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A NOTE ON EFFICIENCY OF ROCKETS.

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ABSTRACT

This paper discusses the dependence of efficiency of a rocket on the expansion ratio and the ratio of the initial mass of the rocket and the mass of the propellant. The relation is illustrated by a table and two graphs.

The efficiency of a rocket, η, is defined as the ratio of the kinetic energy of the rocket at the end of burning to the heat energy of the propellant. Thus (Lakatos, 1941)

$$\eta = \frac{\frac{1}{2}(M-m)v^2}{J.H.\,m} \tag{1}$$

where M is the initial mass of the rocket,

m is the mass of the propellant,

v_b is the velocity of the rocket at the end of burning,

H is the heat of combustion of the propellant and J is the mechanical equivalent of heat.

The velocity at the end of burning is given by

$$v_b = v_e \log_e \frac{M}{M-m} \tag{2}$$

where v_e is the effective exhaust velocity given by

$$v_e = \frac{2\gamma RT}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{3}$$

where P_e and P_c are the exit and chamber pressures respectively.

From equations (1), (2) and (3) Lakatos (1941) arrives at the following expression for the efficiency of rockets

$$\eta = f\left(\frac{P_e}{P_c}\right)\cdot\phi(\alpha) \tag{4}$$

where $\alpha = \frac{M}{m}$

$$\phi(\alpha) = (\alpha-1)\left[\log_e \frac{\alpha}{\alpha-1}\right]^2 \tag{5}$$

$$f\left(\frac{P_e}{P_c}\right) = \left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{6}$$

Thus the variation of the efficiency with P_e/P_c for various values of α can be studied with the help of eqn. (4). However, P_e/P_c is not specified for a given rocket while it is related to a fundamental specification A_e/A_t (A_e and A_t are the areas of the exit and throat of the nozzle) by

$$\frac{A_t}{A_e}=\left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}\left(\frac{P_e}{P_c}\right)^{\frac{1}{\gamma}}\sqrt{\left(\frac{\gamma+1}{\gamma-1}\right)\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad (7)$$

In this paper the authors have given a table and a graph expressing the variation of the efficiency with the fundamental specification $\frac{A_e}{A_t}$ of the rocket for various values of α which will be more useful. Taking $\gamma=1\cdot25$, for different values of α and P_c/P the authors have calculated both A_e/A_t and the efficiency η. Table 1 gives the variation of the efficiency with α and A_e/A_t which is illustrated graphically by figures I and II.

TABLE 1

Variation of efficiency η with A_e/A_t and α

A_e/A_t \ α	3	4	5	6	7	8	9
2·16	0·1213	0·0915	0·0735	0·0614	0·0526	0·0461	0·0410
2·81	0·1375	0·1036	0·0833	0·0695	0·0596	0·0522	0·0460
3·40	0·1482	0·1117	0·0898	0·0750	0·0643	0·0563	0·0500
4·50	0·1623	0·1223	0·0983	0·0821	0·0704	0·0616	0·0548
5·51	0·1716	0·1293	0·1039	0·0868	0·0744	0·0652	0·0579
6·33	0·1785	0·1345	0·1081	0·0902	0·0774	0·0678	0·0602
7·36	0·1838	0·1386	0·1135	0·0930	0·0797	0·0698	0·0620
9·07	0·1920	0·1447	0·1163	0·0971	0·0833	0·0729	0·0648

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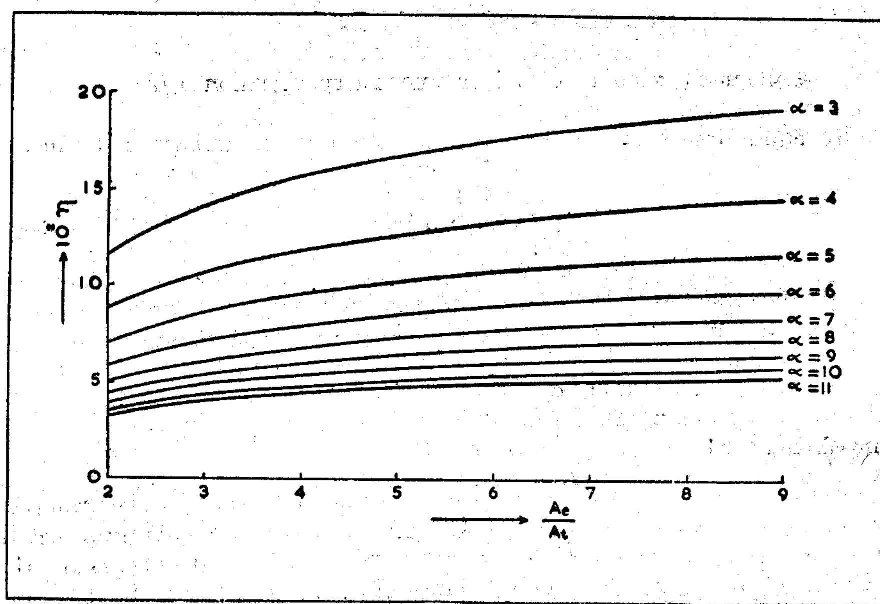
20
15
10
5
0
$10^3\eta$
$\alpha = 3$
$\alpha = 4$
$\alpha = 5$
$\alpha = 6$
$\alpha = 7$
$\alpha = 8$
$\alpha = 9$
$\alpha = 10$
$\alpha = 11$
2
3
4
5
6
7
8
9
A_e/A_t

Fig. I

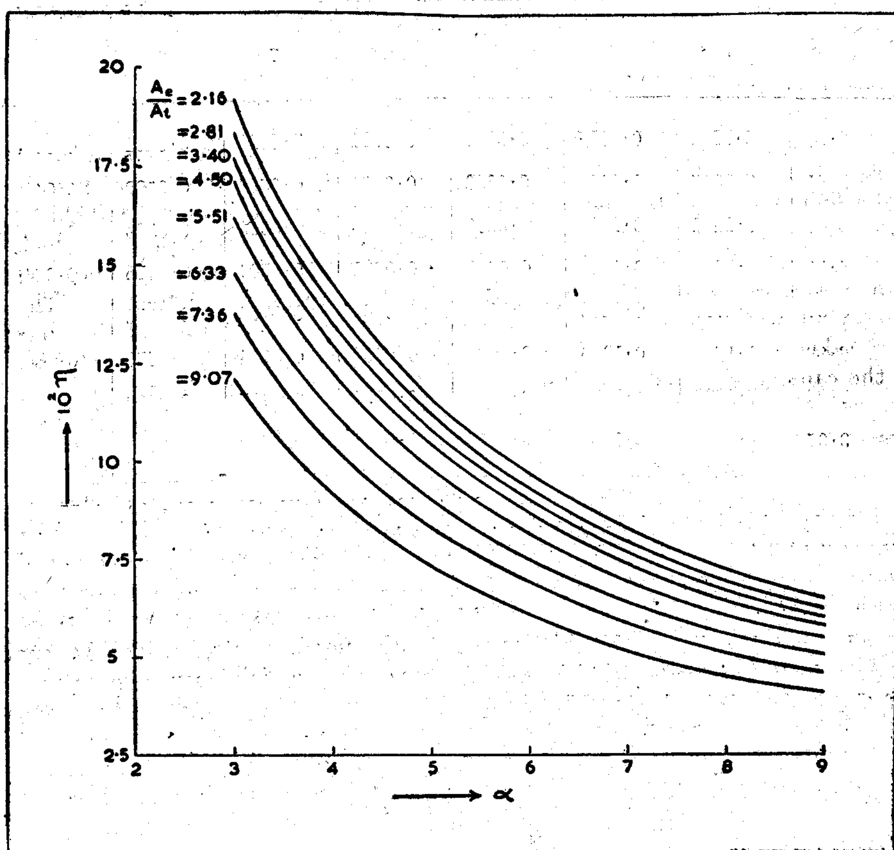
20
17·5
15
12·5
10
7·5
5
2·5
$10^3\eta$
$A_e/A_t = 2{\cdot}16$
$= 2{\cdot}81$
$= 3{\cdot}40$
$= 4{\cdot}50$
$= 5{\cdot}51$
$= 6{\cdot}33$
$= 7{\cdot}36$
$= 9{\cdot}07$
2
3
4
5
6
7
8
9
α

Fig. II