & position)			Desired terminal conditions tolerance bound (0.01°) ≈1100(m)		Achieved terminal conditions (No atmospheric reentry dispersion correction trajectory)		Achieved terminal conditions (ARDCAG trajectory)	
				Latitude (deg)	Longitude (deg)	Latitude (deg)	Longitude (deg)	Latitude (deg)	Longitude (deg)
1	12.5	4760	65,13,148	41.488637	87.088686	41.403161	87.087074	41.489057	87.087713
				(Desired -Achieved) latitude & longitude		0.085476	0.001443	0.000420	0.000973
				Downrange error		7568 (m)		37.18 (m)	
2	12.7	4740	65,13,879	41.488637	87.088686	41.410232	87.087318	41.488035	87.087713
				(Desired -Achieved) latitude & longitude		0.078404	0.0013679	0.000601	0.000973
				Downrange error		6942 (m)		53.30 (m)	
3	12.9	4720	65,14,618	41.488637	87.088686	41.413949	87.087242	41.489240	87.087679
				(Desired -Achieved) latitude & longitude		0.074687	0.001611	0.000603	0.001007
				Downrange error		6613(m)		53.39(m)	

Case 1

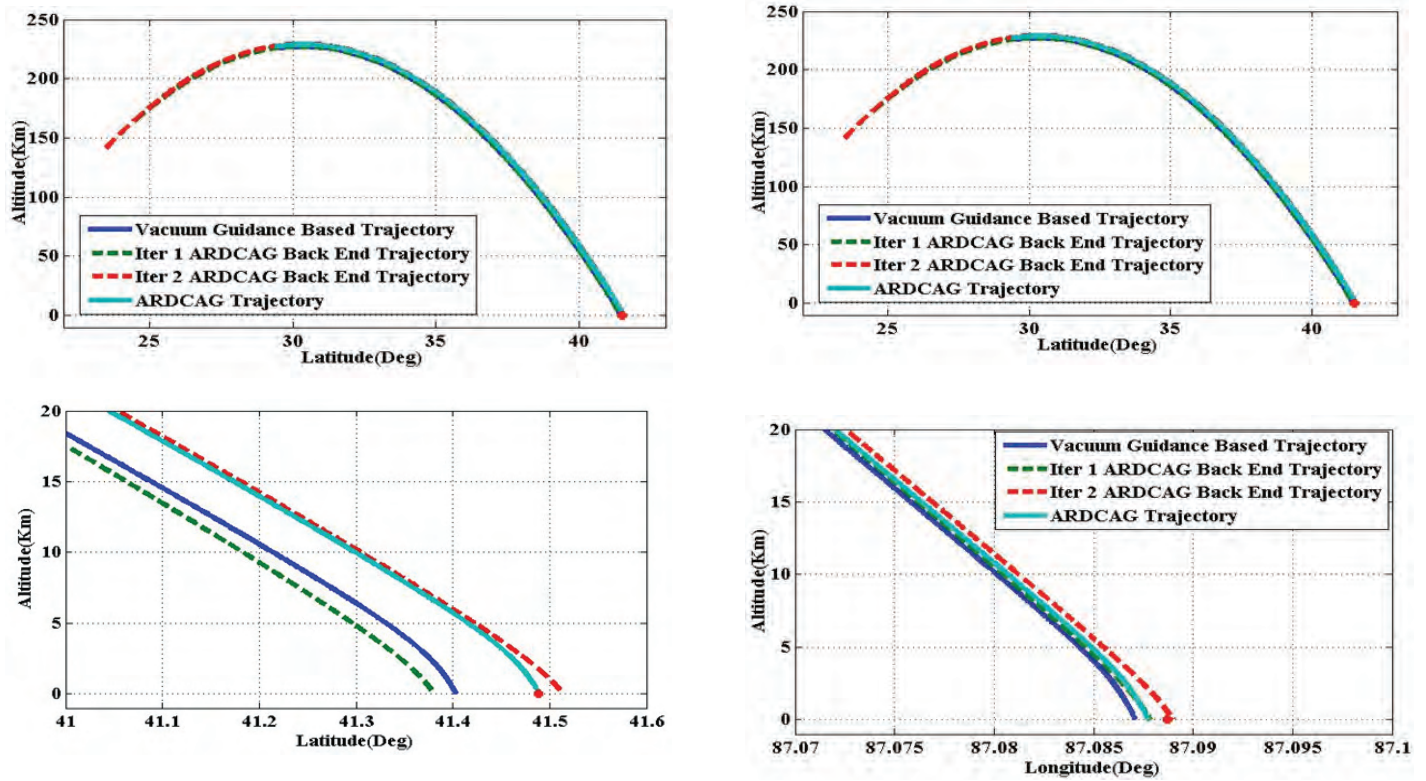


Figure 4. Case 1 altitude vs latitude and longitude trajectory.

Case 2

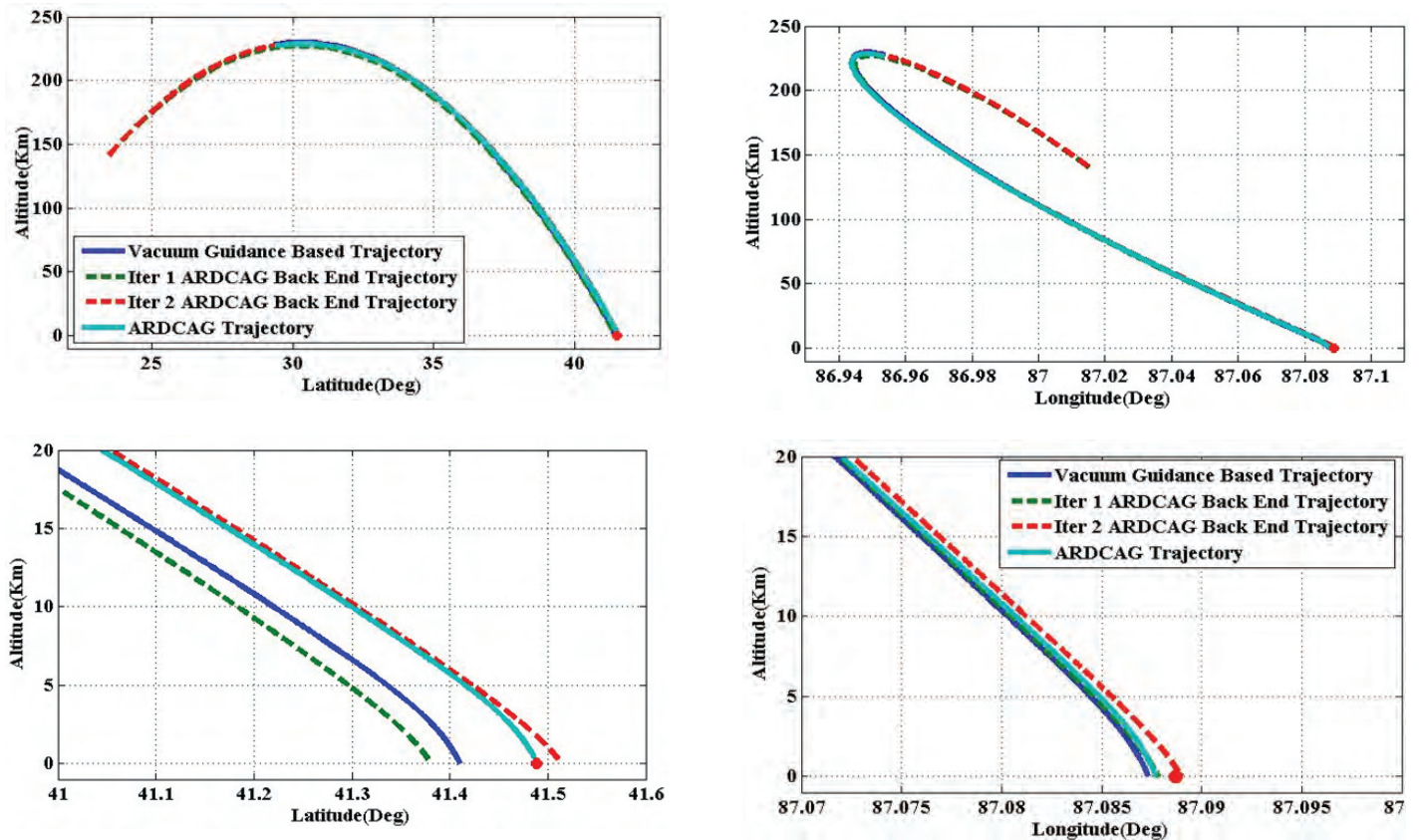


Figure 5. Case 2 altitude vs latitude and longitude trajectory.

Case 3

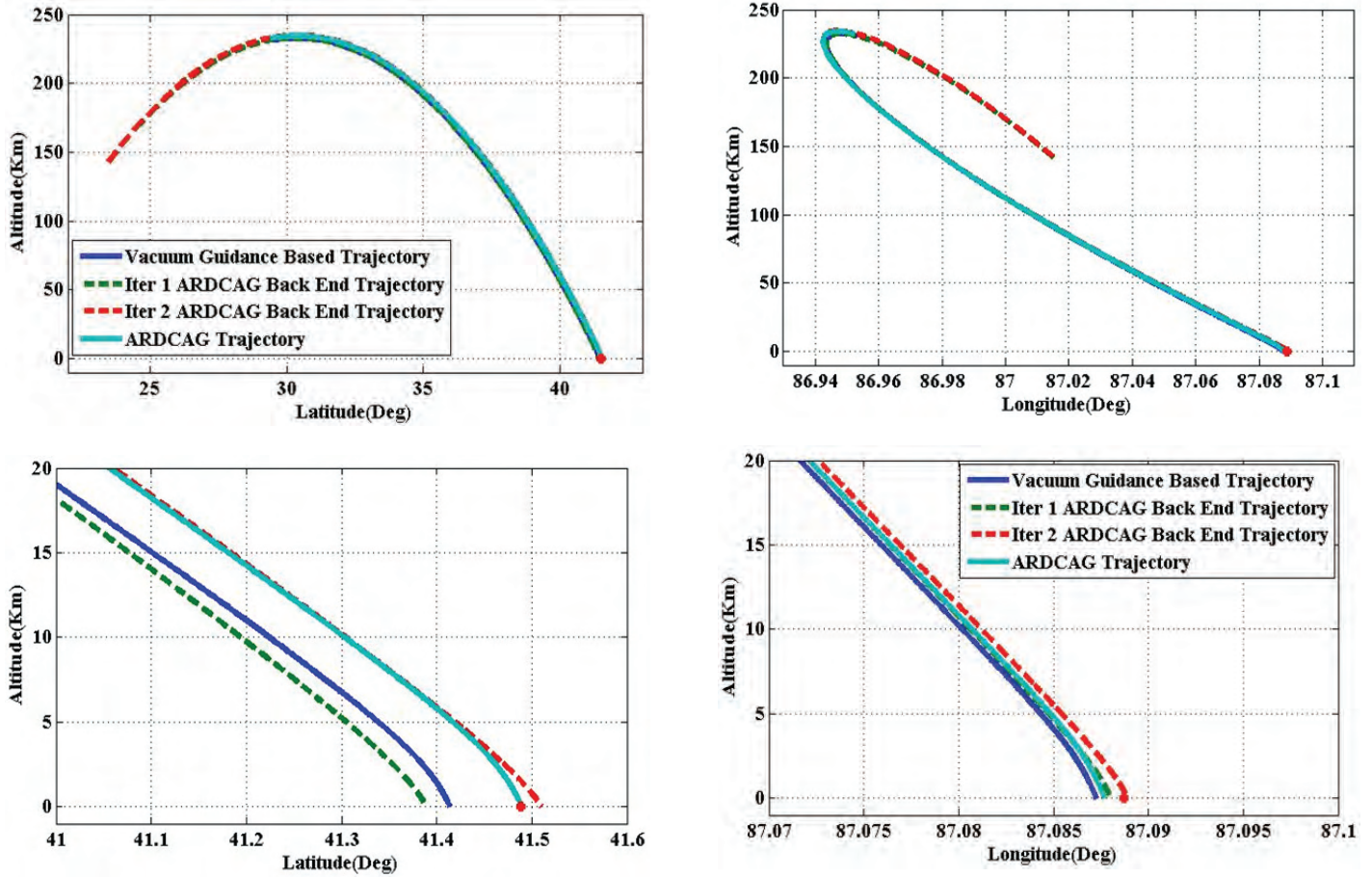


Figure 6. Case 3 altitude vs latitude and longitude trajectory.

defined in the inertial frame (Earth centred inertial). The evolution of the trajectory i.e., instantaneous latitude and longitude with respect to the altitude are shown in Figs. 4-6. Vacuum guidance based trajectory is the one which is obtained with vacuum guidance equations^{8,9}, i.e. without considering reentry atmospheric effects during the ascent phase guidance. Figures 4-6 show that the dispersion at the impact is well above the acceptable tolerance bounds because of non-consideration of reentry atmospheric effects during the ascent phase guidance. ARDCAG based trajectories are also shown in Figs. 4-6. Iteration 1 and Iteration 2 background 6-DOF trajectories provide the augmented coordinate input to the front end trajectory, by virtue of which the final front end trajectory converges to the desired coordinates at the impact point. The above scheme is easily implementable in real time embedded systems.

The robustness of the proposed algorithm under model uncertainty is studied by perturbing the wind, atmosphere and aero models. The case studies are listed in Table 2. The burnout state vector consider for the simulation studies shown in Table 3.

Table 4 shows the desired target point location, achieved terminal point location without reentry dispersion correction (vacuum guidance based trajectory), achieved terminal target point location with ARDCAG without any uncertainty in the

Table 2. Robustness of the proposed algorithm

Case	Percentage of variation on the nominal wind, density and drag (assumed model)		
	Wind (%)	Density (%)	Drag coefficient (%)
4	50	5	2
5	-50	-5	-2

Table 3. Burnout state vector for the simulation studies

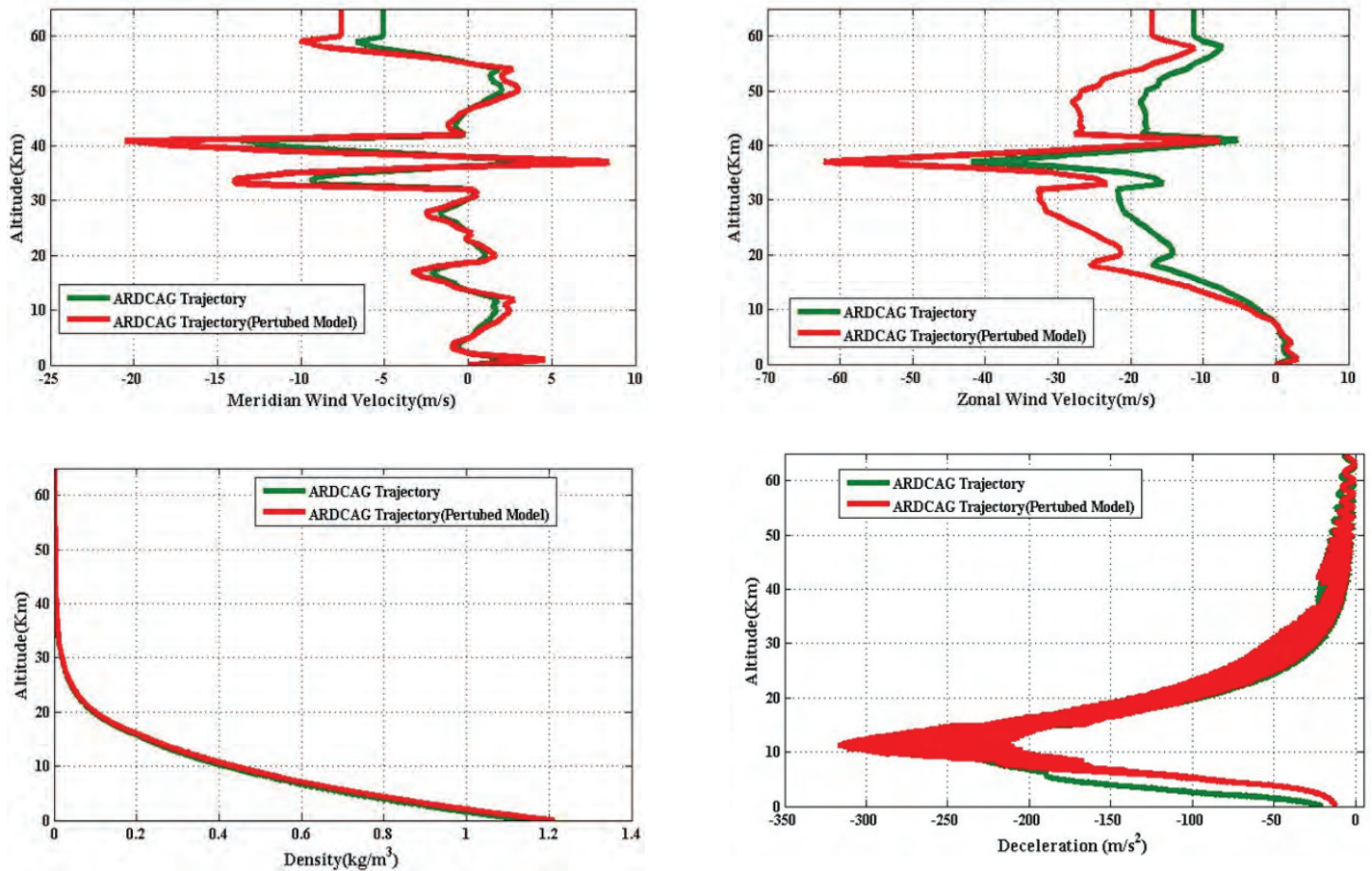
γ_{BO} (Angle between velocity vector and the local horizontal)	V_{BO}	P_{BO}
12.5 (D)	4760 (m/s)	6513148 (m)

models considered for simulation and achieved terminal target point location with ARDCAG trajectory with uncertainty in the models considered for simulation.

Figures 7 and 9 show the wind, density and drag variation with respect to the altitude. Figure 7 shows the wind model considered for background and the actual trajectory is perturbed by 50 per cent, density perturbed by 5 per cent and drag perturbed by 2 per cent. Figure 9 shows the wind

Table 4. Desired target point location, achieved terminal point location without reentry dispersion correction

Case	Desired terminal conditions		Achieved terminal conditions with ideal models (No atmospheric reentry dispersion correction trajectory)		Achieved terminal conditions with ideal models (ARDCAG trajectory)		Achieved terminal conditions (ARDCAG trajectory with perturbed models)	
	Latitude (deg)	Longitude (deg)	Latitude (deg)	Longitude (deg)	Latitude (deg)	Longitude (deg)	Latitude (deg)	Longitude (deg)
4	41.488637	87.088686	41.404718	87.085981	41.491414	87.086688	41.483460	87.085888
	(Desired -Achieved) latitude & longitude		0.0839	0.0027	-0.0028	0.0020	0.0052	0.0028
	Downrange error		6976 (m)		282 (m)		490 (m)	
5	41.488637	87.088686	41.404718	87.085981	41.491414	87.086688	41.498093	87.088395
	(Desired -Achieved) latitude & longitude		0.0839	0.0027	-0.0028	0.0020	-0.0095	0.0002
	Downrange error		6976 (m)		282 (m)		789 (m)	

Case 4**Figure 7. Case 4 wind, density and drag variation vs altitude (with and without perturbation).**

model considered for background and the actual trajectory is perturbed by -50 per cent, density perturbed by -5 per cent and drag perturbed by -2 per cent. Figs. 8 and 9 shows the latitude and longitude achieved when dispersion correction is

not considered, and when dispersion is considered (ARDCAG) with and without uncertainty in the models. From the simulation results it clear that as the uncertainty band is increasing the order of the error is increasing. But the order of the error at the

Case 4

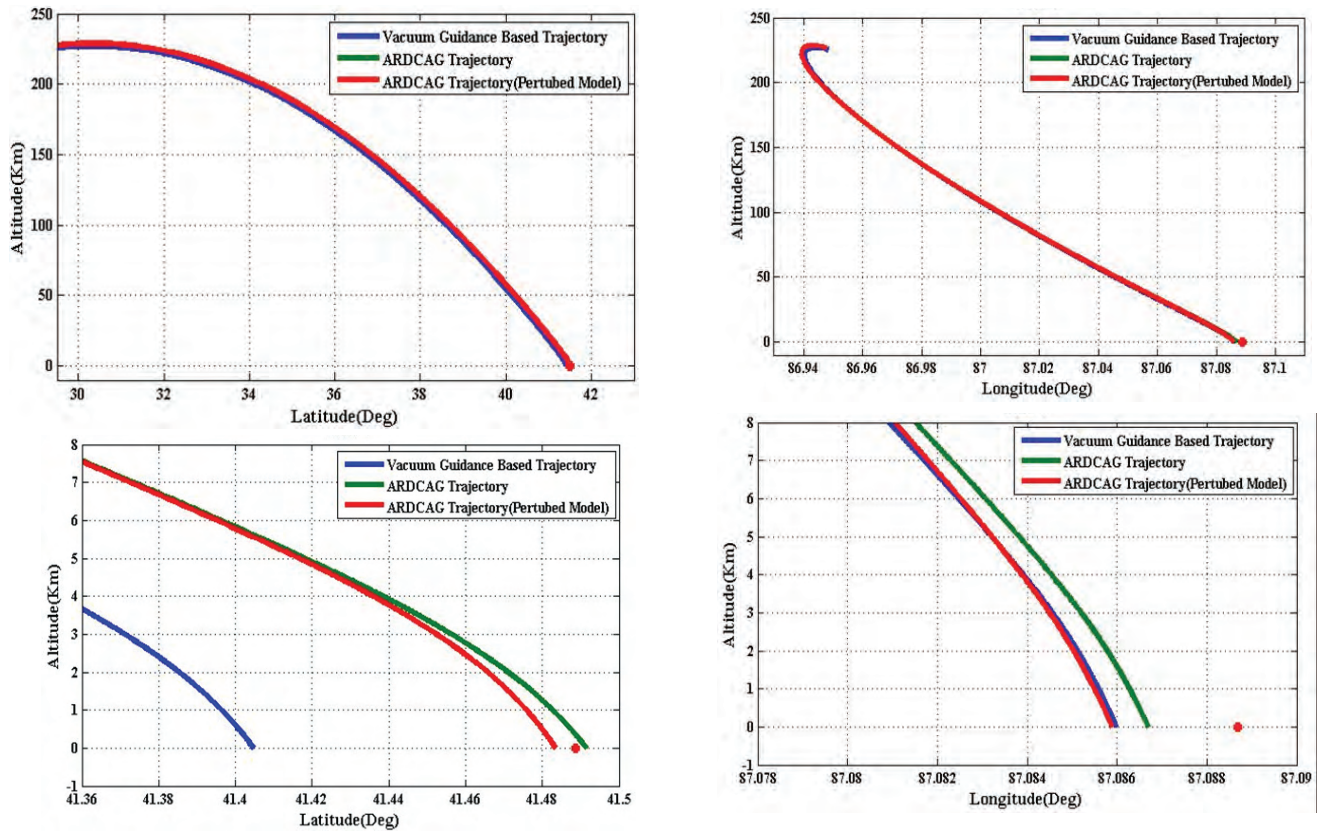


Figure 8. Case 4 altitude vs latitude and longitude trajectory.

Case 5

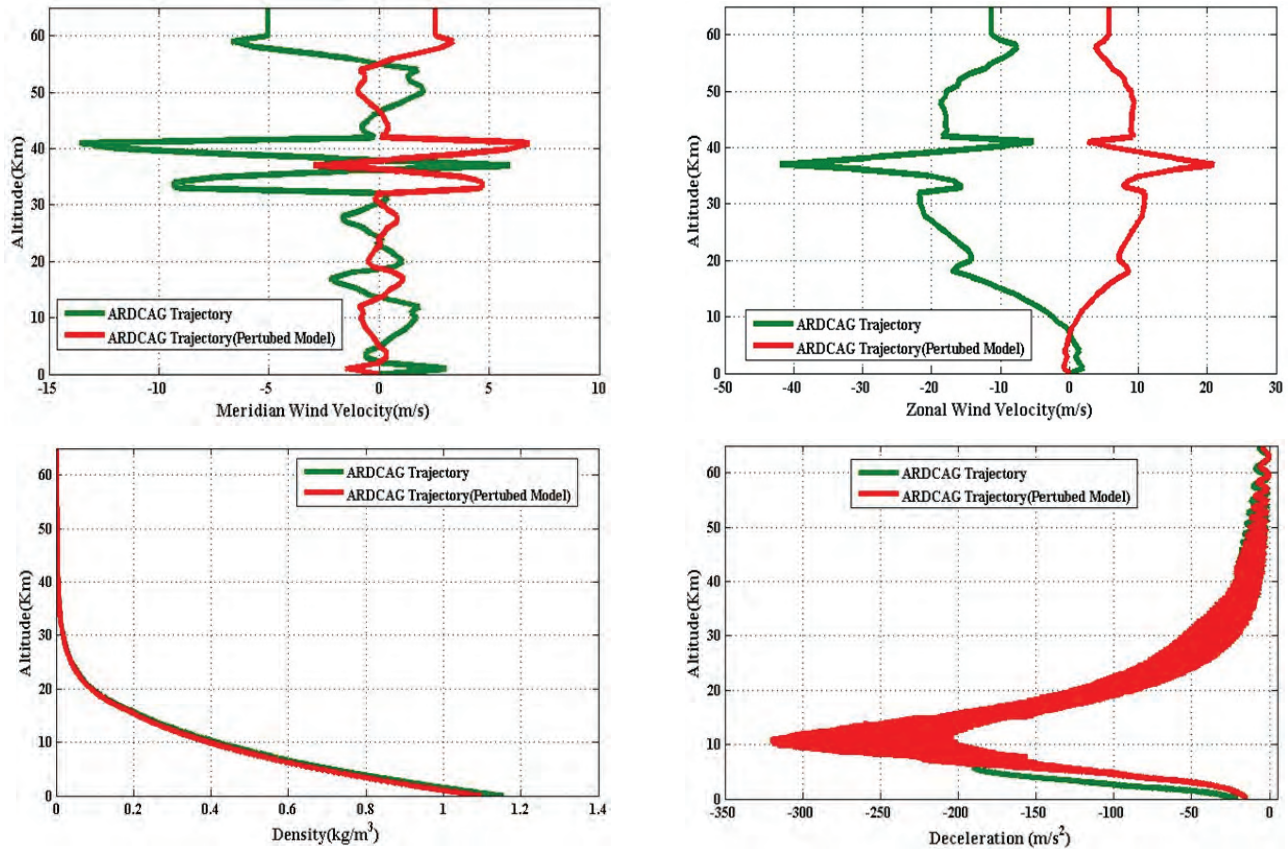


Figure 9. Case 5 wind, density and drag variation vs altitude (with and without perturbation).

Case 5

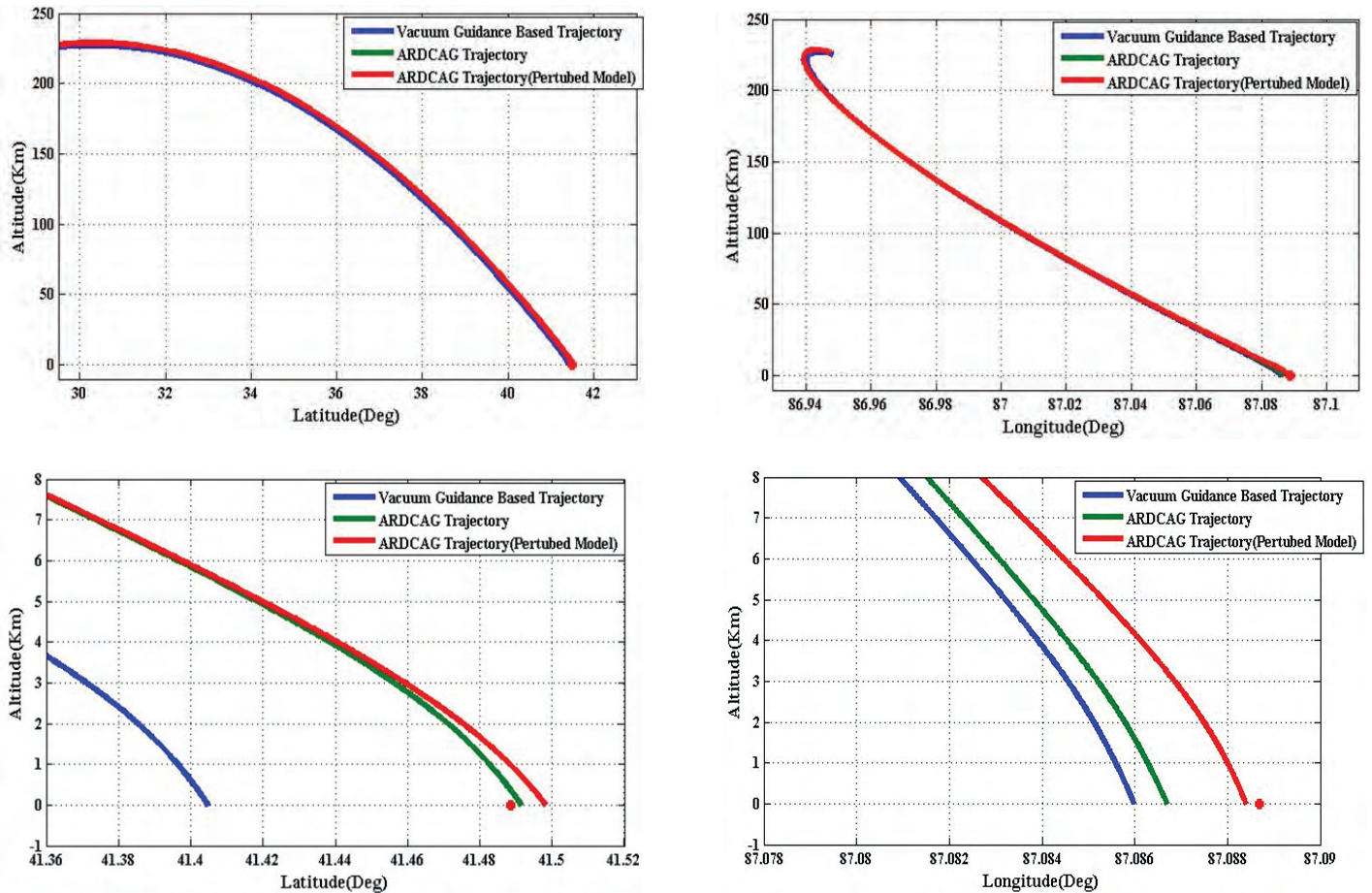


Figure 10. Case 5 altitude vs latitude and longitude trajectory.

final target achieved by ARDCAG trajectory due to uncertainty in the modelling is considerably less as compared to the error caused by the trajectory which is generated using vacuum based guidance strategy. Deterministic dispersion based on inflight achieved state vector are dominant than the variable/random/uncertainty effects due to wind, atmospheric, aerodynamic coefficient modeling. Randomly varying terms such as wind, atmospheric density, drag uncertainty cause second order variations for which covariance error propagation can be used to assess ultimate CEP¹⁵.

5. CONCLUSION

A practically implementable atmospheric reentry dispersion correction ascent phase (ARDCAG) guidance is proposed through which the target point is achieved accurately. The robustness of the algorithm is validated with different burnout conditions and the results are tabulated. The final terminal accuracy depends on the models (aero, atmospheric, earth oblateness, etc.) considered in the 6-DOF simulation used in back ground and the accuracy by which the final burnout conditions are predicted and achieved. Robustness of the proposed algorithm is validated with uncertainty studies, which shows the algorithm is effective under modelling uncertainties. The proposed algorithm can be used for a multi-stage vehicle guidance, where the vehicle is guided in multiple

stages.

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