Defence Science Journal, Vol. 4, No.1, January 1954, pp. 15-18

ON THE ADIABATIC EXPANSION OF THE REACTION PRODUCTS INSIDE THE GUN

By Mr. M. P. Murgai, Defence Science Organisation

ABSTRACT

The change in composition of the products of reaction inside the gun, during their adiabatic expansion, from the position of the all burnt to the muzzle has been considered and the temperature of the products, when they reach the muzzle, determined. While the maximum change of composition is 10 per cent., the difference of temperature at the muzzle from that calculated on the basis of constant composition is only 1 per cent.

Introduction

The solution of the fundamental equations of internal ballistics leads to the knowledge of the position of the all burnt and the pressure of the gases at that time. The temperature does not enter these equations as such and becomes known from the above and an assumed equation of state. From this follows the composition of the gases at the instant of all burnt. After the whole of the charge has been burnt away, the products expand more or less adiabatically, and their subsequent temperature is generally determined from the consideration of this adiabatic expansion, assuming the composition to remain fixed. The temperature and composition are however linked, one depending upon the other, and it is worth while therefore finding out the effect of the change of composition on temperature during the expansion process. The present is an attempt at calculating the temperature at the muzzle taking into account the change in the composition of the product gases. The calculations apply to the propellant W. T. The temperature during the expansion does not admit of any dissociation of the products, and if no Nitrogen compounds are assumed to be found, the amount of nitrogen remaining constant, the composition is determined by the water gas equilibrium.

DETERMINATION OF THE INITIAL CONDITIONS.

2.1 Pressure and Volume of the Gases at the All Burnt :—

The distance of the position of the all burnt from the Chamber end for the approximation $B = o$ ($B = b - \frac{1}{\delta}$, b being the co-volume of the product gases, δ the Sp. gravity of the propellant) and the pressure of the products of explosion at that time are given in terms of the dimensionless variables ξ'_2 $\left[\xi_2 = l + \frac{x_2}{b} \right.$ x_2 being the distance of the all burnt position from chamber end, and $l = \frac{K_o - \frac{c}{\delta}}{A}$ ko the chamber volume, c the charge wt. A is the average bore cross-section. If v_b is the bore volume and S_T the shot travel, A is given by $\left. (v_b / S_T) \right]$ and ξ_2 $(= p \frac{FC}{Al}$, p being the pressure and F the force constant of the propellant) by

$$\xi_2'^{(1-\gamma)} = \left[1-(\gamma-1)\ MJ-Ky \right] \text{ and}$$

$$\xi_2 = \frac{\phi'}{(1-BC'_2)(\xi'_2)^r}$$

J and K being functions of ξ_o (dimension-less constant proportional to shot start pressure) and θ the form factor and Y a function of $M(\gamma-1)$; M being the central Ballistic constant $= \frac{A^2D^2}{FC\beta^2W_1}$ in which D is the webb size of the propellant, β the rate of burning constant, w_1, the corrected shot weight.

Φ' and C'_2 are given from

$$\frac{1}{\Phi} = 1-(\gamma-1)MI+Gy$$

in which I and G are functions of ξ_o and θ and

$$C_2' = \frac{(1+\theta)^2+2\cdot5M\rho_o}{[1+\theta+M(\cdot28-0\cdot086\theta)]^2}$$

The various functions are tabulated in (HMSP)[1].

From the above the volume and pressure of the gases at the time of all burnt is given by

$$v = Ko+x_2A$$

$$p = \frac{Al}{FC}\xi_2'$$

Total no. of Gm. Moles (n) of the Gaseous Mixture.

It turns out that under the assumptions outlined above the total number of gm. moles n, is independent of temperature and pressure corrections to the equilibrium constant, and depends only on the composition of the propellant being given by

$$n = x + \frac{y}{2} + \frac{w}{2}\text{ ; x,y,w being}$$

the no. of gm. atoms of C, H and N in the whole charge.

Temperature at the All Burnt.

With the knowledge of p, v and n, T can be found from an assumed equation of state. For the pressure prevalent inside the gun it is fairly accurate to use the following equation

$$\frac{pv}{nRT} = 1 + \frac{b}{v} + \frac{nc}{v^2}$$

in which b and c are second and third virial coefficient for the mixture $= \Sigma$ ni B and Σ ni c

For the purpose of calculation of temperature however b and c may be but$=n\overline{B}_1$ and $n\overline{C}_1$ respectively; $\overline{B}_1$ and $\overline{C}_1$ being the average values for the constituent mixture at 2500°K. Their values have been tabulated by Corner[2].

Gas Composition :—

The gas composition at any temperature is given by the following equations.

$$\alpha+\beta=x$$
$$2\alpha+\beta+\gamma=z$$
$$2\gamma+2\delta=y$$
$$2\varepsilon=w$$
$$\frac{\beta\gamma}{\alpha\delta}=K'=K_c f(v),$$

Where α, β, γ, δ and ε are the no. of gm. moles of Co_2, Co, H_2o, H_2 and N_2 respectively, K′ the equilibrium constant for the water-gas reaction and f(v), a function of v and a correction for Ko to get K′. f(v) can be calculated from the theory of chemical equilibrium and is given by

$$f(v) = \exp\left[-n\,\frac{\triangle B}{v}-n^2\,\frac{\triangle c}{2v^2}\right]$$

△ B and △ C being the difference in the second and third coefficient of the water gas.

These equations lead to $\alpha^2(K'-1)+\alpha\left[K\left(x+\frac{y}{2}-z\right)+z\right]+x(x-z)=o$

from which α and hence β, γ and δ can be found out.

3. ADIABATIC EXPANSION.

The entropy S of gaseous mixture in terms of V and T is given by

$$S=\int_V^{\infty}\left[\frac{\Sigma n_i\;R}{v}-\left(\frac{\partial p}{\partial T}\right)_{vn}\right]dv+\Sigma\, ni\, R\ln\frac{v}{n_i R T_\circ}+\Sigma\, niS^{\circ}_{i}$$

where

$$S_i{}^{\circ}=S^{\circ}_{\circ i}+\int_{To}^{T}\frac{\overset{x}{Cv_i}}{T}\,dT+R\ln\frac{T}{T_\circ}$$

in which $S^{\circ}_{\circ i}$ is the entropy of the constituent i at the reference temperature To (in this case=300°k) and $\overset{x}{Cvi}$ is the Sp. heat at constant volume of the constituent i at infinite volume. If $\overset{-x}{Cv_i}$ is the average value of the sp. heat between $T_\circ$ and T, $\int_{To}^{T}\frac{\overset{x}{Cvi}}{T}dT=\overset{-x}{Cvi}\log\frac{T}{T_\circ}$. Evaluating the first integral,

$$S-\Sigma niS^{\circ}_{\circ i}=\frac{1}{\cdot 4343}\Big[R\left\{n\log v-n\log R-\Sigma\;ni\log ni\right\}$$
$$+\Sigma ni\;\overset{-x}{Cv},\;\log\;T-\log T_\circ\,(nR+\Sigma\, ni\,\overset{-x}{Cvi})\Big]-nR\left(\frac{b}{v}+\frac{1}{2}\frac{nc}{v2}\right)$$

When the products reach the muzzle the volume of the gases and a suitable T has to be found by trial and error so that S is the same.

CALCULATIONS FOR 25 pr.

The relevant data for a 25 pr gun for super charge W. T. is given below :—

Calibre .. .. .. = 3·45″
Chamber Capacity K_o .. = 151 c. inches.
Bore Capacity .. .. = 896 c. inches.
Shot travel—l .. .. = 77·905 inches.
Average bore area—A .. .. = 11·50 sq. inches.
Four factor θ .. .. = 0
Sp. gravity of the propellant δ .. = 1/17·3
Connected Pressure index β_1 .. = 1·08
Charge weight C .. .. = 2·531 lbs.
Force Constant F .. .. = 2010 inch/tons/lbs.
Web size D .. .. .. = ·053 inches.
Corrected Shot Wt W1 .. = 27·094 lbs.
γ .. .. .. = 1·239.
Shot Start Pressure, Po .. = 2 tons/sq″.
M .. .. .. = 1·693.
ξ'_2 .. .. .. = 4·655.
ξ_2 .. .. .. = 4·343.
Position of all burnt X_2 .. = 31·163″.
Φ' .. .. .. = 1·1817.
ξ_2 .. .. .. = 0·1979.
Pressure at the all burnt p_2 .. = 9.391 tons/sq″.

Atomic composition of the propellant in gm. atoms/gm. of the propellant.

x=0·02156 y = ·02800 z = ·03548 w = ·01036.
Temperature at the all burnt = 2718
Entropy of the products = 674·37 cal/degree.
Volume of the gases at the muzzle = 1047 c. inches.
The result of the calculations is tabulated below

TABLE VI

Temperature °K	Composition in no. of gm. moles				
	CO_2	CO	H_2	HO	N_2
2718 (All burnt)	5·12	19·63	10·66	5·22	5·95
2280 (Muzzle)	5·70	19·08	10·42	5·80	5·95

The temperature at the muzzle calculated from the formula $T(v—b)^{\gamma-1}$ constant, comes out to bw 2250°K

REFERENCES.

1. Internal Ballistics—His Majesty's Stationery Office Publication, 1951.
2. Corner J ; Proc. Phy. Soc. 58 1946, 737.