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ON PRESSURE RISE IN ROCKETS

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ABSTRACT

The phenomenon of pressure rise in rockets has been discussed on the basis of $r=CP^n$ as the law of burning. An explicit expression for the chamber pressure at any instant has been derived and illustrated by a table and a graph.

INTRODUCTION

During burning in a rocket the mass rate of burning is equal to the mass rate of discharge plus the mass rate of accumulation in the chamber, that is, (Kershner, 1944)

$$S\rho r = C_D \ A_t \ P + \frac{d}{dt}\left(V\rho \right) \qquad (1)$$

where S is the area of the burning surface of the propellant.

ρ is the density of the propellant which is about ·057—·059 lb/in^3 for most of the propellants.

r is the rate of surface regression in in/sec.

C_D is the discharge coefficient which is about ·007 per sec. for most rockets using smokeless powder as propellant.

A_t is the area of the throat,

P is the chamber pressure,

ρ_g is the density of the gas in the chamber.

V is the volume of the chamber available to the gas. Putting $\frac{dv}{dt}=Sr$ and $\frac{d}{dt}\left(\rho_g\right)=0$ for a steady state and assuming $r=CP^n$ as the law of burning we get the steady state solution of equation (1) as

$$P_{eq}=\left(\frac{K\rho' C}{C}\right)^{\frac{1}{1-n}} \qquad (2)$$

where P_{eq} is the equilibrium pressure,

K is S/A_t and $\rho'=\rho-\rho_g$

Kershner (1944) has discussed the phenomenon of pressure rise in rockets, assuming a linear law of burning. In this communication the authors have discussed the phenomenon on another equally valid law of burning viz. $r=CP^n$.

Pressure Rise in Rockets

We may assume $\rho_g = BP$ i.e. a constant chamber temperature as has been done by Kershner (1944) in his calculations based on the linear law of burning. Putting $r = CP^n$ and

$\frac{dv}{dt} = Sr$ in equation (1) we get

$$S\rho' CP^n = C_D A_t P + BV \frac{dP}{dt} \quad (3)$$

or $$\frac{BV}{C_D A_t} \frac{1}{P^n} \frac{dP}{dt} + \frac{1}{P^{n-1}} = \frac{K\rho' C}{C_D} = P_{eq}^{1-n} \text{ from equation 2}$$

putting $Z = P^{(1-n)}$

$$\frac{BV}{(1-n) C A_t} \frac{dZ}{dt} + Z = P_{eq}^{1-n}$$

or $$\frac{dZ}{dt} + aZ = aP_{eq}^{1-n} \quad \text{where} \quad a = \frac{C_D (1-n)}{BV}$$

The solution of the above differential equation is

$$Ze^{at} = P_{eq}^{1-n} e^{at} + A$$

where A is the constant of integration.

Applying the boundary condition

$P = 0$ when $t = 0$,

$$Z = P^{1-n} = P_{eq}^{1-n}\left(1 - e^{-at}\right)$$

$$P/P_{eq} = \left(1 - e^{-at}\right)^{1/(1-n)} \quad (4)$$

Putting $$b = \frac{a}{1-n} = \frac{C_D A_t}{BV} \quad (5)$$

$$P/P_{eq} = \left(1 - e^{-(1-n)\,bt}\right)^{1/(1-n)} \quad (6)$$

The variation of P/P_{eq} with bt for various common values of n (0·40—0·80) is illustrated by table I and Figure I.

TABLE I

Values of P/P_{eq} for various values of bt and n

n \ bt	0·5	1·0	2·0	3·0	4·0	6·0	8·0	10·0	15·0	20·0	25·0
·4	·1053	·2655	·5503	·7401	·8535	·9550	·9863	·9972	1	1	1
·5	·0489	·1548	·3996	·6037	·7478	·9028	·9638	·9863	·9991	1	1
·6	·0140	·0624	·2250	·4083	·5692	·7883	·9010	·9550	·9936	·9995	1
·7	·0014	·0114	·0705	·1757	·3030	·5479	·7283	·8433	·9636	·9915	·9984
·75	·0002	·0024	·0240	·0775	·1600	·3643	·5591	·7099	·9093	·9735	·9952
·8	0	·0002	·0039	·0187	·0506	·1667	·3240	·4837	·7745	·9120	·9661

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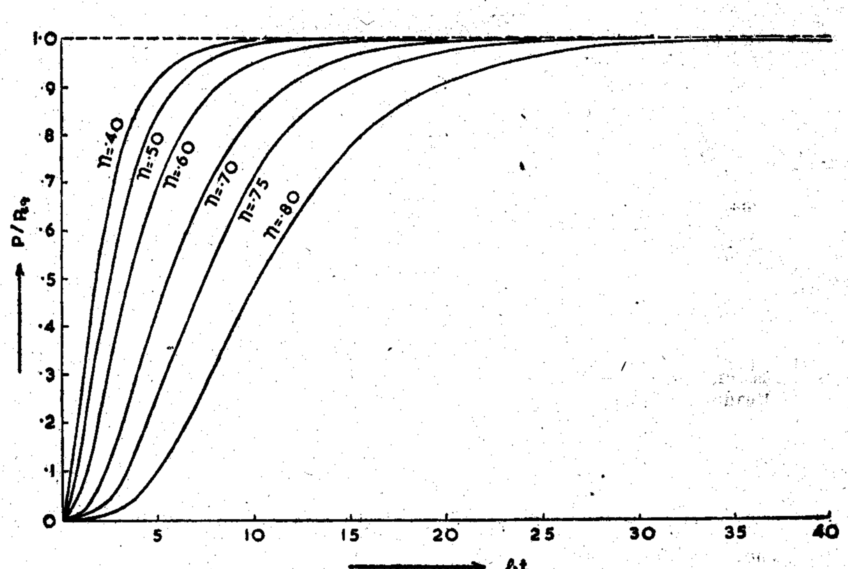

FIG.1 DEPENDENCE OF P/Peq ON bt AND n