Defence Science Journal, Vol. 2, No. 4, October 1952, pp.201-207

THE INTERNAL BALLISTICS OF A LEAKING GUN*

1. Introduction.

The problems arising in the internal ballistics of guns may be classified under two broad categories :— (*a*) physico-chemical problems, (*b*) dynamical problem. The former deals with such questions as the thermochemistry of the propellant gases and the rate of burning of a propellant ; how the rate of burning depends on the composition, initial temperature and pressure. Among the various theories proposed, we may mention here the theory of Rice and Boys and Corner treating the burning of the propellant as an advancing flame zone in a gas. By incorporating certain hydrodynamical considerations on turbulence into the theory, Corner was able to arrive at an estimate of the effect of "erosion" of the propellant gases on the walls of the gun.

The dynamical problem of the motion of the shot in the gun, which otherwise would be simple, is rendered complicated as it has to take account of—

(1) the irreversible processes in the gun such as the physical shot—start pressure, the bore resistance and the heat—loss to the barrel, and

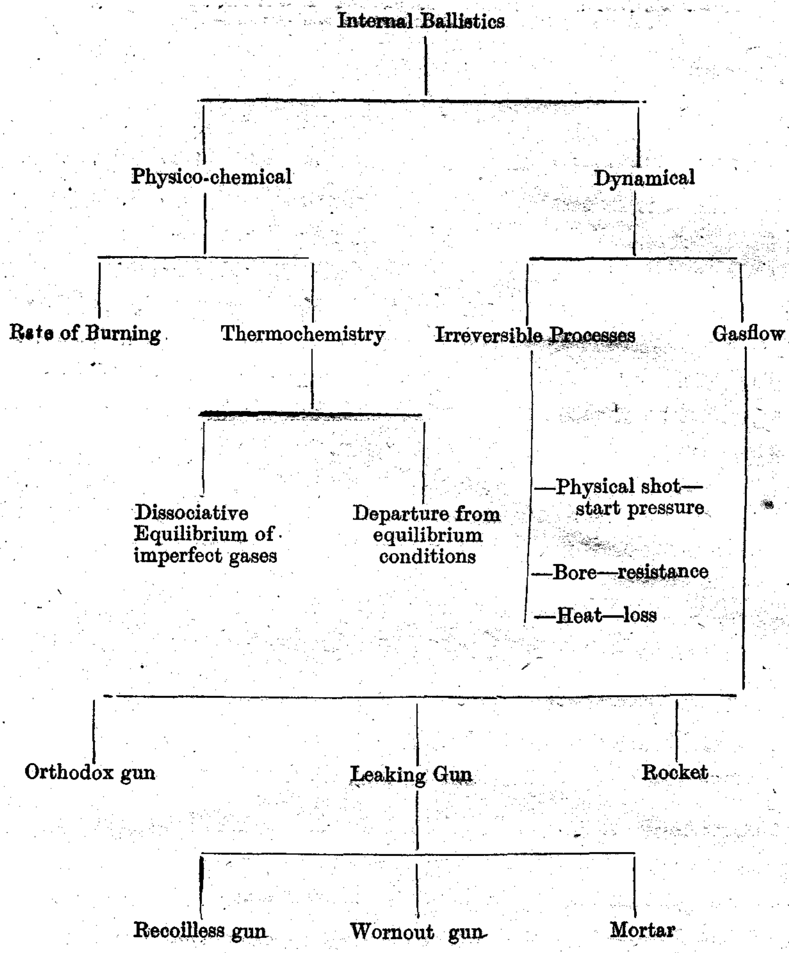
(2) the flow of the gas behind the shot.
In the theory of an orthodox gun, the effect of the gas-flow is taken account of by the so-called " Lagrange corrections ".

The problem of gas-flow assumes a greater importance in the theory of leaking guns and of rockets. In recoilless guns and rockets, a suitably designed venturi at the rear end of the chamber allows the gases to escape, thus providing automatically for the absorption of recoil in guns and for providing the necessary forward thrust in rockets. Again, in a worn gun in which there is a long "runup" before the band engages the rifling, and in a smooth-bore mortar in which the diameter of the projectile is less than the caliber, there is a leakage of gas past the projectile. In all these cases, the problem of the gas flow is then the familiar hydrodynamical problem of nozzle-flow.

In this article a brief account is given of the theory of internal ballistics of leaking guns. The following schematic diagram will serve to make clear the interconnections of the various branches of the subject, attention to which has been drawn in the proceeding paragraphs.

---

* Based on a talk given in the Defence Science Organisation by Dr. D. S. Kothari, Scientific Adviser to the Ministry of Defence.



§ 2. **Basic Equations.**—As in the theory of orthodox guns, we assume

(1) rate of burning equal to $\beta\, p^{\alpha}$

(2) inclusion of covolume independent of temperature

(3) neglect of bore resistance, replacing it by either a shot-start pressure P_0 or an equivalent correction to β.

In addition we introduce the following assumptions peculiar to a leaking gun :

(4) No unburnt propellant is lost through the nozzle.

(5) The gas-flow may be treated as quasi-steady.
(*i.e.* the flow would be the same under the instantaneous pressures as under steady pressures).

(6) The setting up of the backward gas flow by the bursting of the sealing disc can be assumed to begin instantaneously at a certain pressure called the nozzle-start pressure (P_N).

The Lagrange corrections mentioned above in the introduction are derived from the conventional approximation which assumes that the density of the gas is independent of position in the gun. For a gun with leakage some allowance must be made for the fact that the quantity of gas in the gun does not increase up to the full value and also for the change in the velocity-distribution in the gas if there is backward leaking. These factors may be taken account of sufficiently by replacing C by kC (N+1—z) where C is the mass of charge, CN is the mass of gas in the gun at any instant and k is a semi-empirical numerical factor. Actually, it is found that it is more convenient to use kCN instead of kC ≈ (N+1—z), this replacement altering the muzzle velocity and the peak pressure by not more than 1% in most cases. We thus find :

$$\left.\begin{array}{l} \text{space-mean pressure} = \dfrac{P}{1 + kCN/6W} \\ \text{Pressure at base of shot} = \dfrac{P}{1 + kCN/2W} \end{array}\right\} \quad .. \quad (1)$$

where P is the maximum pressure anywhere inside the gun at the instant considered.

The classical theory of nozzle flow furnishes the following expression for the mass-flux :

$$Q = \psi p_r \, S \, (R\, T_r)^{-\frac{1}{2}} \quad .. \quad .. \quad (2)$$

where p_r, T_r represent the pressure and temperature in the reservoir, S is the area of the throat, and ψ is a numerical factor depending on γ and whose value for all service propellants is about 0·66. Allowing for friction and heat-loss, the value of ψ is taken to be 0·63. We indentify p_r with P and T_r with T. We have then the following equations—

Equation of gas-flux :

$$C \frac{dN}{dt} = C \frac{dz}{dt} - \psi \frac{SP}{(RT)^{\frac{1}{2}}} \quad .. \quad .. \quad (3)$$

*Energy equation** :

$$\frac{d}{dt}(NT) = -(\bar{\gamma}-1)\frac{AP}{CR}\frac{dx}{dt} + T_0\frac{dz}{dt} - [\gamma + (\gamma-1)\epsilon]\,\psi\frac{SP}{CR}(RT)^{\frac{1}{2}} \quad (4)$$

where $\varepsilon = \eta\, \rho_r/(1-\eta\rho_r)$, η = Covolume.

The term in ϵ can be omitted with a relative error of 7 % at $\varepsilon = 0{\cdot}35$. In the first term on the right of the energy equation $\bar{\gamma}-1$ has been written, as usual, instead of $\gamma-1$ in order to take into account heat losses. Omitting the ε-term, the energy equation becomes :

$$\frac{d}{dt}\left(NT\right) = -\left(\bar{\gamma}-1\right)\frac{AP}{CR}\frac{dx}{dt} + T_0\frac{dz}{dt} - \frac{\gamma\psi SP}{RC}(RT)^{\frac{1}{2}} \quad .. \quad (4a)$$

Equation of state :

$$P\left(K_0 + A\,x - \frac{C}{\delta}\right) = NCRT\left(1 + \frac{kCN}{6W}\right) \quad .. \quad (5)$$

Here a term $(z/\delta - N\eta)$ has been omitted as being negligible for recoilless guns. (K_0 = initial volume of the chamber).

---

* A simple derivation of the energy equation is given in the Appendix.

Equation of motion of the shot : $W_1 \frac{d^2x}{dt^2} = AP$ .... (6)

$$W_1 = W + \tfrac{1}{2} kCN$$

Rate of burning equation : $D \frac{df}{dt} = -\beta p^{\alpha}$ .... (7)

Form equation : $z = (1-f)(1+\theta f)$ .... (8)

Before the nozzle opens, S=0 ; before the shot starts, x=0 ; after "all burnt" z=1 and equation (5) has to be replaced by

$$P\left(K_0 + Ax - CN\eta\right) = NCRT\left(1 + \frac{kCN}{6W}\right) \quad .. \quad (9)$$

This set of equations can always be integrated numerically. The numerical integration is facilitated by writing the equations in terms of suitably defined non-dimensional variables. For the case when $\gamma=1$ (linear rate of burning) Corner has given a more rapid, semi-analytical solution, the initial conditions being represented by nozzle-start and shot-start pressures. The method depends upon replacing certain quantities that occur in the integration by suitable approximations which have been suggested by comparison with a large number of numerical integrations.

Reduction to equivalent non-leaking gun.

We now turn to a very useful simplification of the equations whereby we may assess the effects of gas leakage by comparison with analogous equations for an orthodox gun. The level of approximation is essentially that of the "isothermal model" in orthodox gun theory. We make the following assumptions :

(1) There is no initial resistance.

(2) The nozzle flow is established at a low pressure.

These assumptions may be partially corrected for by adjusting β,

(3) $\alpha = 1$; (4) S = Constant.

For recoilless guns with nozzles of reasonably good shapes S/A lies near 0·65 and in such cases, the temperature shows a rapid drop after the nozzle opens, thereafter flattens out, and later shows the increasing rate of fall characteristic of normal guns. For most of the period of burning, the gas temperature in the recoilless gun lies near 0·9 of the mean temperature in the corresponding period of an ordinary gun. We may therefore approximate to both leaking and non-leaking guns by giving temperature a mean-value during the period of burning. For a normal gun this mean is about 0·9 T_0, while for recoilless guns this mean is about 0·85 T_0 at S/A =0·7. Thus one effect of leakage may be expressed as a decrease in the effective force constant.

Let λ = mean value of RT.

$$\Psi = \frac{\psi SD}{\beta C \lambda^{\frac{1}{2}}} \quad .... \quad (10)$$

Then we have from (3), (5) and (7) with $\alpha = 1$

$$P\left(K_0 + Ax - \frac{C}{\delta}\right) = C\lambda(1-f)(1-\Psi+\theta f) \quad .... \quad (11)$$

where a factor $1 + \frac{kCN}{6W}$ has been omitted on the right, as its value is never far from unity.

The dimensionless parameter $\overline{\Psi}$ is the fundamental quantity expressing the effect of leaking on the internal ballistics. For smooth-bore guns and mortars $\overline{\Psi} \sim 0{\cdot}1$. For recoilless guns with tubular propellant $\overline{\Psi} \sim 0{\cdot}4$—$0{\cdot}6$. We may write (11) as.

$$P\left(K_o + Ax - \frac{C}{\delta}\right) = C\,(1-\overline{\Psi})\,\lambda\,(1-f)\,(1+\theta' f) \quad \ldots\ldots\ldots\ldots (12)$$

where $$\theta' = \theta/\,(1-\overline{\Psi}) \quad \ldots\ldots\ldots\ldots (13)$$

The corresponding equation for an orthodox gun with a charge $C\,(1-\overline{\Psi})$ of propellant with mean force constant λ and form factor θ' would be

$$P\left[K_o + Az - \frac{C\,(1-\overline{\Psi})}{\delta}\right] = C\,(1-\overline{\Psi})\,\lambda\,(1-f)\,(1+\theta' f) \ldots\ldots (14)$$

This differs from (12) by a term which is important only for high densities of loading.

Thus, up to "allburnt", the leaking gun behaves almost as if it were an orthodox gun with the same dimensions, the smaller charge $C\,(1-\overline{\Psi})$, the bigger form factor $\theta' = \theta/\,(1-\overline{\Psi})$ and a force constant reduced as described previously.

The "effective charge" $C' = C\,(1-\overline{\Psi})$

$$= C - \frac{\psi SD}{\beta\lambda^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots (15)$$

If V_B is the velocity at "burnt" and V_E the muzzle-velocity, then it can be shown that,

$$C' \sim \frac{3}{8}\,\frac{W_1 V_E^{\,2}}{\lambda} + 0{\cdot}8\,\frac{\psi S W V_E}{A\lambda^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots (16)$$

For a recoilless gun, $S/A \sim 0{\cdot}65$, $\psi \sim 0{\cdot}63$ so that

$$C' \sim \frac{3}{8}\,\frac{W_1 V_E^{\,2}}{\lambda} + 0.35\,\frac{W_1 V_E}{\lambda^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots (17)$$

$= C_1 + C_2$, say C_1 may be regarded as the "part that pushes" and C_2 "the part that leaks". In other words C_1 gives energy to the projectile, while C_2 is used up in preventing recoil.

The following formula has been found fairly satisfactory up to nuzzle-velocities of 2,000 ft./sec.:—

$$\frac{C}{N} = \frac{y^2}{2} + \frac{y}{2.3}, \quad y = \frac{V_E}{\lambda^{\frac{1}{2}}}$$

In an orthodox gun, the muzzle velocity is proportional to (charge ^{n}, for small changes in the charge weight.

Thus, $V_E \propto (1-\overline{\Psi})^n \sim (1-n\,\overline{\Psi})$

Similarly, peak pressure $\propto (1-\overline{\Psi})^2 \sim 1-2\,\overline{\Psi}$

and central ballistic parameter $M \propto \dfrac{1}{1-\Psi} \sim 1+\overline{\Psi}$

An important and useful point is that these ballistic quantities are linear in Ψ.

Influence of design variables for a recoilless gun

One has to consider two types of variations : (*a*) changes at constant throat area which are of use in studying the regularity of the weapon, and (*b*) changes at zero recoil, which are important in deciding the design for best standard performance.

At constant nozzle throat area, the peak pressure and the position of "all burnt" exhibit a marked discontinuity of slope on crossing the boundary $P_N = P_0$. A similar discontinuity is exhibited by the muzzle-velocity for zero recoil, but not at constant throat area. The results show that, for example, the maximum pressure is practically independent of nozzle-start pressure (at constant throat area), if $P_N < P_0$ but when $P_N > P_0$, the maximum pressure becomes sensitive to P_N. This provides a method of determining the relative timing of nozzle opening and shot start. With a series of cartridge cases of stronger and stronger discs, we plot the peak pressures against thickness. The position of discontinuity in the curve gives the thickness at which $P_0 = P_N$ and hence we can estimate the relation between these pressures for the standard disc-thickness. The smaller the value of P_N, the smaller are likely to be the variations in muzzle-velocity and maximum pressure due to round-to-round variations in the strength of the disc.

The effects of changes in web-size, of shot weight, or of total shot travel in a recoilless gun are all similar to the corresponding results for a normal gun.

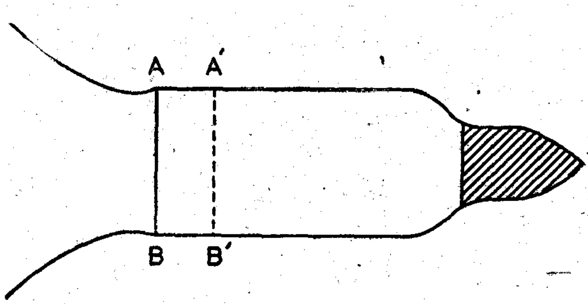
Other applications.

The theory explained in the previous paragraphs can be applied to two other problems : (1) the ballistics of a worn orthodox gun, (2) leakage in a smooth bore mortar. In the case of the former the theory enables us to estimate the loss of ballistics due to leakage in a gun in any stage of wear. In the case of the mortar, the theory has been of value in determining how the tolerances on the weight and dimensions of the bomb are connected with the dispersions of muzzle velocity and the range.

Appendix

The mass of gas that flows through the nozzle in time dt is Qdt.

Let this mass of gas be that contained in the Section ABB'A' of the chamber.



The energy added in time dt

$$= C \frac{dz}{dt} dt (T_O - T) c_v$$

$$= \frac{CR}{\gamma - 1} (T_O - T) \frac{dz}{dt} dt$$

This is utilised partly in increasing the internal energy of the gas and partly in the work done.

The increase in internal energy $= CN \frac{R}{\gamma - 1} \frac{dT}{dt} dt$

The work done = work done on the shot side (W′) + work done on the nozzle side (W″) ;

$$W' = AP \frac{dx}{dt} .dt. \frac{1 + \frac{kCN}{3W_1}}{1 + \frac{kCN}{2W_1}} \backsim \frac{APV\,dt}{1 + \frac{kCN}{6W_1}}$$

$$W'' = P. \frac{Qdt}{\rho} \quad \text{(Pressure} \times \text{change of volume)}$$

Now $P (1/\rho - \eta) = RT$

or $1/\rho = \frac{RT}{P} + \eta$

$$\therefore W'' = (RT + P\eta)\, Qdt = \left\{ RT + \frac{RT\eta}{1/\rho - \eta} \right\} Qdt$$

$$= (1 + \varepsilon)\, RTQ\, dt.$$

Hence we have

$$\frac{CR}{\gamma - 1} (T_O - T) \frac{dz}{dt} dt = CN \frac{R}{\gamma - 1} \frac{dt}{dt} dt + \frac{APV\,dt}{1 + \frac{kCN}{6W_1}} + (1 + \varepsilon)\, QRT.\, dt$$

$$i.e.\ (T_O - T) \frac{dz}{dt} = N \frac{dT}{dt} + \frac{\gamma - 1}{CR} \frac{APV}{1 + \frac{kCN}{6W_1}} + \frac{(\gamma - 1\,(1 + \varepsilon)}{C} QR \quad .. \quad (1,a$$

The equation (3) of the text gives

$$T \frac{dz}{dt} = T \frac{dN}{dt} + \frac{QT}{C} \quad .. \quad .. \quad .. \quad (1,b)$$

Combining (1, a) and (1, b) we get

$$T_O \frac{dz}{dt} = \frac{d}{dt} (NT) + \frac{\gamma - 1}{CR} \frac{APV}{1 + \frac{kCN}{6W_1}} + [1 + (\gamma - 1)(1 + \varepsilon)] \frac{QT}{C} \quad .. \quad (1,c)$$

If, as usual, we replace $\gamma - 1$ by $\bar{\gamma} - 1$ in the second term on the right in order to take account of heat loss, we get

$$T_O \frac{dz}{dt} = \frac{d}{dt} (NT) + \frac{\bar{\gamma} - 1}{CR} \frac{APV}{1 + \frac{kCN}{6W_1}} + [1 + (\gamma - 1)(1 + \varepsilon)] \frac{QT}{C}$$

which is identical with the energy equation [Eq (4)] given in the text.