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Estimation of Stability & Control Derivatives from Flight Test Data of Fighter Aircraft

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ABSTRACT

Longitudinal stability and control derivatives of a fighter aircraft are estimated by output error method for different types of input excitation. The uncertainties in the parameters are computed by correcting Cramer-Rao bounds using fudge factor. In general, the step input response data is not used for estimating the derivatives. Therefore, step response time history trajectories were cross-validated using the estimated derivatives for standard inputs like doublet and 3211. This proves that the model parameters are estimated with high confidence. By appropriately choosing the mathematical model and using the corrected flight data for bias and scale factor errors by compatability check for parameter estimation proves beyond doubt that such a procedure can be adopted for estimating stability and control derivatives of any aircraft.

NOMENCLATURE

α	Angle of attack
θ	Pitch angle, parameter vector
p, q, r	Roll, pitch and yaw rates
δ_e	Elevator deflection angle
A,B,C,D	System matrices
a_x, n_x	Longitudinal acceleration (positive forward)
a_z	Normal acceleration (positive down)
M_α	Dimensional pitching moment coefficient due to α
M_q	Dimensional pitching moment coefficient due to pitch rate
M_{δ_e}	Dimensional pitching moment coefficient due to δ_e
Z_α	Z force derivative due to α
Z_{δ_e}	Z force derivative due to δ_e
h	Altitude
g	Acceleration due to gravity
ω	Frequency of aircraft mode
ζ	Damping ratio

1. INTRODUCTION

Estimation of aircraft stability and control derivatives from flight test data is of growing importance in the testing and certification of a modern fighter aircraft. The present study was undertaken by FMCD, National Aerospace Laboratories (NAL) for the Aircraft & Systems Testing Establishment (ASTE) flight test engineers curriculum programme. This programme is aimed to estimate scale factors and biases in measured data for different types of excitation. Further, these estimated scale factors and biases are used to correct the flight data and the corrected flight data

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was used for estimating the parameters of the aircraft.

This paper presents the data compatibility and parameter estimation results obtained from analyses of the flight test data of a fighter aircraft. The flight tests were conducted at ASTE, as a part of a project, for studying the longitudinal short period dynamics of the aircraft. The consistency between the various measured signals during flight tests was initially checked and the corrected data was used for parameter estimation using output error method (OEM) software package

2. FLIGHT TESTS

The basic fighter aircraft was an all-metal mid-wing monoplane, with a delta planform swept back tail. The aircraft primary flying controls were hydraulic-powered all-moving tail plane and frise-type ailerons. The rudders were mechanically operated. The aircraft was used for advanced training of pilots for flying combat missions at subsonic and supersonic speeds both at low and high altitudes. The aircraft was fully instrumented, and flight tests were conducted at an altitude of 3 km at two different Mach numbers (0.65M and 0.85M) using open loop control inputs like doublet, 3211 and step-input. All sorties were flown in clean configuration. The sampling time for the analysis was chosen to be 0.03125 s. The effects of location uncertainty and vane correction are dealt with[1].

2.1 Choice of Inputs

Identifiability of the derivatives depends on the frequency content of the input signal. To determine the particular derivatives, one should have an *a priori* knowledge of which frequencies should be included in the input signal. By properly choosing the input signal, one can excite the required modes of the aircraft and hence estimate the respective derivatives.

This paper describes the estimation of longitudinal derivatives of the aircraft (under study) for different types of input excitation. Since the flight tests were not as per system identification requirements, conducted altogether for a different purpose. The estimated derivatives may not be very accurate. Still an attempt has been made to compare the derivatives for various types of input excitation.

3. DATA COMPATIBILITY CHECK

The measured responses of aircraft are generally to be corrupted with errors due to measurement noise and scale factor errors in the sensor mounting and calibration. The accuracy of the estimated parameters depends on the quality of the flight-measured data. Essentially, the kinematic consistency checking utilises the measured signals like linear accelerations and angular rates as control inputs to the Math-model. Using OEM, the biases and scale factors in the measured data are estimated using the following 5-DOF coupled kinematic equations:

State equation is as follows:

$$
\begin{aligned}
\dot{u} &= (a_x - \Delta a_x) + (r - \Delta r)v - (q - \Delta q)w - g\sin\theta \\
\dot{v} &= (a_y - \Delta a_y) + (p - \Delta p)w - (r - \Delta r)u + g\cos\theta\sin\phi \\
\dot{w} &= (a_z - \Delta a_z) + (q - \Delta q)u - (p - \Delta p)v + g\cos\theta\cos\phi \\
\dot{\phi} &= (p - \Delta p) + (q - \Delta q)\sin\phi\tan\theta + (r - \Delta r)\cos\phi\tan\theta \\
\dot{\theta} &= (q - \Delta q)\cos\phi - (r - \Delta r)\sin\phi
\end{aligned} \tag{1}
$$

where a_x, a_y and a_z are forward, lateral and normal accelerations at sensor location; p, q and r are roll, pitch and yaw rates, respectively; θ and ϕ are pitch and bank angles, respectively; and Δa_x, Δa_y and Δa_z, Δp, Δq and Δr are biases in the corresponding measured signals, respectively.

Measurement equation becomes:

$$
\begin{aligned}
V_m &= V_n + \Delta v \\
\alpha_m &= K_\alpha \tan^{-1}(w_n / u_n) + \Delta\alpha \\
\beta_m &= K_\beta \tan^{-1}(v_n / u_n) + \Delta\beta \\
\phi_m &= K_\phi \phi + \Delta\phi \\
\theta_m &= K_\theta \theta + \Delta\theta
\end{aligned} \tag{2}
$$

where m signifies the measured signal and n refers to corrected signal due to sensor position. K_α, K_β, K_ϕ and K_θ are scale factors to be estimated. Δv, $\Delta\alpha$, $\Delta\beta$, $\Delta\phi$ and $\Delta\theta$ are biases in respective measured signals. True air speed at nose boom is calculated using the relation:

$$V_n = \sqrt{u_n^2 + v_n^2 + w_n^2} \quad (3)$$

where

$$\begin{aligned} u_n &= u-(r-\Delta r)\,y_n + (q-\Delta q)\,z_n \\ v_n &= v-(p-\Delta p)\,z_n + (r-\Delta r)\,x_n \\ z_n &= w-(q-\Delta q)\,x_n + (p-\Delta p)\,y_n \end{aligned} \quad (4)$$

where x_n, y_n and z_n are the distances along x, y and z-axes, respectively from C.G. to pressure port and u, v and w are the velocity components along x, y and z-axes, respectively. The bias and scale factors are estimated using OEM. Typical results of data compatibility check for various types of input excitation are summarised in Table 1. It is evident from Table 1 that the estimated bias for A_Y is very large in comparison with its total magnitude. This is because the data compatibility check is being done using a coupled model for a purely longitudinal manoeuvre. Removing the bias term from the model

Table 1. Results of data compatability check (coupled model)

Parameters	Doublet (sorties)			Input 321			Step			
	1	2	3		2	3				
Altitude (m)	2870.5	2898.9	2927.5	2913.2	2934.6	2934.7	2956.16	2906.1	2920.36	2934.7
U_0 (m/s)	304.8	233.33	230.54	229.8	232.32	234.8	239.5	236.81	232.32	305.85
K_α	2.2405	2.0147	2.0685	2.0333	2.0145	2.0337	1.9493	1.9241	1.9055	2.1524
	(0.465)	(0.82)	(0.87)	(0.307)	(0.63)	(0.46)	(0.61)	(0.42)	(0.725)	(0.56)
[illegible]	0.7013	0.9887	1.1760	1.2135	1.6579	0.9691	1.0432	1.3746	0.96686	1.4356
	(1.515)	(1.22)	(1.47)	(0.48)	(0.75)	(0.66)	(0.75)	(0.54)	(0.77)	(0.86)
K_ϕ	-1.032	-0.987	-0.938	-1.006	-0.985	-0.985	-0.9474	-1.015	-0.9931	-0.9492
	(0.58)	(0.25)	(0.84)	(0.18)	(0.27)	(0.25)	(0.25)	(0.143)	(0.087)	(0.344)
	1.0293	1.1265	1.1284	1.080	1.0972		1.1334	1.0852	1.21386	1.2399
	(1.312)	(0.97)	(0.80)	(0.34)	(0.34)	(0.41)	(0.42)	(0.313)	(0.214)	(0.49)
ΔA	0.5546	-0.077	-0.054	0.1146	0.3688	-0.076	0.1779	0.0378	0.28842	-0.4768
	(12.2)	(64.5)	(56.1)	(15.76)	(5.89)	(25.4)	(9.76)	(63.7)	(7.21)	(4.63)
ΔA_y	11.844	2.0925	16.588	-3.447	-1.773	1.4970	-1.4294	-0.549	-1.7868	1.8866
	(20.1)	(54.1)	(5.09)	(5.81)	(16.2)	(28.8)	(14.68)	(23.74)	(5.78)	(14.7)
ΔA	-0.574	-0.581	-0.579	-1.208	-1.552	-1.465	-1.6707	-1.946	-1.8805	-3.5229
	(40.68)	(33.5)	(6.78)	(1.47)	(1.33)	(3.43)	(2.64)	(1.72)	(3.54)	(1.52)
Δp	-0.0025	0.0002	-0.004	0.002	-0.001	-0.001	0.0030	0.0044	-0.0050	-0.0011
	(9.17)	(86.5)	(6.8)	(4.55)	(13.2)	(20.8)	(3.83)	(4.93)	(2.55)	(13.7)
Δq	0.0131	0.0138	0.0143	0.0174	0.018	0.0165	0.0181	0.0176	0.02170	0.0249
	(5.9)	(5.5)	(1.077)	(0.44)	(0.29)	(1.25)	(0.85)	(0.75)	(1.06)	(0.699)
Δr	0.0409	0.0114	0.0684	-0.018	-0.011	0.0092	-0.0019	-0.002	-0.0018	0.0053
	(19.07)	(39.6)	(5.19)	(4.88)	(11.4)	(19.7)	(40.9)	(26.8)	(21.9)	(16.95)
Δv	-0.7219	-18.64	-7.307	2.7859	-4.018	-6.493	-28.083	15.825	-14.427	-2.0026
	(50.19)	(3.59)	(7.85)	(17.47)	(14.8)	(8.21)	(2.47)	(3.12)	(4.25)	(17.52)
$\Delta\alpha$	0.0048	-0.064	-0.128	-0.051	-0.012	-0.075	-0.05817	-0.047	-0.0430	-0.0963
	(36.28)	(5.06)	(1.96)	(4.13)	(24.5)	(3.87)	(3.72)	(2.12)	(3.8)	(2.187)
$\Delta\beta$	0.0215	0.0443	0.0316	0.0352	-0.038	-0.0003	-0.0121	0.0297	-0.0244	0.0224
	(4.87)	(2.15)	(2.99)	(3.11)	(5.30)	(577)	(11.54)	(2.99)	(4.83)	(70.77)
$\Delta\phi$	-0.0989	-0.212	-0.092	0.0018	-0.044	-0.0950	-0.01574	-0.073	-0.2887	-0.0357
	(1.42)	(0.41)	(1.32)	(75.4)	(4.34)	(1.39)	(1.89)	(6.8)	(0.756)	(3.93)
$\Delta\theta$	0.1420	0.0986	0.0836	0.1221	0.1289	0.1255	0.1237	0.1487	0.13823	0.1723
	(1.00)	(2.25)	(1.66)	(1.21)	(1.40)	(1.39)	(1.29)	(0.97)	(0.922)	(0.897)

for data compatibility check results in convergence problems, and hence, this term is retained in the model. However, this value is not used for correcting the data.

Data compatibility check-time history match between estimated and flight data trajectories along with control inputs for different types of input (doublet, 3211 and step) is shown in Figs 1(a), 1(b) and 1(c). The acquired flight data is corrected for bias and calibration errors and then is used for parameter estimation.

4. PARAMETER ESTIMATION

Analysis of flight test data includes the mathematical model of the aircraft and an estimation criterion. By iterative computational algorithm, estimation criterion adjust *a priori* estimates of the parameters until a set of best parameter estimates is obtained which minimises the response error[2]. The general representation for the physical system for nonlinear systems with measurement noise has been considered.

$$\dot{x} = f(x,u,\theta) \text{ where } x(0) \text{ is known or estimated.}$$
$$y = f(x,u,\theta)$$
$$z(t) = y(t) + \text{noise}$$

Using N sampled values of input and output time history, the maximum likelihood problem can be formulated in a probabilistic manner by defining the likelihood function as the conditional probability density function of the measurements $z(t)$ given R and θ (R is the measurement noise covariance matrix and q is the parameters vector). The likelihood function can be maximised by minimising negative log-likelihood function.

$$L = \tfrac{1}{2}\sum_{i=1}^{N}\left[z(t)-y(t)\right] \; R \; \left[z(t)-y(t)\right] + (N/2)\, ln|R| \tag{6}$$

The OEM performs this minimisation and yields the estimates of the parameters and initial conditions. In addition, it generates the predicted model response. Using OEM for the reconstructed flight data, the longitudinal aircraft parameters are estimated using the following 3-DOF model:

State equations

$$\begin{aligned}\dot{\alpha} &= (Z_\alpha/U_0)\alpha + (Z_{\delta_e}/U_0)\delta_e + q \text{ bias}\\ \dot{q} &= M_\alpha\,\alpha + M_q\,q + M_{\delta_e}\,\delta_e + \text{bias2}\\ \dot{\theta} &= q + \text{bias3}\end{aligned} \tag{7}$$

where Z_α, M_α, M_q are aircraft's dimensional stability derivatives; Z_{δ_e}, M_{δ_e} are aircraft's dimensional control derivatives; U_0 trim longitudinal velocity and bias1, bias2, bias3 are biases in corresponding states.

Observation equations

$$\begin{aligned}\alpha &= \alpha + \text{bias4}\\ q_m &= q + \text{bias5}\\ \theta_m &= \theta + \text{bias6}\\ nz_m &= (Z_\alpha/g)\alpha + \text{bias7}\end{aligned} \tag{8}$$

The short period natural frequency and damping are calculated using

$$\begin{aligned} w_{sp} &= \sqrt{\left(-M_\alpha + (Z_\alpha M_q/U_0\right)}\\ \zeta_{sp} &= -\frac{M_q + (Z_\alpha/U_0)}{2W_{sp}}\end{aligned} \tag{9}$$

5.

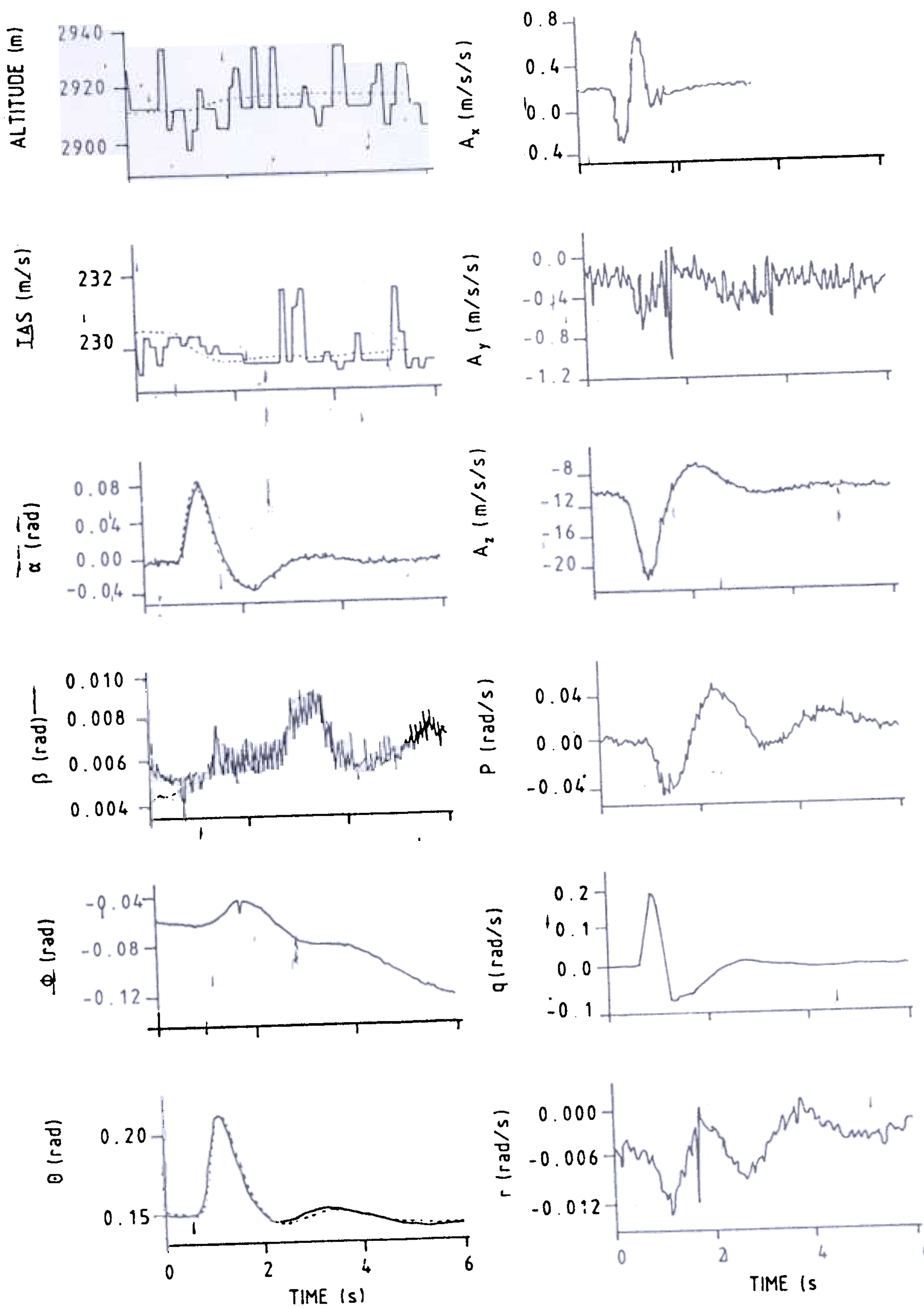

Figure 1(a). Data compatibility check (coupled model); (doublet input, __ FLT, --- ESTM)

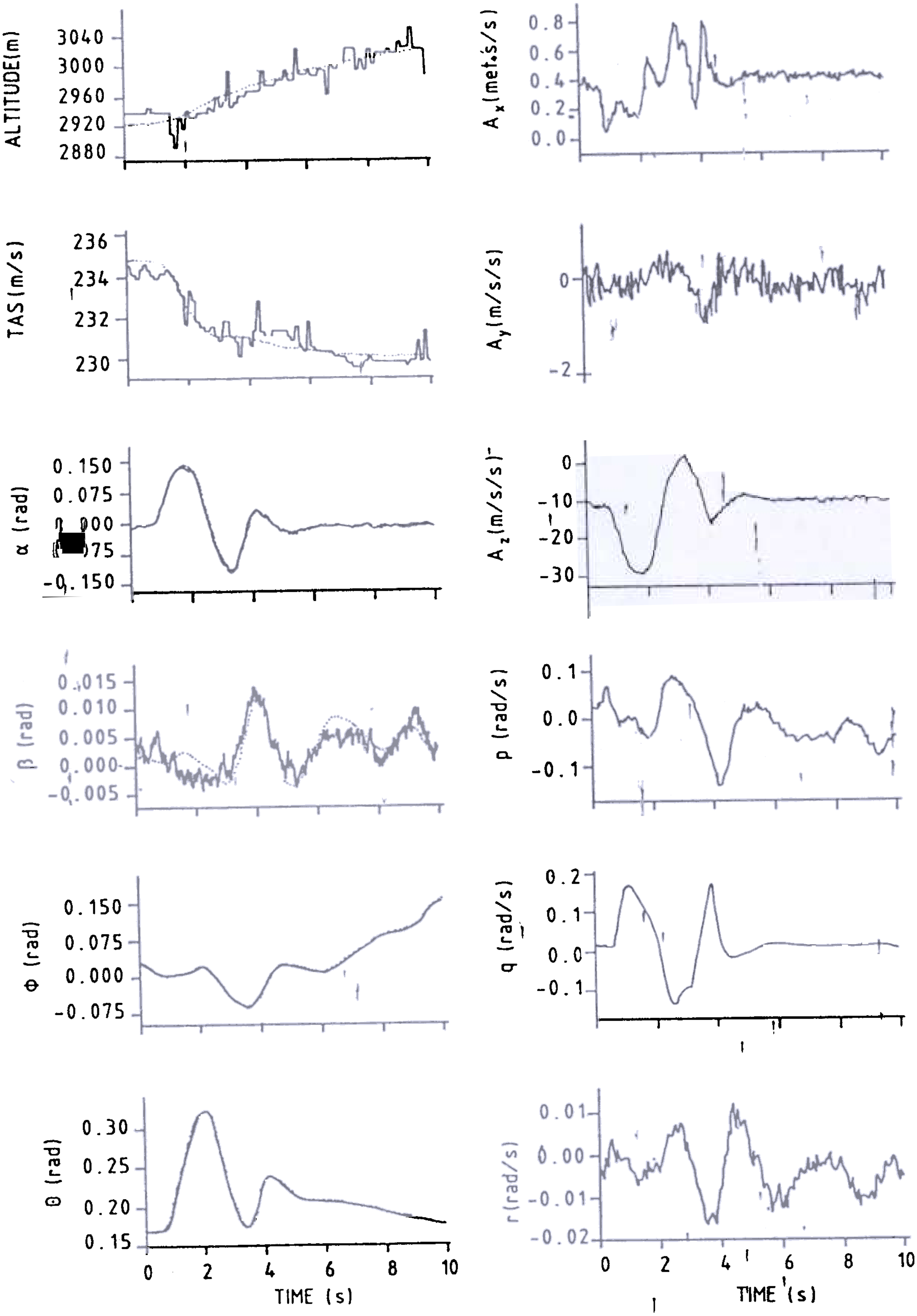

Figure 1(b). Data compatibility check (coupled model); (3211 input, __ FLT, --- ESTM)

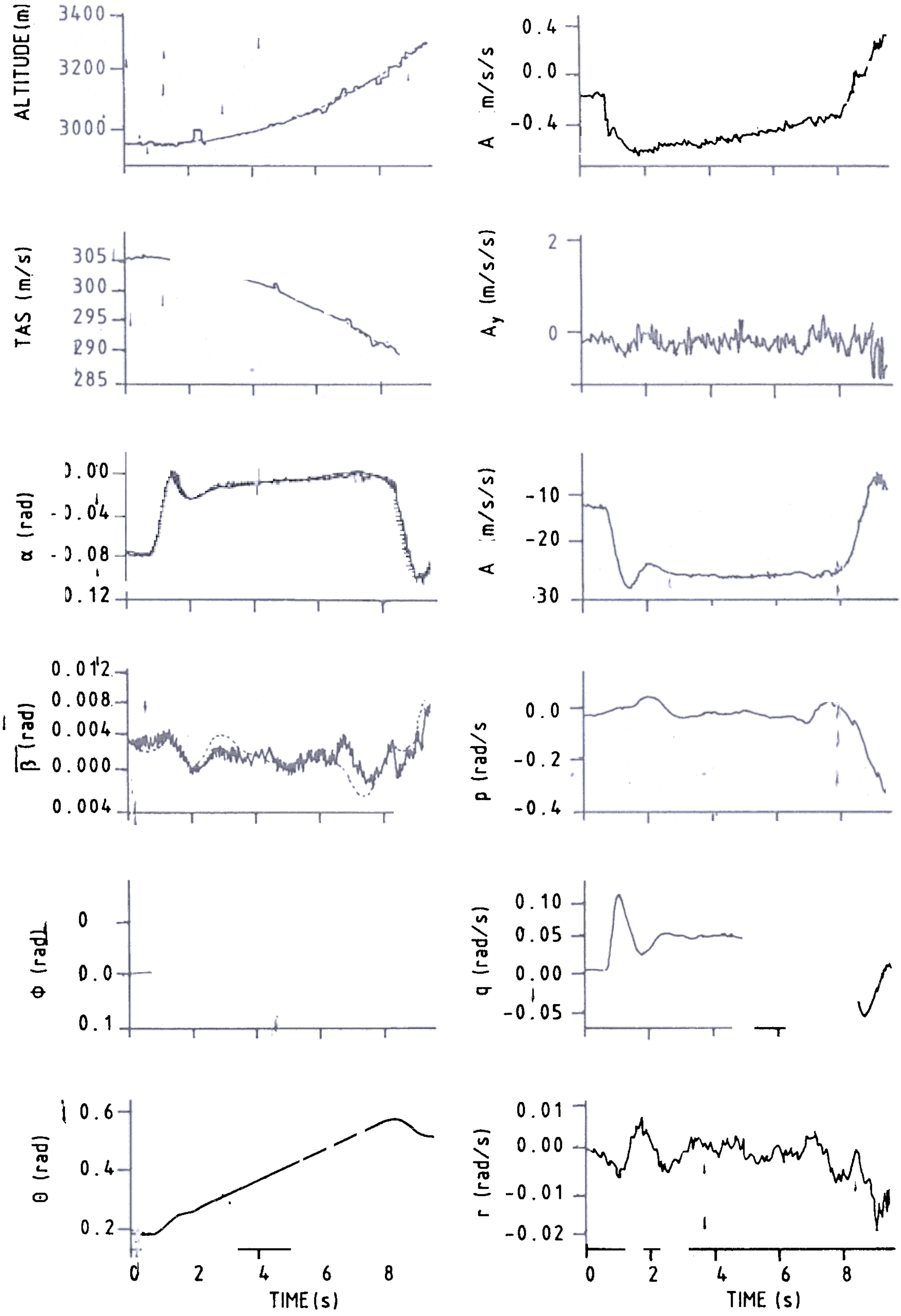

Figure 1(c). Data compatibility check (coupled model); (step input, FLT, ESTM)

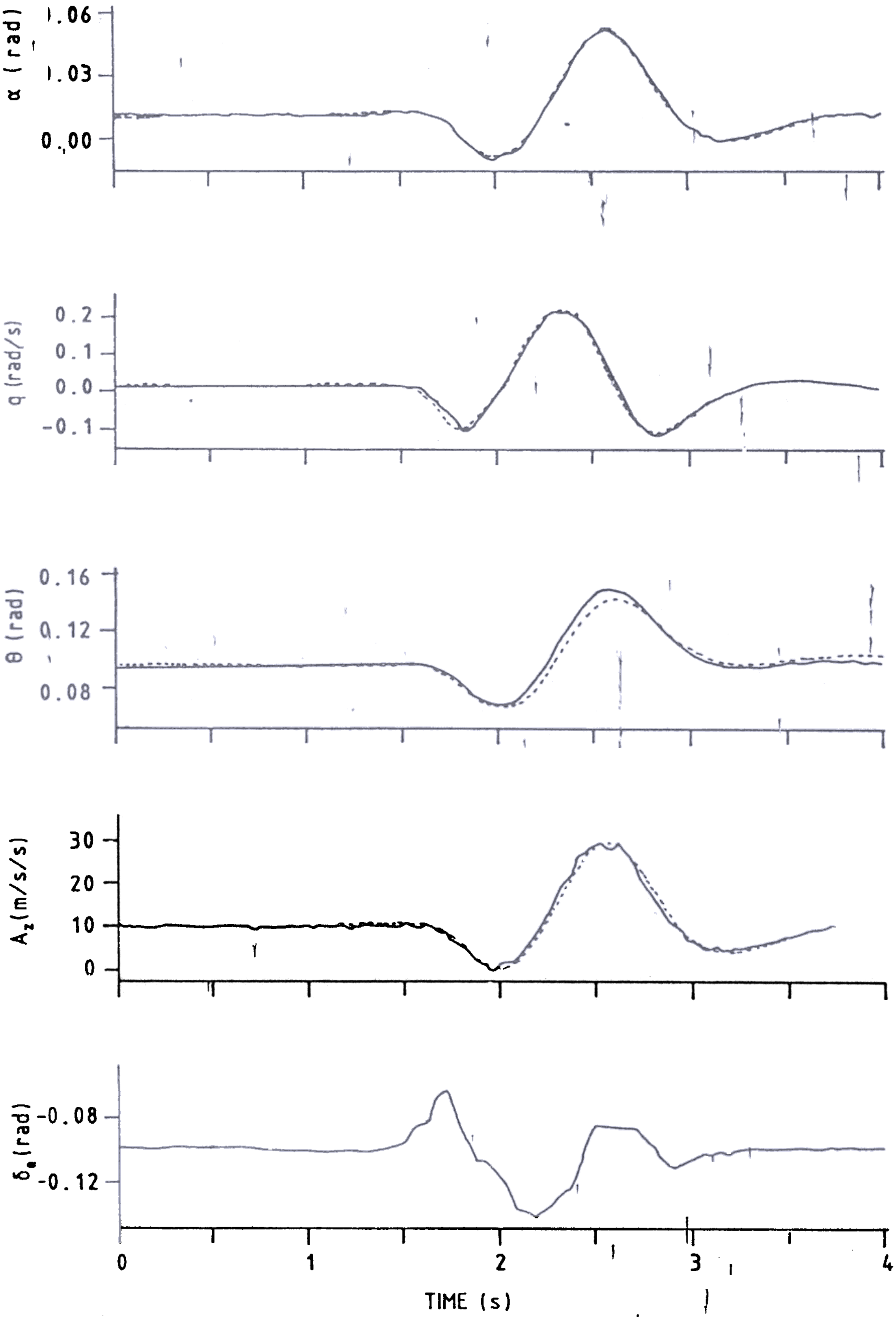

Figure 2(a). Time history match (parameter estimation, short period) (doublet input, FLT, .,. ESTM)

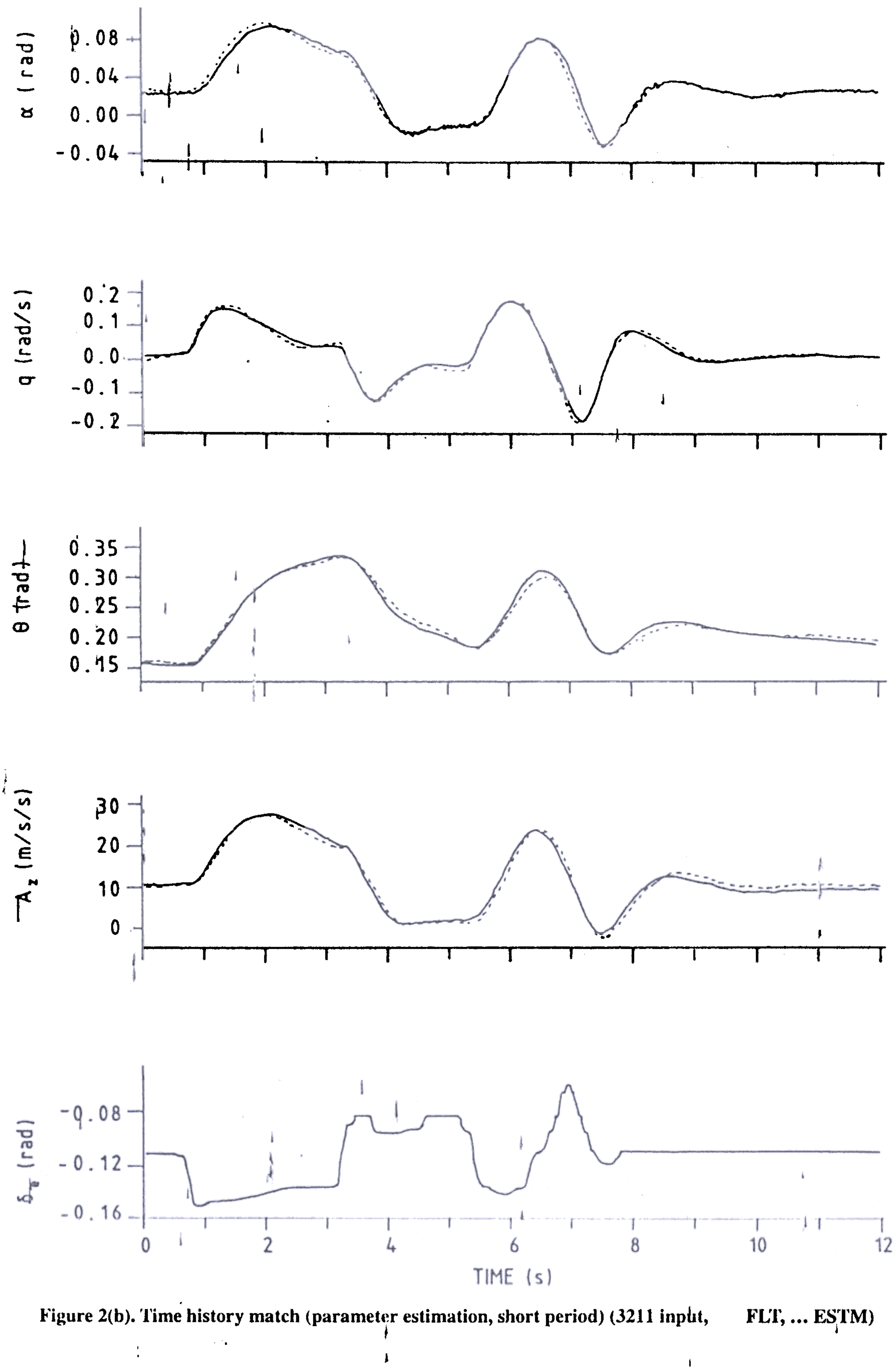

Figure 2(b). Time history match (parameter estimation, short period) (3211 input, FLT, ... ESTM)

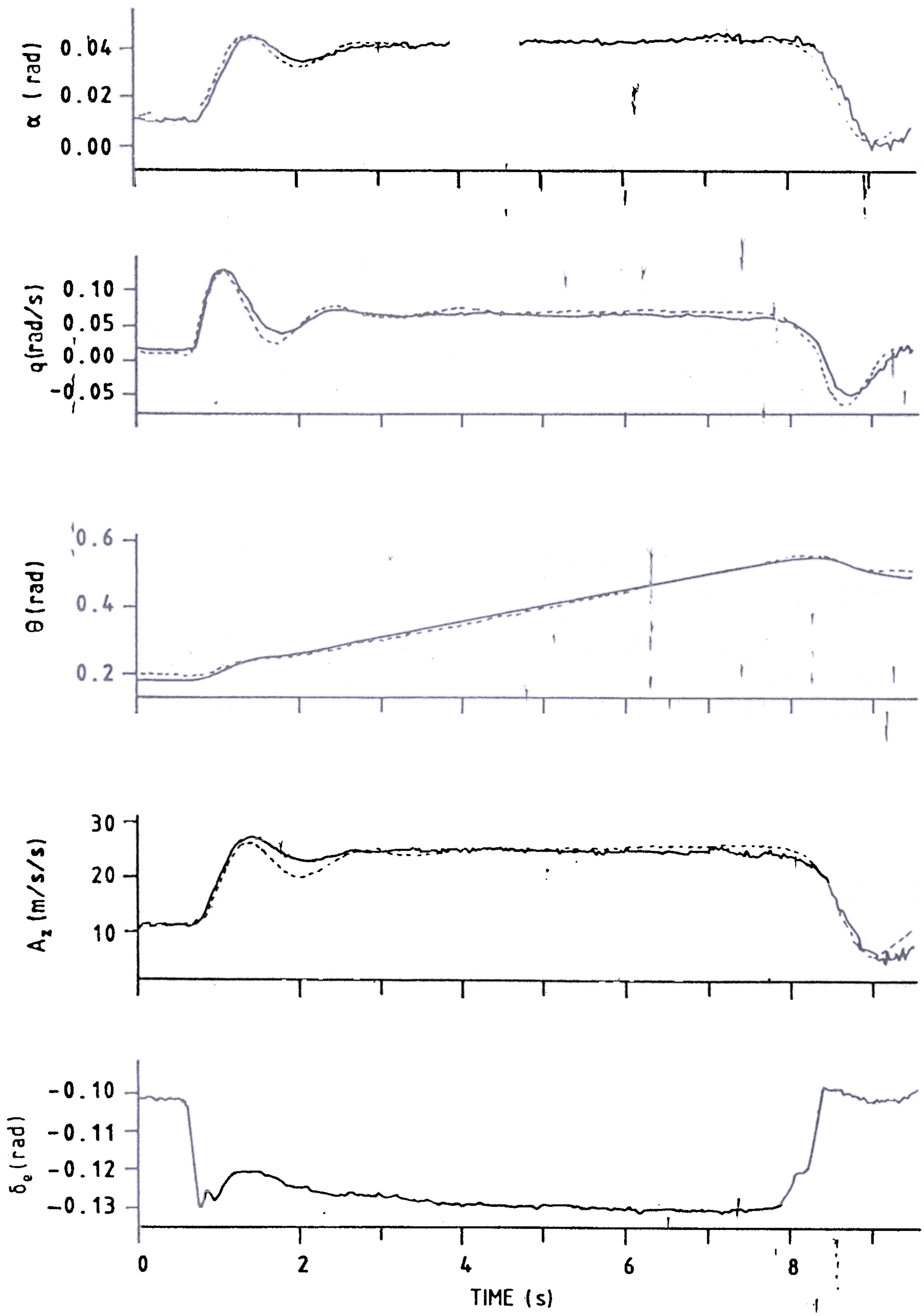

Figure 3. Time history match (parameter estimation, step input) (cross-validation, ___ FLT, ... ESTM)

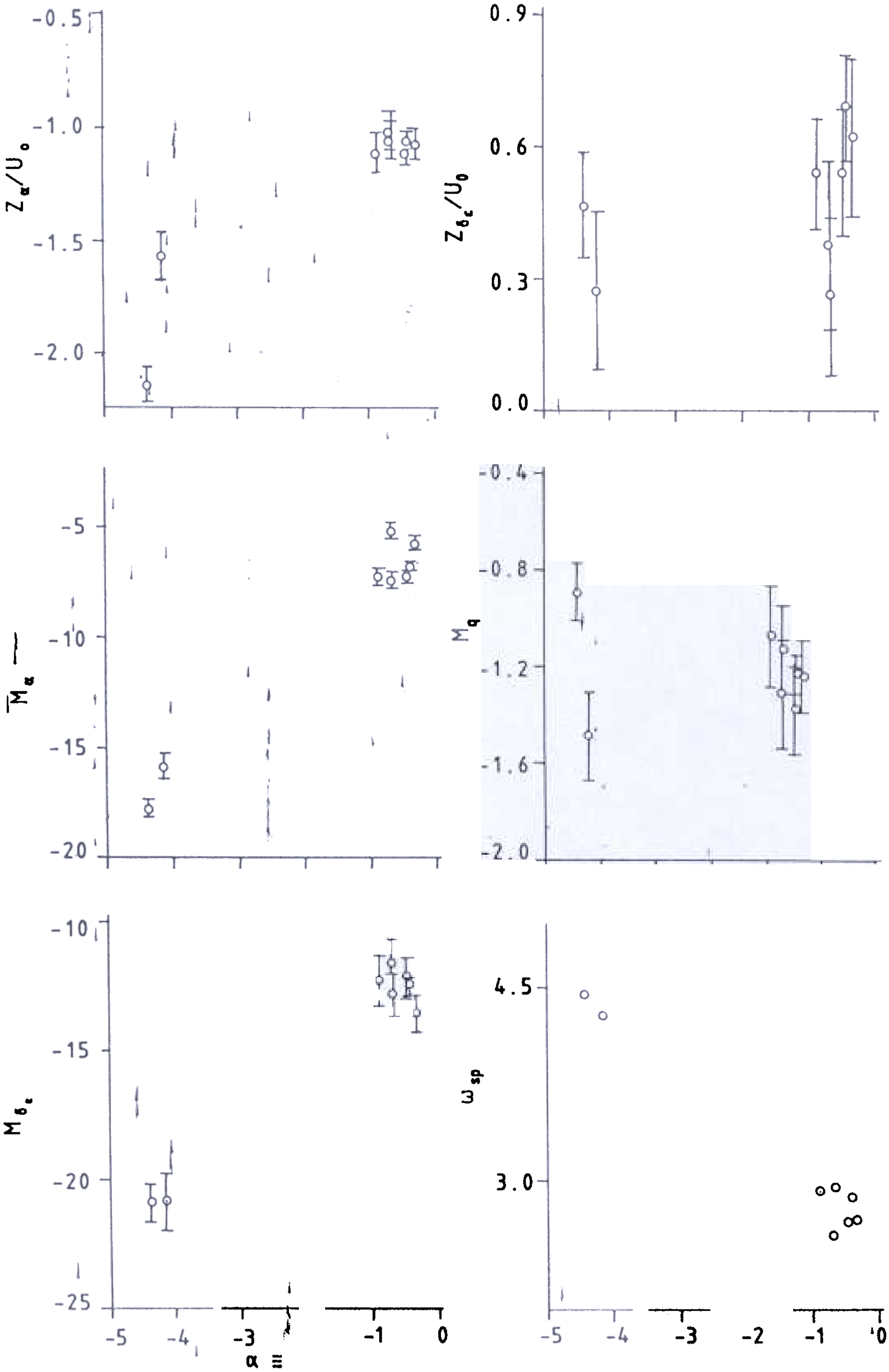

Figure 4. Variation of derivatives with angle of attack (α)

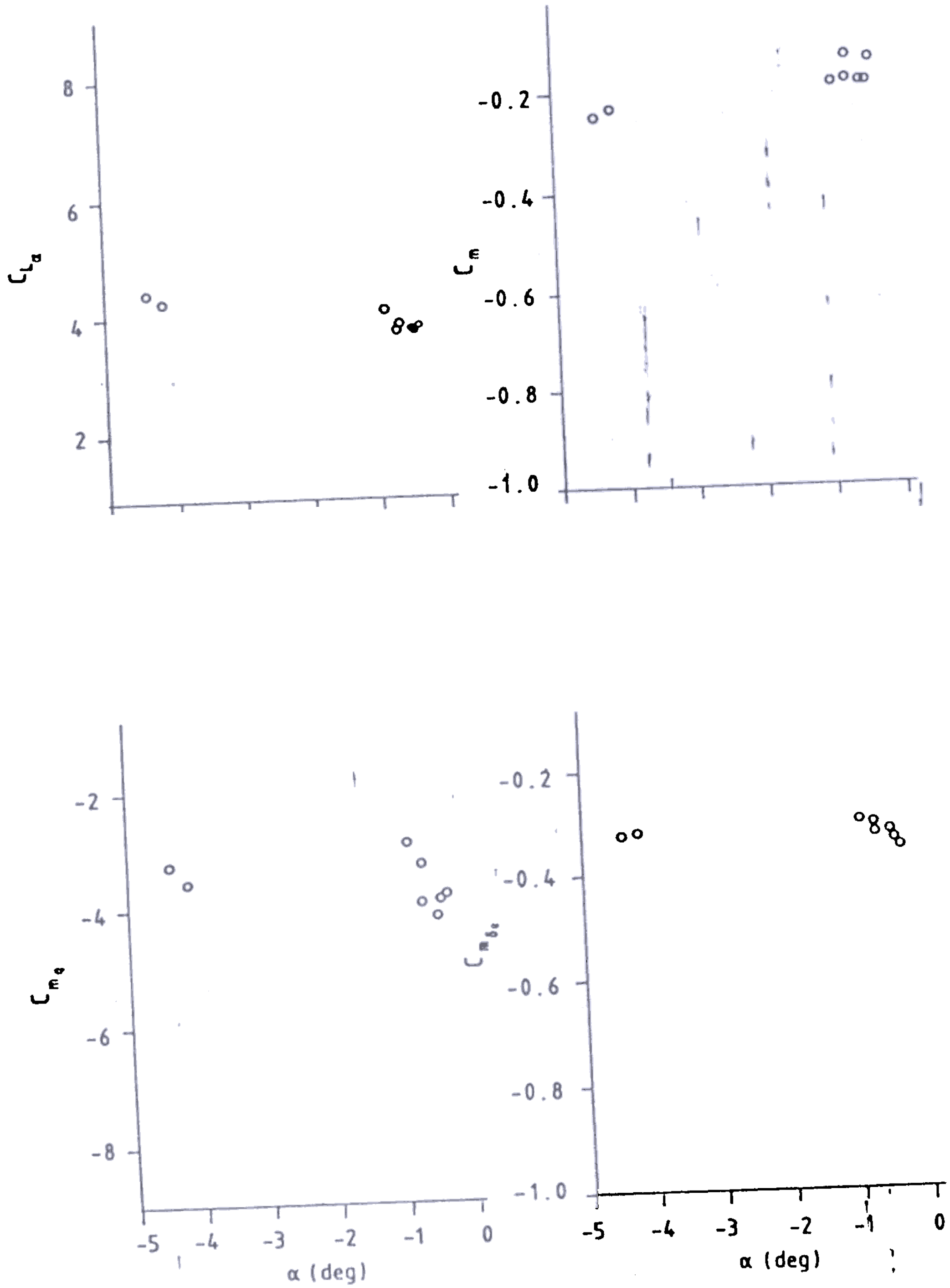

Figure 5. Variation of non-dimensional derivatives with angle of attack (α)

$$FF = \sqrt{SF/(2RBW} \qquad (10)$$

Table 2 also lists the fudge factor along with the derivatives.

In general, the step input response data is not used for estimating the stability and control derivatives. Therefore, by using the estimated derivatives for standard inputs like doublet or 3211 step input response data was cross-validated. Figure 3 shows the time history match between the estimated and flight test data trajectories for step input (cross-validation plot). This shows that the model parameters are estimated with high confidence.

Figures 4(a) and 4(b) show the variation of estimated derivatives plotted with the corrected standard deviations (using fudge factor to account for coloured residuals) and short period frequency with angle of attack (trim α, obtained from the time history at initial time). For the sake of completeness, estimated derivatives presented in Table 2 are converted to non-dimensional form, and are put together and plotted with α in Fig. 5.

6. CONCLUSION

Longitudinal stability and control derivatives of fighter aircraft were estimated by OEM for different types of input excitation. The uncertainties in the parameters were computed by correcting Cramer-Rao bounds using fudge factor. The step input response data is cross-validated using the estimated derivatives for standard inputs like doublet. The results generated by the procedure of correcting the data by kinematic consistency check and parameter estimation using OEM after formulating an appropriate Math-model outlined in this paper clearly proves that the same can be used for estimating stability and control derivatives of any stable aircraft.

Table 2. Short period analysis results (3-DOF dimensional model) – estimated derivatives

Parameters	Doublet					3211		
		2				.2	3	4
Trim α (deg) +	-4.156	-0.417	-4.39	-0.32	-0.458	-0.69	-0.889	-0.67
Altitude (m)	2870.5	2898.5	2927.5	2927.5	2913.2	2934.6	2934.6	2956.16
Velocity (m/s)	304.83	233.33	305.52	230.54	229.78	232.32	234.84	239.53
Z_α/U_{0l}	-1.5718 (0.017)*	-1.0649 (0.01)	-2.1444 (0.019)	-1.0779 (0.0115)	-1.0191 (0.007)	-1.0212 (0.0095)	-1.1135 (0.012)	-1.06235 (0.0099)
Z_{δ_e}/U_0	0.2728 (0.033)	0.6881 (0.0285)	0.4658 (0.0294)	0.6192 (0.0296)	0.2641 (0.0163)	0.3761 (0.0234)	0.5389 (0.026)	0.2612 (0.021)
M_α	-15.8687 (0.1013)	-6.8212 (0.0487)	-17.763 (0.0923)	-5.7533 (0.0468)	-5.7049 (0.030)	-5.1929 (0.041)	-7.2357 (0.054)	-7.4292 (0.039)
M_a	-1.4999 (0.0338)	-1.2375 (0.0202)	-0.9027 (0.0337)	-1.2439 (0.025)	-1.3861 (0.02)	-1.3199 (0.0263)	-1.0797 (0.031)	-1.1367 (0.02)
M_{δ_e}	-20.9058 (0.1830)	-12.498 (0.099)	-20.958 (0.182)	-13.5443 (0.125)	-12.1848 (0.087)	-11.6676 (0.1137)	-12.281 (0.145)	-12.8263 (0.09)
Fudge factor	6.339	4.241	4.2587	6.4215	9.09	9.07	7.15	9.07
	4.2692	2.8529	4.4343	2.6635	2.6678	2.5575		2.9385
ξ_{sp}	0.3598	0.4035	0.3391	0	0.4508	0.4577		0.3741

* Absolute standard deviation

\+ Trim α obtained directly from the flight data

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