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INFLUENCE OF COMPOSITE CHARGES ON MAXIMUM PRESSURE AND MUZZLE VELOCITY

By Shri N.S. Venkatesan, Defence Science Organisation, Ministry of Defence, New Delhi, and Shri G.C. Patni, Maharaja's College Jaipur.

Introduction

The problem of the composite charges has been discussed in the " Theory of the Interior Ballistics of Guns " by Corner (1) and in the " Internal Ballistics"— His Majesty's Stationery Office Publication (2) by reducing the problem to one of an equivalent single charge. In (2) only a particular case of two charges with different shapes and web sizes of the same propellent is considered. An approximate solution is obtained by making use of a modified form factor. Corner (1) considers the more general problem of two charges of different shapes, sizes, and composition. The problem is reduced to that of a single equivalent charge with adjusted parameters.

In this paper we give a direct treatment of the general problem based on the Hunt-Hinds system. We derive expressions for the maximum pressure and muzzle velocity and study the variation in these with the ratio of the charge weights for a given total charge. In order to obtain closed expressions we make the usual assumption regarding Co-volume, viz., that the Co-volume of the gases equals the specific volume for each propellant. It is also assumed that $\gamma_1=\gamma_2$ since γ is practically the same for most propellants.

Basic Equations

The basic equations with a single charge have been derived in H.M.S Publication [2] and in our case they become

$$\frac{F_1C_1z_1 + F_2C_2z_2}{Al} = p\left(1+\frac{x}{l}\right) + \frac{\gamma-1}{2\,Al} w_1 v^2 \qquad \text{......(1)}$$

$$\text{where } w_1 = 1{\cdot}05\, w + \frac{C_1}{3} + \frac{C_2}{3} \qquad \text{......(2)}$$

$$w_1 \frac{dv}{dt} = A\,p \qquad \text{......(3)}$$

$$z_1 = (1-f_1)(1+\theta_1 f_1) \qquad \text{......(4A)}$$

$$z_2 = (1-f_2)(1+\theta_2 f_2) \qquad \text{......(4B)}$$

$$D_1\frac{df_1}{dt} = -\beta_1\, p \qquad \text{......(5A)}$$

$$D_2\frac{df_2}{dt} = -\beta_2\, p \qquad \text{......(5B)}$$

where C_1, F_1, β_1, D_1, θ_1, f_1, z_1 refer to first charge

and C_2, F_2, β_2, D_2, θ_2, f_2, z_2 refer to second charge.

Initially when the shot starts

$$x = 0 ;\ v = 0 ;\ f_1 = f_{10} ;\ f_2 = f_{20} ;\ z_1 = z_{10} ;\ z_2 = z_{20}$$

Let

$$\left.\begin{aligned} \frac{\beta_1}{D_1} &= \beta' \\ \frac{\beta_2}{D_2} &= \beta'' \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots(6)$$

From (3), and (5A), on integration and using initial conditions we get

$$v = \frac{A}{\beta' w_1} (f_{10} - f_1) \qquad \ldots\ldots\ldots\ldots(7A)$$

Similarly from (3) and (5B) we get

$$v = \frac{A}{\beta'' w_1} (f_{20} - f_2) \qquad \ldots\ldots\ldots\ldots(7B)$$

Hence

$$\beta' f_2 - \beta'' f_1 = \beta' f_{20} - \beta'' f_{10} \qquad \ldots\ldots\ldots\ldots(8)$$

From (4A) and (7A) we get

$$z_1 = z_{10} + \frac{\beta' w_1}{A} v (1 - \theta_1 + 2 \theta_1 f_{10}) - \frac{\beta'^2 w_1^2}{A^2} \theta_1 v^2 \qquad \ldots\ldots\ldots(9A)$$

Similarly

$$z_2 = z_{20} + \frac{\beta'' w_1}{A} v (1 - \theta_2 + 2 \theta_2 f_{20}) - \frac{\beta''^2 w_1^2}{A^2} \theta_2 v^2 \qquad \ldots(9B)$$

Let us now define our new variable ξ

$$\xi = 1 + \frac{x}{l} \qquad \ldots\ldots\ldots\ldots(10)$$

From (3)

$$w_1 v \frac{dv}{dx} = A p$$

$$i.e.\ \frac{w_1}{A l} v \frac{dv}{d\xi} = p \qquad \ldots\ldots\ldots\ldots(11)$$

Using (9A), (9B), (10) and (11) in (1) and simplifying we get

$$\xi v \frac{dv}{d\xi} = K (a - v)(b + v) \qquad \ldots\ldots\ldots(12)$$

where

$$K = \frac{F_1 C_1 \beta'^2 w_1}{A^2} \theta_1 + \frac{F_2 C_2 \beta''^2 w_1}{A^2} \theta_2 + \text{[illegible]} \qquad \ldots\ldots\ldots(12A)$$

$$a - b = \frac{1}{K}\left[\frac{F_1 C_1 \beta'}{A}(1 - \theta_1 + 2\,\theta_1 f_{10}) + \frac{F_2 C_2 \beta''}{A}(1 - \theta_2 + 2\,\theta_2 f_{20})\right] \quad \ldots\ldots(12B)$$

$$ab = \frac{1}{K}\,\frac{F_1 C_1 z_{10} + F_2 C_2 z_{20}}{w_1} \quad \ldots\ldots(12C)$$

$$K' = \frac{\gamma - 1}{2} \quad \ldots\ldots(12D)$$

Integrating (12) and using the initial conditions, viz., $v = 0$, $z = 1$, we get

$$z = \left[\left(\frac{a}{a - v}\right)^{a}\left(\frac{b}{b + v}\right)^{b}\right]^{\frac{1}{K(a+b)}} \quad \ldots\ldots(13)$$

From (11) and (12) we have

$$p = \frac{w_1}{A\,l}\,\frac{K(a - v)(b + v)}{z} \quad \ldots\ldots(14)$$

These equations are valid so long as both the propellants are burning.

From (8) we see that if

(1) $\beta^1 f_{20} > \beta'' f_{10}$, then f_2 cannot become zero before f_1. Hence charge C_1, must be burnt out earlier.

(2) $\beta' f_{20} < \beta'' f_{10}$, then f_2 cannot become zero before f_2. Hence charge C_2 must be burnt out earlier.

(3) $\beta' f_{20} = \beta'' f_{10}$, then both the charges will have to be burnt out simultaneously.

Hence we see that two different cases arise, *viz.,*

(*I*) The two propellants burn out at different times.

(*II*) Both the propellants burn out simultaneously.

Let us for the sake of definiteness call that propellant which will burn first as C_1.

Case I.—We have to consider this in two parts

(*i*) when only C_2 is burning.

(*ii*) when C_2 is also burnt out.

I(*i*). Eqn (1) becomes

$$\frac{F_1 C_1 + F_2 C_2 z_2}{A\,l} = p\left(1 + \frac{x}{l}\right) + \frac{\gamma - 1}{2\,A\,l}\,w_1 v^2 \quad \ldots\ldots(15)$$

Proceeding as above we have

$$v \frac{dv}{d\xi} = K_1 (a_1 - v)(b_1 + v) \qquad \text{............(16)}$$

$$\text{where } K_1 = \frac{F_2 C_2 \beta''^2 w_1}{A^2} \theta_2 + K' \qquad \text{..........(16A)}$$

$$a_1 - b_1 = \frac{1}{K_1} \left[\frac{F_2 C_2 \beta''}{A} (1 - \theta_2 + 2\theta_2 f_{20}) \right] \qquad \text{..........(16B)}$$

$$a_1 b_1 = \frac{F_1 C_1 + F_2 C_2 z_{20}}{w_1 K_1} \qquad \text{............(16C)}$$

Let the suffix (2,1) denote the position when the charge C_1 is just burnt out.

$$\text{From (7 A) } v_{2,1} = \frac{A}{\beta' w_1} f_{10} \qquad \text{............(17)}$$

Then $\xi_{2,1}$ is given by equation (13) with $v = v_{2,1}$

Hence

$$\xi = \xi_{2,1} \left[\left(\frac{a_1 - v_{2,1}}{a_1 - v} \right)^{a_1} \left(\frac{b_1 + v_{2,1}}{b_1 + v} \right)^{b_1} \right]^{\frac{1}{K_1(a_1 + b_1)}} \qquad \text{......(18)}$$

From (11) and (16) we have in this case

$$p = \frac{w_1}{Al} \frac{K_1 (a_1 - v)(b_1 + v)}{\xi} \qquad \text{.........(19)}$$

Equations (18) and (19) give us the velocity and pressure at any point between the position when C_1 is just burnt out and the position when C_2 is also just burnt out (*i.e.*, the " all—burnt " position).

I (*ii*). Eqn (1) now becomes

$$\frac{F_1 C_1 + F_2 C_2}{Al} = p\left(1 + \frac{x}{l}\right) + \frac{\gamma - 1}{2Al} w_1 v^2 \qquad \text{............(20)}$$

Proceeding in the same way as in case I(i) we get

$$\xi v \frac{dv}{d\xi} = K' \left[L - v^2 \right] \qquad \text{............(21)}$$

where

$$L = \frac{F_1 C_1 + F_2 C_2}{K' w_1} \qquad \text{............(21A)}$$

Initial conditions for this case are that $v=v_2$ when $\xi=\xi_2$, the suffix 2 denoting the values at "all—burnt".

From (7B)

$$v_2=\frac{A}{\beta'' \ w_1} f_{20} \qquad \ldots\ldots\ldots\ldots (22)$$

and ξ_2 is given by equation (18) with $v=v_2$.

Hence

$$\xi=\xi_2\left[\frac{L-v_2^2}{L-v^2}\right]^{\frac{1}{2K'}} \qquad \ldots\ldots\ldots\ldots (23)$$

From (11) and (21) in this case

$$p=\frac{w_1}{Al}\ \frac{K'\ [L-v^2]}{\xi} \qquad \ldots\ldots\ldots\ldots (24)$$

Hence if v_3 and ξ_3 denote the values at the muzzle we have the muzzle velocity given by the relation

$$v_3^2=L\left[1-\left(\frac{\xi_2}{\xi_3}\right)^{2K'}\right]+v_2^2\left(\frac{\xi_2}{\xi_3}\right)^{2K'} \qquad \ldots\ldots\ldots\ldots (25)$$

II. $\beta' f_{20}=\beta'' f_{10}$ *i.e.*, when both the charges burn out simultaneously. At "all—burnt" the velocity is given by

$$v_2=\frac{A}{\beta' \ w_1} f_{10}=\frac{A}{\beta'' \ w_1} f_{20} \qquad \ldots\ldots\ldots\ldots (26A)$$

and ξ_2 is given by (13).

$$i.e., \ \xi_2=\left[\left(\frac{a}{a-v_2}\right)^a \left(\frac{b}{b+v_2}\right)^b\right]^{\frac{1}{K(a+b)}} \qquad \ldots\ldots\ldots\ldots (26B)$$

After all—burnt the equations are the same as in I (*ii*) and hence

$$\xi=\xi_2\left[\frac{L-v_2^2}{L-v^2}\right]^{\frac{1}{2K'}} \qquad \ldots\ldots\ldots\ldots (27)$$

$$p=\frac{w_1}{Al}\ \frac{K'[L-v^2]}{\xi} \qquad \ldots\ldots\ldots\ldots (28)$$

and hence $$v_3^2=L\left[1-\left(\frac{\xi_2}{\xi_3}\right)^{2K'}\right]+v_2^2\left(\frac{\xi_2}{\xi_3}\right)^{2K'} \qquad \ldots\ldots\ldots\ldots (29)$$

where v_2 & ξ_2 are given by (26A) and (26B) respectively.

Maximum Pressure

The following cases may arise, *viz.*, that the maximum pressure occurs when

(*a*) both the charges are burning.

(*b*) C_1 is burnt out and C_2 is burning.

(*c*) at the position of "all—burnt",

Case (*a*). From (14) we have

$$p = \frac{w_1}{Al}\,\frac{K(a-v)\,(b+v)}{\xi}$$

At the position of maximum pressure dp=O. So differentiating, simplifying and comparing with (12) we get

$$\frac{(a-v_1)\,(b+v_1)}{a-b-2v_1} = \frac{K(a-v_1)\,(b+v_1)}{v_1}$$

where the suffix 1 denotes the values at the position of maximum pressure.

$$\therefore\ v_1 = \frac{K(a-b)}{2K+1}\ or\ v_1 = a\ or\ v_1 = -b.$$

The values $v_1 = a$ and $v_1 =\quad b$ are inadmissible.

Hence

$$v_1 = \frac{K(a-b)}{2K+1} \qquad \text{............(30A)}$$

From (13)

$$\xi_1 = \left[\left(\frac{a}{a-v_1}\right)^a \left(\frac{b}{b+v_1}\right)^b\right]^{\frac{1}{K(a+b)}} \qquad \text{............(30B)}$$

and

$$p_1 = \frac{w_1}{Al}\,\frac{K(a-v_1)\,(b+v_1)}{\xi_1} \qquad \text{............(30C)}$$

The conditions for the occurrence of the maximum pressure in this position are

$f_{11} > 0$

$f_{21} > 0$

Hence from (7A) and (7B) these reduce to

$$\left.\begin{aligned} f_{10} &> \frac{\beta' w_1}{A}\,\frac{K(a-b)}{2K+1} \\ f_{20} &> \frac{\beta'' w_1}{A}\,\frac{K(a-b)}{2K+1} \end{aligned}\right\} \qquad \text{............(30D)}$$

Case (*b*) :—

$\beta' f_{20} > \beta'' f_{10}$

Equations (18) and (19) give and p in this case.

$$p = \frac{w_1}{Al}\,\frac{K_1(a_1-v)\,(b_1+v)}{} \qquad \text{............(19)}$$

Differentiating (19), putting dp=O, simplifying and comparing with (16) we get

$$v_1 = \frac{K_1(a_1-b_1)}{2K_1+1} \qquad \text{.........(31A}$$

Then

$$ {}_1 = t_{2,1}\left[\left(\frac{a_1-v_{2,1}}{a_1-v_1}\right)^{a_1}\left(\frac{b_1+v_{2,1}}{b_1+v_1}\right)^{b_1}\right]^{\frac{1}{K_1(a_1+b_1)}} \qquad \text{............(31B)} $$

and

$$ p_1=\frac{w_1}{Al}\;\frac{K_1(a_1-v_1)\,(b_1+v_1)}{t_1} \qquad \text{............(31C)} $$

The conditions for the occurrence of the maximum pressure in this position are

$f_{11}=0\,;f_{21}\geqslant 0$

Hence

$$ f_{10}=\frac{\beta' w_1}{A}\;\frac{K_1(a_1-b_1)}{2K_1+1} $$

$$ f_{20}>\frac{\beta'' w_1}{A}\;\frac{K_1(a_1-b_1)}{2K_1+1} $$

$\beta' f_{20}>\beta'' f_{10}$.

But in view of the last condition, *viz*, $\beta' f_{20}>\beta'' f_{10}$, f_{21} cannot be equal to zero. Hence the conditions are

$$ \left.\begin{aligned} &\beta' f_{20}>\beta'' f_{10}\\ &f_{10}=\frac{\beta'' w_1}{A}\;\frac{K_1(a_1-b_1)}{2K_1+1}\\ &f_{20}>\frac{\beta'' w_1}{A}\;\frac{K_1(a_1-b_1)}{2K_1+1} \end{aligned}\right\} \qquad \text{..........(31D)} $$

Case (*C*). $\beta' f_{20}=\beta'' f_{20}$. In this case the maximum pressure can occur (*i*) when both the propellants are burning or (*ii*) at the position of "all—burnt".

(*i*) has already been dealt with in case (*a*).

(*ii*) The conditions for this case are

$f_{11}=0\,;\,f_{21}=0$ which reduce to

$$ \begin{aligned} f_{10}&=\frac{\beta' w_1}{A}\;\frac{K(a-b)}{2K+1}\\ f_{20}&=\frac{\beta'' w_1}{A}\;\frac{K(a-b)}{2K+1} \end{aligned} \qquad \text{.......(32)} $$

It is now our aim to find out how the muzzle velocity and maximum pressure change when the total mass of the propellant is kept constant and the proportion of the charges is varied. Hence let us put

$$ C_1+C_2=C \qquad \text{............(33A)} $$

$$ \frac{C_1}{C}=\lambda \qquad \text{............(33B)} $$

$$ \therefore\; C_1=\lambda C \qquad \text{............(34A)} $$

$$ C_2=C\,(1-\lambda) \qquad \text{............(34B)} $$

Calculations have been made on a typical gun. Since the purpose of the calculations is only to illustrate the theory, the calculations have been made simple, by some assumptions. Hence the shot—start pressure has been taken to be zero and the form factors θ_1 and θ_2 have also been put equal to zero. Further only one propellant characteristic has been varied by 5% at one time, the other things being assumed to be the same.

The changes in M.V. & Max. Pressure are given below :—

$$F_2 = 1 \cdot 05\, F_1\, ;\; \beta'' = \beta'\, ;\; \theta_2 = \theta_1 = 0\, ;\; p_0 = 0$$

λ	Muzzle velocity ft/sec.	Maximum Pressure tons/sq.in.
0·0	1383·3	7·370
0·2	1372·1	7·231
0·4	1360·7	7·092
0·5	1355·7	7·023
0·6	1349·3	6·955
0·8	1337·7	6·819
1·0	1326·1	6·685

$$F_2 = F_1\, ;\; \beta'' = 1 \cdot 05\, \beta'\, ;\; \theta_1 = \theta_2 = 0\, ;\; p_0 = 0$$

λ	Muzzle velocity ft/sec.	Maximum Pressure tons/sq.in.
0·0	1371·9	7·370
0·2	1363·2	7·231
0·4	1353·7	7·092
0·5	1349·9	7·023
0·6	1345·2	6·955
0·8	1335·2	6·819
1·0	1326·1	6·685

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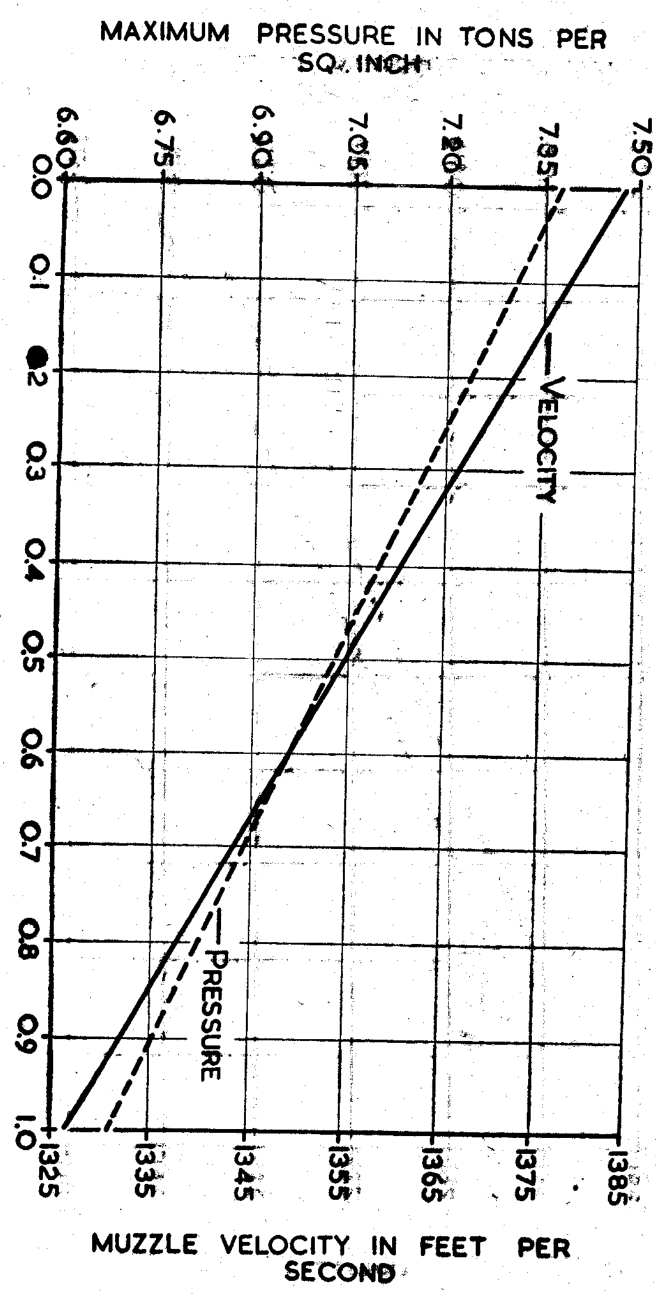
References

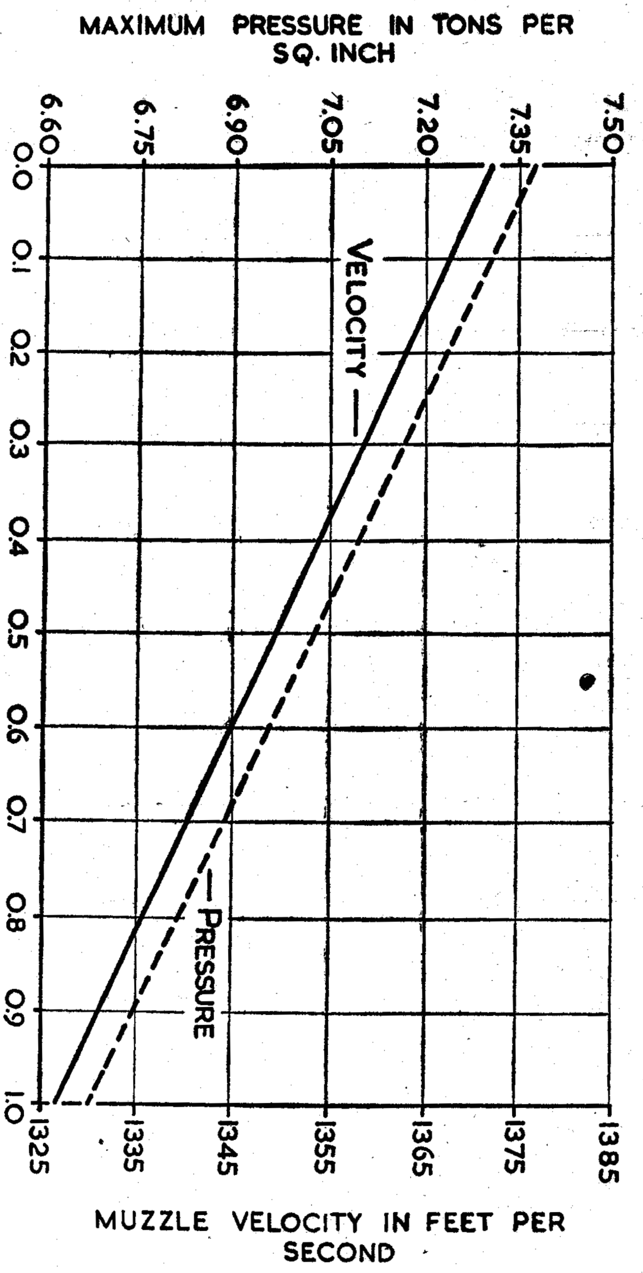
1. Corner, J., Theory of the Interior Ballistics of Guns—John Wiley, New York, 1950.

2. H.M. Stationery Office. Internal Ballistics 1951.


MAXIMUM PRESSURE IN TONS PER SQ. INCH
6.60
6.75
6.90
7.05
7.20
7.35
7.50
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
VELOCITY
PRESSURE
1325
1335
1345
1355
1365
1375
1385
MUZZLE VELOCITY IN FEET PER SECOND

λ →

FIG. 1



$\lambda \rightarrow$

FIG.2