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Predictive Terminal Guidance with Tuning of Prediction Horizon & Constrained Control

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ABSTRACT

Continuous time-predictive control approach is employed to formulate an output tracking nonlinear, optimal, terminal guidance law for re-entry vehicles. The notable features of this formulation are that the system equations are not linearised and the evaluation of the guidance equations does not need the information of vehicle parameters, such as drag and mass. The formulation allows to impose the physical constrains on the control inputs, i.e. on the demanded lateral accelerations through a saturation mapping and the controls are obtained using a fixed point iteration algorithm which converges typically in a few iterations. Further, a simple method of tuning the prediction horizon needed in the guidance equations is presented. Numerical simulations show that the guidance law achieves almost zero terminal errors in all states despite large errors in initial conditions.

1. INTRODUCTION

Extensive research has been carried out in the area of re-entry guidance and control, and a variety of schemes for re-entry guidance[1] as well as for terminal guidance of re-entry vehicles[2-4] have appeared in the literature. The guidance law designs[2-4] are essentially based on linear control theory. The performance of these and similar guidance laws gets deteriorated whenever the assumptions of linearisation are violated. Further, in general, the guidance law designs based on linear techniques do not address the problem of control input saturation.

Certain schemes, such as proportional guidance and cross product law, which do not require a reference trajectory, have also been considered for terminal guidance[5]. However, achieving proper terminal values for the dynamic variables with these techniques are not easy and also the resulting control may not be optimal.

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In this study, a recently developed continuous time-predictive control[6,7] approach is employed to formulate an optimal, nonlinear terminal guidance law with constrained control inputs for re-entry vehicles. In this approach, the state or output response of the nonlinear system is predicted by appropriate expansion, and the guidance law is obtained by pointwise minimising a quadratic performance measure based on the error between the predicted response and the desired response. Since the system equations are not linearised, the drawbacks of the linear techniques are overcome. The reason for formulating an output tracking guidance law instead of the state tracking one is that, firstly, in a state tracking formulation, the evaluation of the guidance equations requires the information about the vehicle parameters, such as drag which is not very precisely measurable and, secondly certain states, such as velocity cannot be effectively regulated[4]. In this study, the re-entry vehicle is considered to be moving in a two-plane (i.e. having downrange as well as crossrange) over the flat earth. The equations of motion have been modified for the derivation of guidance law by choosing the downrange as an independent variable in place of time, as time as an independent variable offers inferior results for the reference riding guidance law[8].

The altitude and crossrange have been chosen as outputs, while the lateral accelerations in vertical and horizontal planes are the inputs. The guidance law thus derived achieves input-output feedback linearisation, necessitating to check the stability of zero dynamics of the system for overall closed-loop stability. Also, the guidance law rquires selection of the prediction horizon which is the interval at which the system response is predicted. This interval need not be constant and can be tuned suitably to achieve better tracking performance.

2. PREDICTIVE CONTROL

The continuous time-predictive control theory results in an optimal and nonlinear feedback control law through minimisation of a performance measure based on the predicted errors between the actual trajectory and the reference trajectory. A brief outline of this approach is presented for the sake of completeness. Consider a nonlinear system which is described as

$$\dot{x} = f(x)+G(x)\ u \qquad)$$

$$y = c(x) \qquad (2)$$

where

$$f \triangleq [f_1 f_2 \quad f_n]^T ;\ G \triangleq [g_1 g_2 \quad g_m] \qquad (3)$$

and $x(t) \in X \subset R^n$ is the state, $u(t) \in U \subset R^m$ represents the control and $y(t) \in R^m$ is the output vector, where X and U are compact sets in R^n and R^m spaces, respectively. The functions f: $R^n \rightarrow R^n$, c: $R^n \rightarrow R^m$ and G: $R^n \rightarrow R^{n \times m}$ are continuously differentiable nonlinear functions. Suppose that the desired output trajectory is specified by $q(t), 0 \le t \le t_f$, which is an outcome of Eqns (1) and (2) for some feasible reference control $u^*(t) \in U$ for all $t \in [0,t_f]$.

Given the present output of the system, $y(t)$, at any instant, $t \in [0,t_f]$, the current control, $u(t)$ determines or influences the output response in the immediate future. To predict the response of the system due to present control input, the following procedure has been followed:

Let λ_i, $i = 1,...,m$, be the lowest order of the derivative of y_i, the i^{th} component of $y(t)$, such that the component of control appears for the first time. Define $z(x(t),\delta) = [z_1(x(t),\delta),...,z_m(x(t),\delta)]^T$

where

$$z_i = \delta L_f(c_i) + \frac{\delta^2}{2!} L_f^2(c_i) + \quad + \frac{\delta^{\lambda_i}}{\lambda_i!} L_f^{\lambda_i}(c_i),$$

$$i = 1,...,m \qquad (4)$$

where c_i is the i^{th} component of vector $c(x)$, $\delta > 0$ is a real number designated as the prediction horizon and $L_f^k(c_i)$ denotes the k^{th} order Lie derivative of c_i wrt f. To expose fully the influence of $u(t)$ on $y_i(t+\delta)$, for a small $\delta > 0$, one may approximate each $y_i(t+\delta)$ by an λ_i^{th} order Taylor series at t. In doing so, one can express $y(t+\delta)$ as a vector function of $u(t)$ in a compact form as

$$y(t + \delta) \approx y(t) + z[x(t),\delta] + \Lambda(\delta) W[(x(t)] u(t)$$

where $\Lambda(\delta) \in R^{m \times m}$ is a diagonal matrix with the elements on the main diagonal being

$$\Lambda_i = \frac{\delta^{\lambda_i}}{\lambda_i!}; \qquad i = \quad ,m$$

and $W(x) \in R^{m \times m}$ has each of its rows in the form of

$$w_i = [L_{g1}\{L_f^{\lambda_i - 1}(c_i)\}, \quad , L_{gm}\{L_f^{\lambda_i - 1}(c_i)\}],\ i = \quad .,m \qquad (7)$$

Similarly, expanding the i^{th} component of $q(t+\delta)$ in the λ_i^{th} order Taylor's series yields:

$$q(t+\delta) \approx q(t) + d(t,\delta) \qquad (8)$$

where the i^{th} component of $d(t,\delta) \in R^m$

$$d_i(t,\delta) = \delta \dot{q}_i(t) + \frac{\delta^2}{2!}\ddot{q}_i(t) + \dots + \frac{\delta^{\lambda_i}}{\lambda_i!} q_i^{(\lambda_i)}(t), \quad i = 1,\dots,m \tag{9}$$

where $q_i^{(\lambda_i)}$ is the λ_i^{th} differentiation of q_i wrt time. To find the current control $u(t)$ that improves the tracking accuracy at next instant, consider pointwise minimisation of the performance index that penalises the output tracking error at $(t+\delta)$ and the current control expenditure $u(t)$.

$$J[u(t)] = \frac{1}{2}\left[y(t+\delta) - q(t+\delta)\right]^T Q \left[y(t+\delta) - q(t+\delta)\right] + \frac{1}{2}u^T(t) R u(t) \tag{10}$$

where $Q \in R^{m\times m}$ is positive definite, and $R \in R^{m\times m}$ is positive semi-definite weighting matrices. Replace $y(t+\delta)$ and $q(t+\delta)$ in Eqn (10) by predictions in Eqns (5) and (8), respectively. The control that minimises the performance index is obtained by setting $\partial J/\partial u$ equal to zero as

$$u(t) = -\left[\{A(\delta)W(x)\}^T Q A(\delta) W(x) + R\right]^{-1} \left[\{A(\delta)W(x)\}^T Q\{e(t) + z(x,\delta) - d(t,\delta)\}\right] \tag{11}$$

where $e(t) \triangleq y(t) - q(t)$ is the current output tracking error. It has been shown[7] that if $W(x)$ as defined in Eqn (7) is of full rank, then the control [Eqn (11)] achieves input-output linearisation and asymptotic tracking of any given output history $q(t)$ for $R = 0$ and for any $\delta > 0$, $Q > 0$ and if the relative degree $\lambda_i \le 4$. The relative degree of a system is equal to the number of times the output must be differentiated to have the control input appear explicitly for the first time. When the relative degree is more than four, it can be shown that the control [Eqn (11)] still achieves the input-output linearisation of the system, but may not guarantee asymptotic tracking of output history. The condition of invertability of W is equivalent to system [Eqns (1) and (2)] having relative degree in the terminology of the geometric control theory[9]. When the control is constrained or bounded, the control command is obtained through the fixed point iteration algorithm[7], as given by the following theorem:

2.1 Theorem

Consider a system of the form of Eqns (1)–(2). Assume that the matrix $P[x(t),\delta] = \{p_{ij}\} = [\{A(\delta)W(x)\}^T Q A(\delta) W(x) + R]$ is nonsingular at $x(t)$. Then for any $\delta > 0$:

- The unique optimal control $u^*(t)$ to [Eqn (10)] exists and is the unique solution of the fixed point equation in u.

$$u(t) = s\left\{\beta\left[(AW)^T Q(d - z - e)\right] - \left[\beta\left((AW)^T QAW + R\right) - I\right]u\right\} \triangleq \rho(u) \tag{12}$$

where all the arguments have been suppressed for clarity, I is an identity matrix,

$$\beta \triangleq \left\{\sum_{i=1}^{m}\sum_{j=1}^{m} p_{ij}^2\right\}^{-1/2} \tag{13}$$

and s represents a saturation mapping on some $\alpha \in R^m$, such as

$$s_i(\alpha) = \begin{cases} U_i(x,t) & \alpha_i \ge U_i(x,t) \\ \alpha_i & L_i(x,t) < \alpha_i < U_i(x,t) \\ L_i(x,t) & \alpha_i \le L_i(x,t) \end{cases} \qquad i = 1,2,\dots,m$$

where L_i and U_i are pre-defined functions

- The fixed point iteration sequence $\{u^k\}$ is generated by:

$$u^k = \rho(u^{k-1}), \quad k = 1,2,\dots \quad \forall u^0 \in R^m \tag{15}$$

converges to $u^*(t)$.

It is straightforward to verify that if the saturation mapping in Eqn (12) is removed, one gets Eqn(11). Thus, controller [Eqn (12)] gives the optimal control in both saturated as well as unsaturated cases. The fixed point algorithm is well-suited for computer implementation, and it converges typically in just a few iterations.

3. GUIDANCE LAW FORMULATION

The continuous time-predictive control is employed to fomulate the terminal guidance law for a point mass re-entry vehicle moving in two-plane over the flat earth. The origin of the coordinate frame is fixed at the projection of the nominal re-entry position of vehicle on ground with X-axis pointing downrange, Z-axis along the local vertical and positive upward, and Y-axis completing the right hand system giving the crossrange. The standard equations of motion[4] are:

$$\left.\begin{aligned} \dot{r} &= V\cos\gamma\cos\psi \\ \dot{y} &= V\cos\gamma\sin\psi \\ \dot{h} &= V\sin\gamma \\ \dot{V} &= -\frac{D}{m} - g\sin\gamma \\ \gamma &= \frac{u_1}{V} - \frac{g\cos\gamma}{V} \\ \psi &= \frac{u_2}{V\cos\gamma} \end{aligned}\right\} \qquad (16)$$

where the state $x = [r\,y\,h\,V\,\gamma\,\psi]^T$ are the downrange, crossrange, altitude, velocity, flight-path angle and azimuth angle of the point mass vehicle, respectively. The state y is not to be confused with the output vector y defined in Section 2. The quantities D and m are the drag and the mass of the vehicle, g is the gravitational acceleration, while u_1 and u_2 are controls, i.e. the lateral accelerations in vertical and horizontal planes, respectively. It has been shown[8] that time as an independent variable for the nominal riding guidance schemes give inferior results and so for the derivation of the guidance law, the downrange, r, was chosen as an independent variable instead of time. To this end, the modified equations of motion with r as independent variable are:

$$y' = \tan\psi \quad ; \quad h' = \tan\gamma\sec\psi$$

$$V' = -\frac{D}{mV\cos\gamma\cos\psi} - \frac{g\tan\gamma}{V\cos\psi}$$

$$\gamma' = \frac{u_1}{V^2\cos\gamma\cos\psi} - \frac{g}{V^2\cos\psi}$$

$$\psi' = \frac{u_2}{V^2\cos^2\gamma\cos\psi} \qquad (17)$$

where $(.)'$ represents the differential of $(.)$ wrt downrange variable r. To derive the guidance law, altitude $h(r)$ and crossrange $y(r)$ were chosen as outputs. Following the method outlined in Section 2, and noting that the relative degree for both the outputs is two, the various quantities required in the control law [Eqn11] are:

$$\Lambda = Diag\left[\frac{\delta^2}{2}, \frac{\delta^2}{2}\right] \qquad (18)$$

$$W = \begin{bmatrix} 0 & \dfrac{}{V^2\cos^2\gamma\cos^3\psi} \\ \dfrac{1}{V^2\cos^3\gamma\cos^2\psi} & \dfrac{\tan\gamma\tan\psi}{V^2\cos^2\gamma\cos^2\psi} \end{bmatrix} \qquad 19)$$

$$e(r) = [e_y(r)\; e_h(r)]^T = [(y - y^*)\;(h - h^*)]^T \qquad (20)$$

$$z = [z_1 z_2]^T = \left[\delta\tan\psi\left(\delta\frac{\tan\gamma}{\cos\psi} - \frac{\delta^2}{2}\frac{g}{V^2\cos^2\gamma\cos^2\psi}\right)\right]^T$$

$$d = [d_1\, d_2]^T = \left[\left(\delta\dot{y}^* + \frac{\delta^2}{2}\ddot{y}^*\right)\left(\delta\dot{h}^* + \frac{\delta^2}{2}\ddot{h}^*\right)\right]^T$$

$$Q = Diag[Q_1\; Q_1] \quad R = Diag[R_1\; R_2$$

One important issue in the predictive controllers is the choice of proper values for the weighting matrices and usually these quantities are selected through trial and error procedure by observing the simulated responses as there does not exist any systematic methodology for their selection. Thus all the quantities required in Eqn (12) are defined, and the guidance command can be obtained by substituting Eqns (18)-(23) in Eqn (12). The saturation functions L_i and U_i are user defined and should reflect the physical constraints on the control magnitude. One can observe that the evaluation of the guidance law does not require information of the vehicle parameters, such as mass, drag and reference area and also atmospheric quantities, such as density of the air. The quantities

$$y^*(r), \dot{y}^*(r), \ddot{y}^*(r) \text{ and } h^*(r), \dot{h}^*(r), \ddot{h}^*(r)$$

are obtained while generating the reference trajectory. In the unsaturated case or when the control input is not constrained, one can set the weighting matrix R to zero in controller [Eqn(11)], if W in Eqn (19) is nonsingular. In that case, i.e. when the control input is not constrained, the guidance commands in scalar form are:

$$\begin{aligned} u_1 = \frac{2}{\delta^2} V^2 \cos^3\gamma \cos^2\psi \Big[& \cos\psi \tan\gamma \tan\psi \left\{ e_y + \delta(\tan\psi - \dot{y}^*) - \frac{\delta^2}{2}\ddot{y}^* \right\} \\ & - \left\{ e_h + \delta\left(\frac{\tan\gamma}{\cos\psi} - \dot{h}^*\right) - \frac{\delta^2}{2}\left(\frac{g}{V^2\cos^2\gamma\cos^2\psi} + \ddot{h}^*\right) \right\} \Big] \\ u_2 = -\frac{2}{\delta^2} V^2 \cos^2\gamma \cos^3\psi & \left[e_y + \delta(\tan\psi - \dot{y}^*) - \frac{\delta^2}{2}\ddot{y}^* \right] \end{aligned} \tag{24}$$

The guidance commands [Eqn (24)] are used to show the closed-loop stability of the system.

4. CLOSE-LOOP STABILITY

No general proof for the closed-loop stability is available in the literature when the control is saturated. For the unsaturated case, the closed-loop stability of the system can be presented for the case when the weighting R is zero. The guidance commands [Eqn (24)] represents the unsaturated version [Eqn (11)] with $R = 0$. To show the closed-loop stability of the system [Eqn (17)] under the guidance law [Eqn (24)], first the authors show the asymptotic tracking of the reference output trajectory. Consider the crossrange and altitude equations as given in Eqn (17):

$$y' = \tan\psi;\ h' = \tan\gamma \sec\psi$$

Differentiating Eqn (25) twice wrt r (as relative degree is two for y as well as h) and substituting Eqn (24), respectively, one gets the tracking error dynamics for the crossrange and altitude as follows:

$$e_y'' + \frac{2}{\delta}e_y' + \frac{2}{\delta^2}e_y = 0 \tag{26}$$

$$e_h'' + \frac{2}{\delta}e_h' + \frac{2}{\delta^2}e_h = 0 \tag{27}$$

Clearly the error dynamics [Eqns (26)-(27)] are given by stable linear differential equations for $\delta > 0$ thus assuring asymptotic tracking of the reference output history for given initial condition errors. As discussed in Section 2, for $R = 0$ in Eqn (11), the guidance commands [Eqn (24)] achieves input-output feedback linearisation[9]. From the results of the geometric control theory, when the summation of the relative degrees of outputs is less than the number of system states, zero dynamics[10] exists. The zero dynamics essentially represents an internal dynamics of the system which becomes unobservable when the system is subjected to the input-output linearising controller. Thus for the closed-loop stability of the system under feedback linearising control laws, it is necessary that the zero dynamics of the system must be stable. In the present case, the sum of the relative degrees of the outputs is four while the number of states as seen from Eqn (17) are five, and thus the closed-loop stability of system [Eqn (17)] under guidance law [Eqn (24)] is subject to the stability of the zero dynamics of the system [Eqn (17)]. To this end, a nonlinear state transformation of the state vector in Eqn (17) is obtained as $z = [z_1\ z_2]^T$ where $z_1 \triangleq [z_{11}\ z_{12}\ z_{13}\ z_{14}]^T = [y y' h h']$ and z_2 represent the states associated with the zero dynamics of the system. Choosing $z_2 = V$, the zero dynamics of the system in the new coordinates is given by:

$$z_2' = \frac{-D\sqrt{z_{12}^2 + z_{14}^2 + 1}}{m z_2} - \frac{g z_{14}}{z_2} \tag{28}$$

The stability of the zero dynamics is assessed[10] by substituting $z_1 = 0$ and one can easily verify that the zero dynamics of the present system is stable, thus assuring closed-loop stability for the complete system [Eqn (17)] under the guidance law [Eqn (24)].

5. TUNING OF PREDICTION HORIZON

The guidance commands [Eqn (24)] requires selection of the prediction horizon δ and it has been shown that the performance of the guidance law is very sensitive to the selection of this parameter. One can note that the prediction horizon represents the time constant of the error dynamics [Eqns (26)-(27)] implying small value of this parameter is desirable for tracking accuracy. However, one can observe that too small value of this parameter causes large control as is obvious from Eqn (24), while large value may result in poor tracking accuracy. This shows the need for tuning this parameter appropriately instead of keeping it constant. In literature, the prediction horizon is usually chosen by carrying out extensive simulations due to the lack of any standard methodology for its selection. Here, a simple but effective way of tuning this parameter for the present problem has been presented.

Consider the error dynamics [Eqns (26)-(27)]. Solving these equations for initial errors in crossrange and altitude yields:

$$e_y(r) = e_y(0)e^{-\frac{r}{\delta}}\left(\cos\frac{r}{\delta} + \sin\frac{r}{\delta}\right) \tag{29}$$

$$e_h(r) = e_h(0)e^{-\frac{r}{\delta}}\left(\cos\frac{r}{\delta} + \sin\frac{r}{\delta}\right) \tag{30}$$

where $e_y(0)$ and $e_h(0)$ represent the initial errors (i.e. at $r = 0$) in the crossrange and altitude from their reference value, respectively. In the present problem, the prediction horizon is in meters as the equations of motion have been modified by choosing downrange as independent variable. Suppose, it is desired to bring the magnitudes of the errors in crossrange and altitude within certain tolerance $\pm \epsilon_y$ and $\pm \epsilon_h$, respectively, by certain value of downrange, r_s, travelled by the vehicle, then from Eqns (29)-(30) one can obtain an approximate closed form solution for δ at $r = 0$ as

$$\delta_y(0) \approx -\frac{r_s}{\ln\left(\dfrac{\epsilon_y}{\sqrt{2}e_y(0)}\right)}$$

$$\delta_h(0) \approx -\frac{r_s}{\ln\left(\dfrac{\epsilon_h}{\sqrt{2}e_h(0)}\right)} \tag{31}$$

and one chooses the smaller one from Eqn (31) as the value of prediction horizon in the guidance law. Further, one can exploit Eqn (31) for continuously tuning the value of the prediction horizon. To this end, defining $\delta_y(r)$ and $\delta_h(r)$ as

$$\delta_y(r) \approx -\frac{(r_f - r)}{\ln\left(\dfrac{\epsilon_y}{\sqrt{2}e_y(r)}\right)}$$

$$\delta_h(r) \approx -\frac{(r_f - r)}{\ln\left(\dfrac{\epsilon_h}{\sqrt{2}e_h(r)}\right)} \tag{32}$$

where r_f represents the downrange-to-go and r is the present downrange travelled by the vehicle. (Here, it is important to note that the downrange, r is zero at the re-entry point). Then, to ensure that the value of δ is feasible (i.e. δ > 0) and also to ensure that a smaller one of the two is used, the following rules are obtained:

When $e_y(r) > 0.707\epsilon$ and $e_h(r) > 0.707\epsilon_h$ simultaneously:

$$\delta(r) = min[\delta_y(r), \delta_h(r)] \tag{33}$$

When at least $e_y(r) > 0.707\epsilon_y$ or $e_h(r) > 0.707\epsilon_h$:

$$\delta(r) = max\ [\delta_y(r), \delta_h(r)] \tag{34}$$

When $e_y(r) \leq 0.707\epsilon$ and $e_h(r) \leq 0.707\epsilon_h$ simultaneously:

$$\delta(r) = \delta_c \tag{35}$$

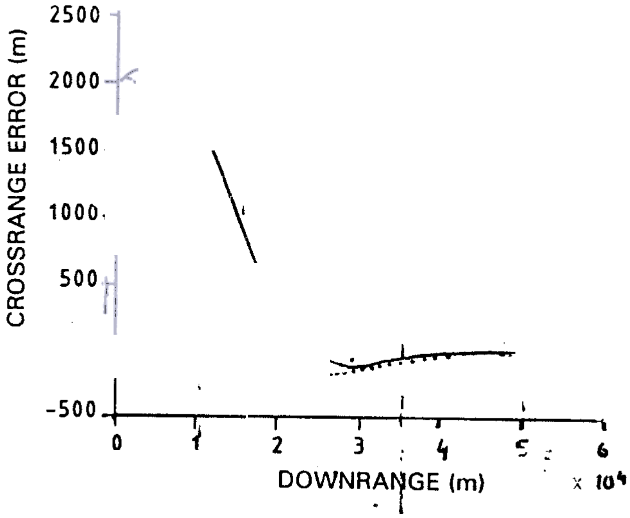

Figure 1. Crossrange error *vs* downrange

and

When $(r_f - r)$ r,

$$\delta(t) = \delta_{c2} \tag{36}$$

where δ_{c1} and δ_{c2} are some positive constants and are used either to ensure that the value of δ is feasible one or to prevent it from becoming very small. The signs of tolerances ϵ_y and ϵ_h are same as the signs of the errors $e_y(r)$ and $e_h(r)$, respectively so that the natural logarithm in Eqns (31)-(32) exists. To avoid δ becoming very small as $r \to r_f$, a constant value of $\delta = \delta_{c2}$ is chosen when $(r_f - r) < r_1$. Through proper selection of the tolerances ϵ_y and ϵ_h, one can achieve better tracking performance.

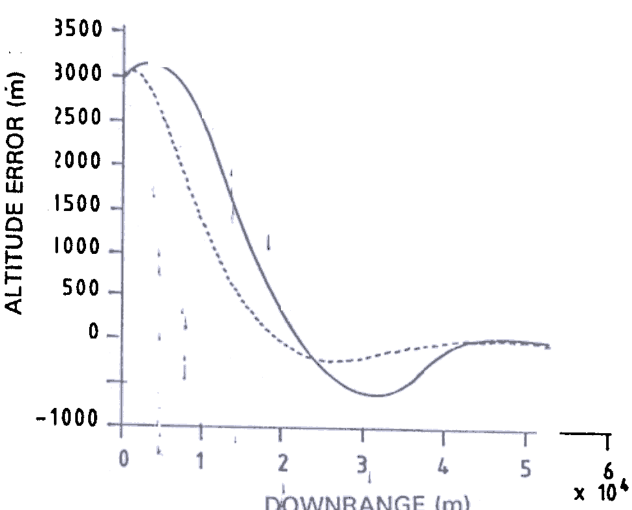

Figure 2. Altitude error *vs* downrange

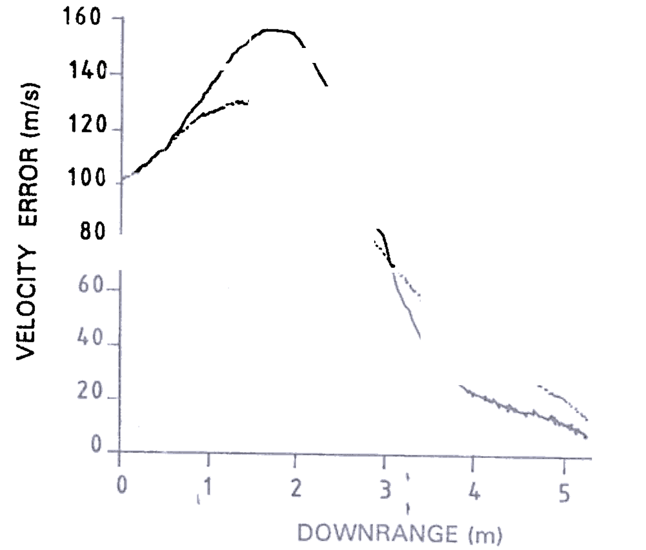

Figure 3. Velocity error *vs* downrange

The tuning of the prediction horizon offers many advantages. Firstly, it ensures large value of δ initially so that the control is not excessive due to large initial tracking errors. It has been shown[6] that the output tracking controller based on the continuous time-predictive control approach offers stability robustness in the face of bounded uncertainty. The tuning of the prediction horizon will result in robustness in performance since the value of prediction horizon at any instance depends upon the traking errors at that instance. Next, the value of the prediction horizon decreases continuously, thus assuring better

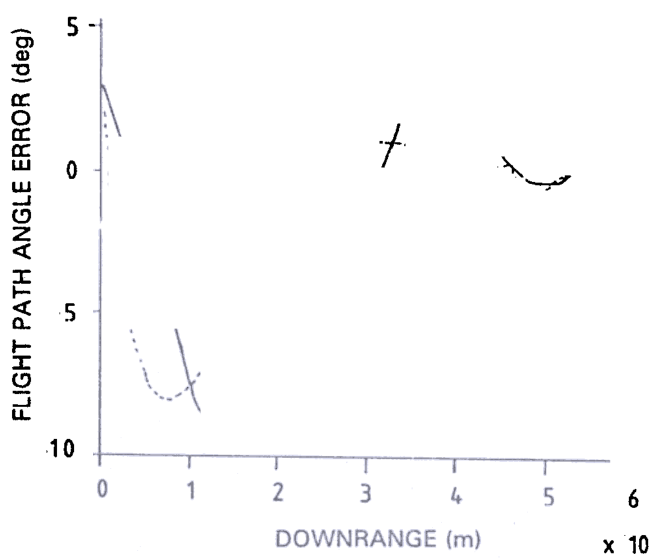

Figure 4. Flight-path angle error *vs* downrange

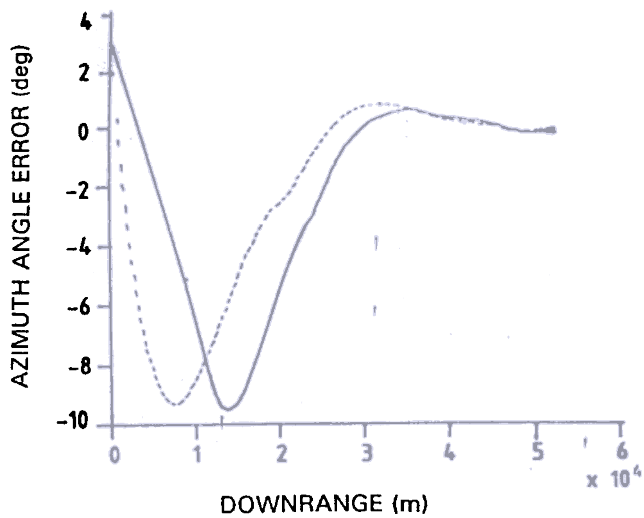

Figure 5. Azimuth angle error *vs* downrange

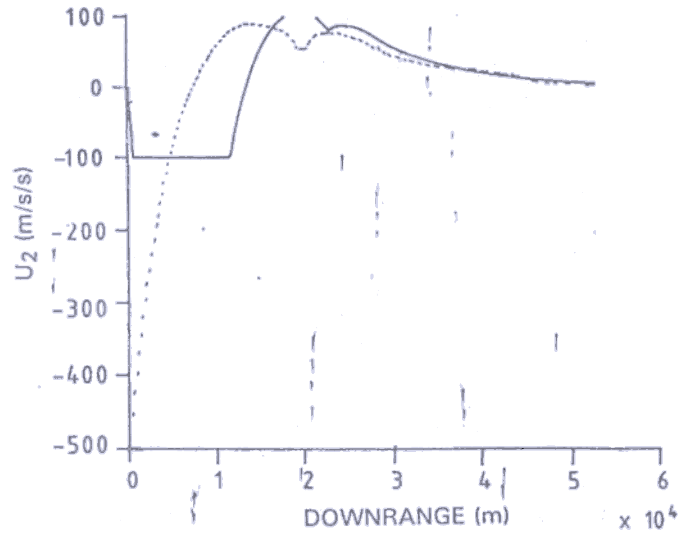

Figure 7. Lateral acceleration in horizontal plane *vs* downrange

tracking accurancy as the vehicle reaches its destination. Although the tuning of the prediction horizon by following Eqns (32)-(33) is for the unsaturated control, the choice of δ obtained through this procedure holds even when the controls are saturated. There is no claim that this is the best way of choosing the prediction horizon. However, in the absence of any standard mehodology to choose this parameter, the rules presented here are quite logical for the present application.

6. SIMULATIONS & RESULTS

To assess the performance of the predictive control-based guidance law, it is necessary to generate a reference trajectory for the re-entry vehicle. The reference trajectory is usually obtained by employing certain optimisation techniques which incorporate various constraints, such as physical constraints of the vehicle and constraints imposed on terminal conditions, while achieving the mission objectives. However, to assess the performance of the guidance law, it is not necessary that the reference trajectory be optimal; instead it needs to be just a feasible solution that satisfies the equations of motion. The vehicle data needed for this purpose was taken from the work of Regan[11]. The reference trajectory was generated by simulating the equations of motion [Eqn (16)] by giving open-loop controls and the

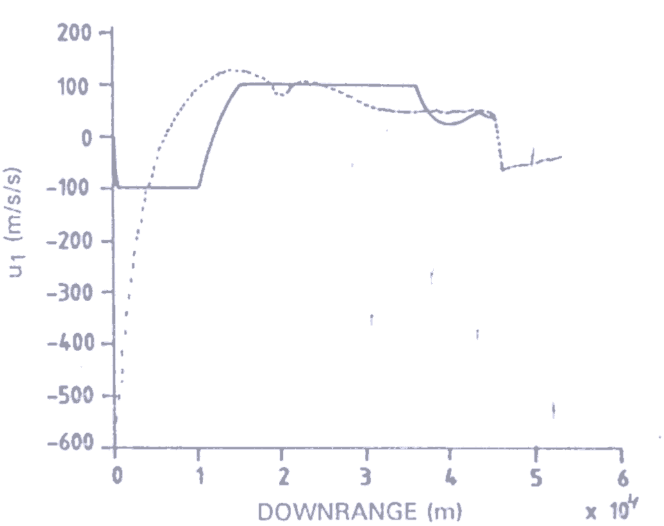

Figure 6. Lateral acceleration in vertical plane *vs* downrange

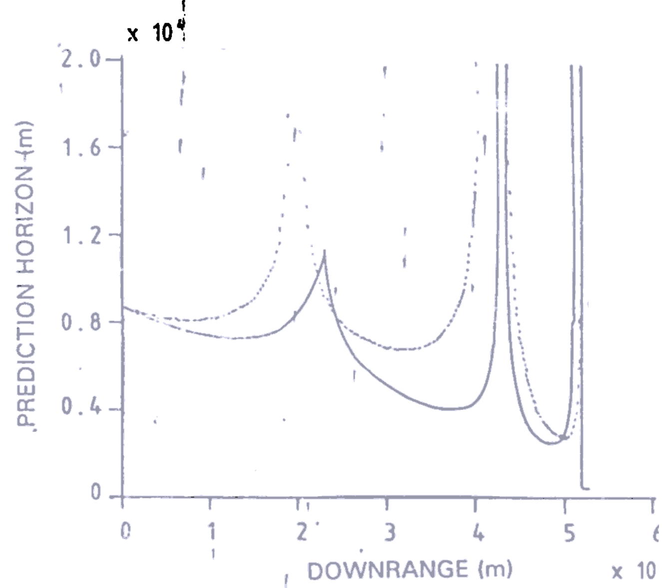

Figure 8. Prediction horizon *vs* downrange

variables required for evaluation of the guidance law were stored as a function of the downrange, r. Since this work deals with a 2-D trajectory, lateral acceleration in X-Z and X-Y planes were needed as inputs for the equations of motion. International standard atmosphere was used for obtaining the density as a function of altitude. The weightings required in the controller [Eqn (12)] were chosen as

$$Q = Diag\ [10, 10];\ R = [0, 0] \tag{37}$$

The prediction horizon was tuned by following the rules presented in Section 5. The nominal downrange, r_f as obtained from the reference trajectory is 52985 m. The other quantities needed for tuning of the prediction horizon were taken as

$\in_y=10$ m, $\in_h=10$ m, $\delta_{c1}=20\,000$ m
$\delta_{c2}=500$ m and $r_t=1000$ m

Upper limit of 20 000 m was enforced for the value of δ. The control magnitudes were constrained through the saturation mapping defined in Eqn (14) with

$L_1 = L_2 = 100$ m/s² and $U_1 = U_2 = 100$ m/s²

Using these values, large number of simulations were carried out for various initial condition errors and the trajectories were analysed for saturated as well as unsaturated controls. For saturated control, Eqn (12) was used, while for unsaturated control, Eqn (11) was used. The simulation results show that the guidance law achieves almost zero terminal errors in both cases, i.e. when control is saturated and also without saturation of controls. It is important to note that the formulation of the guidance law is output tracking and not the state tracking one. Thus, regulation of the outputs y and h has resulted in regulation of the other states as well resulting in satisfactory terminal errors. Secondly, it was observed that the initial lateral acceleration demands were significantly large for unsaturated control which underlines the need for imposing saturations on the controls. The large demands for control are logical since the tracking errors are maximum initially. When the control saturation is considered, the controls are obtained using Eqn (12) and it was observed that the fixed point equation usually converges in just two to five iterations, making it useful even for online implementation. The results of one such simulation are presented in Figs (1)-(8) for the following initial condition errors:

$$y(0) = y_n(0) + 2000 \text{ m}$$
$$h(0) = h_n(0) + 3000 \text{ m}$$
$$V(0) = V_n(0) + 100 \text{ m/s}$$
$$\gamma(0) = \gamma_n(0) + 3°$$
$$\psi(0) = \psi_n(0) + 3°$$

where the reference values of the corresponding variables are given by suffix n. Figures (1)-(5) show the state errors, i.e. $y-y_n$, $h-h_n$, $V-V_n$, $\gamma-\gamma_n$, and $\psi-\psi_n$ as a function of downrange for both cases, i.e. when the saturation is imposed and without saturation on control magnitudes. In all the figures, the solid curves represent the results of the saturating case (i.e. when the control input is constrained), while the dotted lines represnt the results when the control is unconstrained. From the figures it can be observed that the state errors reduces smoothly almost to zero as the destination is reached. However, the instantaneous tracking errors are found to be larger for the saturated case which is obvious. Figures (6)-(7) give control histories, i.e. lateral acceleration in vertical plane (u_1) and horizontal plane (u_2), respectively and it is clear that for the controller without saturation, the initial demand is extremely large which reduces subsequently while for the constrained control case, the controls get saturated within the prescribed limits. Figure (8) shows the corresponding history of prediction horizon as obtained from the rules stated in Section 5 by which the prediction horizon is tuned between the limits of 200-20 000 m. From this figure, one can infer that the variation of this parameter does not follow any standard pattern, such as linear or exponential decrement.

The pertubations in the re-entry conditions considered for the results might be on higher side for certain realistic missions. However, the large values were chosen to show that the predictive control-based guidence algorithm can handle comparatively larger initial errors while delivering satisfactory performance. The performance of the guidance law will certainly be satisfactory if the initial errors are smaller than what has been considered.

7. CONCLUSION

A continuous time-predictive control approach is applied to formulate a output tracking terminal guidance law with constrained control for re-entry vehicles. The notable feature of this formulation is that the evaluation of the resulting guidance equations does not require the knowledge of the vehicle parameters, such as drag and mass. A simple method for tuning the prediction horizon required in the guidance law to achieve better performance is presented. Simulations were carried out for a variety of errors in initial conditions and results have been presented for one such case. The results show that the guidance law achieves almost zero terminal errors with constrained control for all the states not withstanding that the formulation of the guidance law was an output tracking and not the state tracking one. The fixed point iteration algorithm used to compute the control commands converges in just few cycles, making it viable for online implementation.

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