Aerodynamic Jump for Long Rod Penetrators

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ABSTRACT

Aerodynamic jump for a non-spinning kinetic energy penetrator is neither a discontinuous change in the direction of motion at the origin of free flight, nor is it the converse, i.e. a cumulative redirection over a domain of infinite extent. Rather aerodynamic jump, for such a projectile, is a localised redirection of the centre of gravity motion, caused by the force of lift due to yaw over the relatively short region from entry into free flight until the yaw reaches its first maximum. The primary objective of this paper is to provide answers to the questions like what is aerodynamic jump, what causes it, and what aspects of the flight trajectory does it refer to, or account for.

NOMENCLATURE

A  Projectile cross-sectional area
\( \bar{D} \)  Aerodynamic drag force
\( \bar{L} \)  Aerodynamic lift force
\( \bar{c}_g \)  Centre of gravity
\( \bar{M} \)  Moment due to forces \( L \) and \( D \) about the projectile centre of gravity
\( \bar{R} \)  Resultant force
\( \bar{Y} \)  Transverse displacement of projectile cg, measured from the swerve axis
AJ  Aerodynamic jump
\( C_D \)  Drag coefficient
FF  Free flight
\( I_t \)  Transverse moment of inertia of (symmetric) projectile
KE  Kinetic energy
LD  Launch disturbance
\( C_L \)  Lift coefficient
\( C_{\alpha L} \)  Derivative of lift coefficient wrt \( \alpha \)
\( C_{\mu L} \)  Derivative of restoring moment coefficient wrt \( \alpha \)

Greek symbols

\( \alpha \)  Projectile yaw angle
\( \varepsilon \)  Angle between cg velocity vector and original line of fire
\( \lambda \)  Wavelength of swerve curve
1. INTRODUCTION

The motion of a projectile can be divided into two general regions: (i) free flight (FF) region, and (ii) launch disturbance (LD) region (prior to FF). For instance, if the projectile is a gun-launched, saboted long rod or kinetic energy (KE) penetrator, then the LD region begins in-bore and extends downrange to the point, where shock waves from the discarding sabot petals no longer interact with the rod. The end of the LD region marks the beginning of the FF region, where the phenomenon known as AJ occurs. The KE penetrator is chosen to facilitate the ensuing discussion and illustrations on the subject of AJ.

Although AJ occurs in the FF region, its magnitude is influenced by events that take place in the LD region. A KE projectile consists of a long rod with an aerodynamically-shaped nose and stabilising tail fins. The high mass-density sub-calibre rod is held centered in the gun bore by a low mass-density full-calibre sabot. The rod can undergo small, lateral, cg displacements and rotations while being propelled longitudinally down the bore. Such in-bore motion permits the projectile cg to exit the barrel with a velocity vector oriented at an \( \angle CG \) wrt the instantaneous bore axis. In addition to the rod moving relative to the bore axis, the barrel itself can be moved. Thus, the rod can be launched with the instantaneous pointing angle of the bore axis, \( \angle PA \), different from the original muzzle sight line. Furthermore, the instantaneous bore axis can have a lateral (crossing) velocity that is transferred to the projectile cg motion. The angular change in the projectile cg velocity due to this barrel crossing motion is denoted by \( \angle CV \). Outside the gun, it is possible for asymmetric sabot discarding to create uneven mechanical and aerodynamic forces on the rod that add yet another transverse cg velocity component, and redirecting angle, \( \angle SD \). The net effect of these four pre-FF LOs can give the projectile cg a cumulative transverse deflection angle, \( \angle LD = \angle CG + \angle PA + \angle CV + \angle SD \), at the point where it enters FF (Fig. 1). Techniques to measure these LO components are discussed by Bornstein et al.

. After transitioning the LD region, side forces can continue to influence the lateral motion of the projectile in the FF region. These side forces are aerodynamic in nature and cause the projectile cg to oscillate (swerve) about a mean FF path (swerve axis), as it travels to the target (Fig. 2). For a typical KE rod (which is statically stable, near-symmetric, and virtually non-rolling), the swerve curve can be approximated by a damped sine wave in both vertical and horizontal directions*. As indicated in Fig. 2, the swerve axis can be, and most often is different from the direction given to the projectile cg as it leaves the LD region. The term AJ, in particular \( \angle AJ \), is used to quantify this change in direction.

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\* The effects of gravity and the Coriolis force on the trajectory are not included in this discussion because they are not aerodynamic in nature; if warranted, their influence can simply be superimposed on the swerve motion.
One of the earliest descriptions of AJ was given by Murphy\(^2\) stating that AJ is the angle between the bore sightline and the average trajectory when the other contributors to jump are neglected. Although this definition describes AJ as an angle, it is actually the tangent of the described angle. However, for small AJ angles (typically the case), the angle and its tangent are nearly one and the same. Neglecting other contributors to jump means setting, or assuming, \(\angle LD = 0\) in the discussion of Fig. 1. In this case, Fig. 2 would transform into Fig. 3.

Figure 3 depicts that the axis of swerve symmetry is closely aligned with the point of impact on a distant target. In fact, when the FF trajectory approaches infinity, AJ and \(\angle AJ\) are defined by

\[
AJ = \lim_{z \to \infty} \left[ \frac{y}{z} \right]; \quad \angle AJ = \tan^{-1} (AJ)_{AJ,LAJ} \approx AJ
\]

where \(y\) represents the transverse cg displacement and \(z\) represents the longitudinal or downrange displacement\(^*\). Both Murphy\(^2\) and Murphy and Bradley\(^3\) begin their discussion of AJ based on Eqn (1). A more detailed expression for AJ, one that does not neglect other contributors to jump, is put forth later by Murphy\(^4\). This more general definition states\(^*\)

\[
AJ = \lim_{z \to \infty} \left[ \frac{y(z) - y_o}{z - z_o} \right] \frac{dy}{dz}_{z_o} \approx \angle AJ = \tan^{-1} (AJ)_{AJ,LAJ} \approx AJ
\]

where \(y_o\) is the transverse cg displacement, and \(dy/dz\) is the tangent to the cg displacement, both at the origin of FF. Figure 4 gives the geometrical interpretation of Eqn (2). Equations (1) and (2) define AJ by calling upon the limit as the trajectory approaches infinity; to some, this may erroneously infer that AJ is an effect that accumulates with downrange distance. An alternative kinematical definition for AJ can be given as one that does not invoke an infinite limit, but rather, attributes AJ to a

** The sign convention for the direction of positive \(y\) in Eqn (1) will determine the sign convention for positive AJ.

# Equation (2) here is actually the single-plane equivalent of combining Murphy's\(^5\) Eqns (9) and (10) with gravitational and Coriol effects neglected.

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Figure 2. Characterisation of cg transition into FF region

Figure 3. Geometrical view of \(\angle AJ\), neglecting \(\angle LD\)

Figure 4. Geometrical rendering of Eqn (2)
relatively short segment of the FF trajectory, less than one-half of one swerve oscillation in length.

2. KINEMATICAL DEFINITION FOR AERODYNAMIC JUMP

From Eqn (2) and Fig. 4, $\angle AJ$ is the angular change between the tangent to the cg trajectory at the end of LD region and the axis of swerve symmetry. Figure 5 shows that the axis of swerve symmetry runs parallel to the tangent to the swerve curve at any and all of the local swerve maxima (positive or negative wrt the swerve axis).

Hence, $\angle AJ$ can also be defined as the angular difference between the tangent to the swerve curve at the origin of FF and the tangent to the swerve curve at the first (or second, or third, etc.) local maximum in the swerving motion. In equation form, it can be expressed as

$$\angle AJ = \tan^{-1} \left( \frac{dy}{dz} \biggl|_{\text{swerve maximum}} \right) - \tan^{-1} \left( \frac{dy}{dz} \biggl|_{\text{orig of line flight}} \right)$$

$$\approx \frac{dy}{dz} \biggl|_{\text{swerve maximum}} - \frac{dy}{dz} \biggl|_{\text{orig of line flight}}$$

(3)

where the subscripts identify the locations at which the derivatives are to be evaluated. It is noted that unlike Eqns (1) and (2), the definition for $\angle AJ$ given in Eqn (3) does not call upon the limit as the trajectory approaches infinity.

Even though $\angle AJ$ can be defined using the tangent line at any of the local maximum, it is clear that the minimum distance needed to establish the orientation of the swerve axis is the distance to the first swerve maximum, $z_1$. Thereafter, the cg motion simply oscillates about this axis, albeit with a damped amplitude.
The limitless definition of Eqn (3) facilitates some additional insight into the kinematic relationship between \( \angle AJ \) and the initial conditions at the origin of FF. Take special cases illustrated in Figs 6(a) and 6(b), unlike Fig. 5, the swerve axes in these two cases are nearly parallel with the LD direction, hence, \( \angle AJ \) is nearly zero. From Fig. 6(a), for instance, the distance between the origin of FF, \( z_0 \), and the first swerve maximum, \( z_1 \), is relatively small, at least in comparison to the wavelength of the swerved curve, \( \lambda \). On the other hand, in Fig. 6(b), \( z_1 - z_0 \) is relatively large, \( \approx \lambda/2 \). Contrasting the larger \( \angle AJ \) of Fig. 5 with that of Figs. 6(a) and 6(b), it can be inferred that the largest \( \angle AJ \) will occur when \( z_1 - z_0 = \lambda/4 \). In fact, if the swerve curve is approximated by a sine wave of the form \( y = A \sin(2\pi[z-z_0]/\lambda) \), at least for the first cycle, then, from Eqn (3), the maximum \( \angle AJ \) would be given by

\[
\angle AJ = \frac{d\{y = A \sin\left(\frac{2\pi[z-z_0]}{\lambda}\right)\}}{d\{z = z_0 + \lambda/4\}}
\]

\[
A \frac{2\pi}{\lambda}
\]

To appreciate the significance of Eqn (4), Fig. 7 illustrates how \( \angle AJ_{\text{max}} \) varies with \( A \) and \( \lambda \) for two cases where \( y \) conforms to \( A \sin(2\pi[z-z_0]/\lambda) \). From the depiction (and Eqn (4)), a larger \( A \) and smaller \( \lambda \) produce a larger \( \angle AJ_{\text{max}} \). For large calibre guns, \( A \) may be of the order of several millimeters, whereas \( \lambda \) is of the order of tens of meters; hence, \( \angle AJ_{\text{max}} \) from Eqn (4), will be small—of the order of milliradians.

Figures 5–7 illustrate that the axis of swerve symmetry is fixed in space by the time the rod reaches its first swerve maximum, as implied by Eqn (3). These also provide visible examples that support the contention that it is not necessary to take the swerving motion to infinity, as called for in Eqns (1) and (2), in order to establish the direction of the swerve axis.

Based on the kinematical developments discussed here, it is a simple matter to derive a dynamical expression for \( AJ \). However, before such an expression can be formulated, it is beneficial to review some basic aerodynamics.

### 3. BASIC AERODYNAMIC FORCES & MOMENTS ACTING ON A NON-SPINNING KE PENETRATOR

The force of friction and drag on the projectile are probably the most fundamental of the aerodynamic forces. It is commonly expressed as

\[
D = \frac{1}{2} C_D \rho A |\vec{u}| \vec{u}
\]  

and, by virtue of the minus sign, drag is in the direction opposite \( \vec{u} \).

The expression for lift is conventionally written as

\[
L = \frac{1}{2} C_L \rho A |\vec{u}| \vec{\hat{l}}
\]

The unit vector direction of the lift force, \( \vec{\hat{l}} \), is perpendicular to the drag force and is in the yaw plane. In this discussion, yaw is the vertical (z–y) plane angle, \( \alpha \), between the projectile's tail-to-nose axis and the tangent to its trajectory (or equally suitable, \( \vec{u} \)). It is assumed here that \( \alpha + \delta \) means the nose of projectile is above \( \vec{u} \).

For small yaw (e.g. \( \alpha < 5^\circ \)),

\[
C_L = C
\]  

Suppose the original direction of fire is defined to be the positive z-axis, with positive y downward, and positive x to the gunner's right. Assume the cg motion is 2-D planar, in particular, assume (for illustrational simplicity) that the motion is confined to the vertical plane, then \( \vec{u} = \hat{z} \hat{z} + \hat{y} \hat{y} \) as depicted in Fig. 8.
Suppressing the effects of gravity and the Coriolis force, Newton’s second law for linear motion in the \( \hat{y} \) direction* dictates that

\[
m \frac{d\hat{y}}{dt} = \vec{L} \hat{y} + \vec{D} \hat{\hat{y}} = -\frac{\alpha}{|\alpha|} \left( |\vec{L}| \cos \varepsilon \right) + |\vec{D}| \sin \varepsilon
\]

where \( \alpha /|\alpha| \) accounts for the positive or negative influence of yaw on lift.

The value of \( \varepsilon \) is always small, and it would expedite the analysis to assume it is zero, however, such an over simplification is not necessary. The cg velocity vector, and hence \( \varepsilon \), oscillates about some mean values, \( \bar{u} \) and \( \bar{\varepsilon} \), respectively (in actuality, \( \bar{u} \) is in the direction of the swerve axis and \( \bar{\varepsilon} \) is the angle of the swerve axis wrt the \( z \)-axis). If the coordinate axis \( \hat{z} \) and \( \hat{y} \) are simply rotated by the angle \( \bar{\varepsilon} \), and thereafter denoted \( \hat{s} \) and \( \hat{\hat{y}} \), respectively, as shown in Fig. 9, then the equation of motion in the \( \hat{\hat{y}} \) direction can be written as:

\[
m \frac{d\hat{Y}(s)}{dt} = \vec{L} \hat{\hat{Y}} + \vec{D} \hat{\hat{\hat{Y}}}
\]

where the unit vector \( \hat{\hat{Y}} \) is nearly parallel with the direction of \( \vec{L} \) and nearly perpendicular to \( \vec{D} \) (even though it may not appear as such in the not-to-scale illustration of Fig. 9, i.e.,

\[
\varepsilon - \bar{\varepsilon} \approx 0. \text{ Similarly, } \hat{u} \approx \bar{u} = \hat{s} \hat{\hat{s}}
\]

Simply stated, Eqn (9) establishes that lift is the primary cause of swerve. From Eqns (6) and (7), lift is proportional to yaw, hence

\[
m \frac{dY(s)}{dt} = -\frac{\alpha}{|\alpha|} \left( |\vec{L}| \cos \varepsilon \right)
\]

The expression for \( \alpha \) must satisfy the torque equation, viz.,

\[
m k^2 \frac{d\alpha}{dt}(\hat{x}) = \vec{M}
\]

where \( k^2 \) is the radius of gyration of the KE rod about its transverse (\( x \)) axis.

* If positive \( y \) had been defined as up, rather than down, the signs on the right in Eqn (8) would be reversed.
In FF region, the axis of the projectile will oscillate about its cg trajectory (i.e. the swerve curve). Just as air opposes the forward motion of the projectile, it will also oppose this oscillating motion. Hence, there will be a resisting torque, known as the damping moment, that varies with the yaw rate. As the name implies, the damping moment causes the yaw magnitude to diminish with time of flight.

However, since it has been argued in Eqn (3) and Fig. 5 that \( \angle AJ \) is established within a relatively short segment of the trajectory, the effect of damping on \( AJ \) can be ignored. In this case, the moment \( \dot{M} \) about the cg will only be due to the resultant force, \( \vec{R} = \vec{L} + \vec{D} \) located at cg (Fig. 8).

Thus, for small \( \alpha \)

\[
\ddot{M} = \left( \alpha |\vec{L}| \right) \cos \alpha (-\hat{x})
\]

\[
\approx \left( \alpha |\vec{L}| \right) (-\hat{x})
\]

(13)

Using Eqns (5-7) in Eqn (13), yields

\[
\ddot{M} = \frac{1}{2} \rho A \vec{u}^2 \left( \alpha |\vec{L}| \right) (-\hat{x})
\]

(14)

where \( C_{ma} \left( = -[C_D + C_{ia}] |\vec{L}| \right) \) is called the derivative of the restoring (overturning, or pitching) moment coefficient wrt \( \alpha \), and \( d \) is the rod diameter. By definition, \( C_{ma} \) is negative for a statically stable projectile. The coefficients \( C_D, C_{ia}, \) and \( C_{maq} \) can all be determined from wind-tunnel measurements or numerically predicted using computational fluid dynamics.

Substituting Eqn (14) into Eqn (12), one has:

\[
mk \gamma \frac{d^2 \theta}{dt^2} = \frac{1}{2} C_{ma} \rho A \vec{u}^2 \left( \alpha \right)
\]

(15)

Since \( C_{ma} \) is negative for KE projectile, this differential equation for \( \alpha \) is of the form \( \alpha \propto -\alpha \).

Such an equation has a sinusoidal solution, which means \( Y(s) \), from Eqn (11), will have a sinusoidal solution (however, \( \alpha \) and \( Y \) will be 180° out of phase). It is now proven that this oscillatory motion, coupled with the lift force, can account for \( AJ \) in the relatively short region from \( z_o \) to \( z_1 \).

4. DYNAMICAL DEFINITION FOR AERODYNAMIC JUMP

From Eqn (3)

\[
\angle AJ = \tan \left( \frac{dy}{dz} \right)_{z_1} - \tan \left( \frac{dy}{dz} \right)_{z_o} \approx \frac{dy}{dz}_{z_1} - \frac{dy}{dz}_{z_o}
\]

(16)

Hence, \( \angle AJ \) can be viewed as a change in slope of the cg trajectory from \( z_o \) to \( z_1 \) [or, it can be viewed as a change in transverse velocity from \( z_o \) to \( z_1 \), non-dimensionalised by the longitudinal velocity (approximately constant from \( z_o \) to \( z_1 \)].

Equations (3) and (16) define \( AJ \) in terms of \( dy/dz \), and to find its equivalent expression in terms of \( dy/ds \), it is necessary to transform from \( y \) and \( z \) to \( Y \) and \( s \). To that end (with the aid of Fig. 9), it can be shown that

\[
y(z) = Y(s) \cos \bar{e} + (s - s_o) \sin \bar{e}
\]

\[
z = z_o = (s - s_o) \cos \bar{e} - Y(s) \sin \bar{e}
\]

(17)

and

\[
\frac{dy(z)}{dz} \frac{dy(s)}{ds} = \frac{dY(s)}{ds} \cos \bar{e} + \sin \bar{e}
\]

\[
\approx \frac{dY(s)}{ds} + \tan \bar{e}, \text{ for } \frac{dY(s)}{ds}, \bar{e} \ll 1
\]

(18)

Combining Eqns (16) and (18), one has
where the subscript notation \( z_i, s_i \) (for example) refers to the point on the swerve curve with coordinate \( z_i \) along the \( z \)-axis and \( s_i \) along the \( s \)-axis (Fig. 9). In effect, Eqn (19) states that the difference in slopes between the two points on the swerve curve does not change if the coordinate system, used to describe the curve, is rotated through an angle \( \theta \).

Combining Eqns (9), (11) and (15), it can be shown that

\[
\frac{d\dot{Y}}{dt} \approx \frac{C_k}{m} \int \left( \frac{\ddot{L}}{\dot{Y}} \right) dt = -\frac{C_k}{\dot{Y}} \frac{d^2\alpha}{dC_{ma}} \frac{d\alpha}{dC_{m_0}} \left( \alpha_1 - \alpha_0 \right)
\]

Denoting \( \dot{\alpha}_0 \) as the yaw rate at entry into FF region, and \( \dot{\alpha}_1 \) as yaw rate at the first local maximum in the swerve curve, then integration of Eqn (20) from entry into FF region until the first local maximum in swerve and in yaw yields

\[
\dot{Y}(s_f) - \dot{Y}(s_0) \approx \frac{1}{m \dot{Y}} \int_{s_0}^{s_f} \left( \frac{\ddot{L}}{\dot{Y}} \right) dt
\]

Equations (19) and (21) can be combined to show

\[
\angle AJ \approx \frac{dY}{ds} \bigg|_{s_0}^{s_1} - \frac{dY}{ds} \bigg|_{s_0}^{s_1} \approx \frac{1}{m \dot{Y}} \int_{s_0}^{s_1} \left( \frac{\ddot{L}}{\dot{Y}} \right) dt = -\frac{C_k}{\dot{Y}} \frac{d^2\alpha}{dC_{ma}} \left( \alpha_1 - \alpha_0 \right)
\]

Equation (22) reveals that \( \angle AJ \) can be viewed as a change in the slope of the cg trajectory from \( s_0 \) to \( s_1 \), or it can be related to a change in angular rates from \( s_0 \) to \( s_1 \). Furthermore, the insertion of the lift correlation in Eqn (20) and its retention in Eqns (21) and (22) underscores the physical explanation that \( \angle AJ \) is due to the (integrated) effect of lift, caused by yaw, from \( s_0 (s_0) \) to \( s_1 (s_1) \).

Equations (19), (21) and (22) could have been simplified by setting \( \dot{Y}/ds \bigg|_{s_0}^{s_1} = 0 \) (and therefore \( \dot{Y}(s_f) = 0 \)), since by definition, \( \dot{Y}(s) \) is at a local maxima at \( s_0, s_f \). This would also mean [from Eqn (18)] that \( \dot{Y}/ds \bigg|_{s_0}^{s_1} = \tan \theta \), as marked in Fig. 9). Moreover, since \( \dot{\alpha} \) and \( \dot{Y} \) are \( 180^\circ \) out of phase, when \( \dot{Y} = 0, \dot{\alpha} = 0 \), hence, Eqn (22) can be simplified to

\[
\angle AJ = -\frac{dY}{ds} \bigg|_{s_0}^{s_1} = -\frac{C_k}{\dot{Y}} \frac{d^2\alpha}{dC_{ma}} \left( \alpha_1 - \alpha_0 \right)
\]

Also from Eqn (22), it can be seen that \( \angle AJ \) will increase if either the integrand (viz., the lift force) or the domain of integration (viz., the lift force action time) increases. In component terms (noting that \( C_{m_0} \) will always be negative for a KE rod, and \( C_{m_0}, k, d, \) and \( |\bar{v}| \) are all positive), \( \angle AJ \) will increase if either (i) \( C_{m_0} \) decreases (so that the lifting force per degree yaw increases), (ii) \( k \) increases (in which case, the rod would rotate slower, and hence the lifting force would act longer), (iii) \( \dot{\alpha}_0 \) increases (so that, once again, it would take more time to
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bring the rod to rest), or (iv) $C_{m_0}$ decreases (so that the overturning moment per degree yaw decreases, again lengthening the action time for the lifting force).

Other equivalent expressions for $AJ$ that can be found in the literature include:

$$
\angle AJ = \frac{C_n I_t}{m d^2 C_{m_0}} \alpha_o' - \frac{C_n}{m \dot{\bar{u}}} \frac{dC_{n_o}}{d\bar{u}} \alpha
$$

where the approximation sign shown in Eqn (23) is discontinued for expediency, $I_t (= m \bar{L}^2)$ is the moment of inertia of the (symmetric) projectile about its transverse axis, and $\alpha_o'$ is the initial FF rate of change of yaw w.r.t. the trajectory arc length, measured in rod diameters (i.e. $\alpha_o' = \frac{d\alpha}{ds}$).

Depending upon the coordinate system used, there may or may not be a negative sign on the right hand side in the equalities/identities of Eqn (24). The convention chosen here (which is also the one most often adopted in the field of ballistics), is to define the positive vertical axis ($y$) as down, and positive yaw ($\alpha$) as up (up, for $\alpha$, means its nose is above the cg velocity vector). However, if the positive vertical axis was defined as up, like that of yaw, it would yield a negative sign in the expressions on the right in Eqn (24). The plus sign form of the expression for $\angle AJ$ is the most common construction\(^{3,4,6,7}\). There is one other sign variation that may appear in the literature, if both the positive vertical axis and positive yaw are defined as down, then the sign is also negative\(^{11,8}\) in Eqn (24). Regardless of the sign convention for the coordinate system used, it is always the case\(^9\) that jump due to $\alpha_o'$ is in the direction of $\alpha_o'$.

5. CONCLUSIONS & DISCUSSION

Equation (3) [or Eqn (16)] provides a limitless kinematic definition for $\angle AJ$, which, reassuringly leads to the traditional dynamic expression for $\angle AJ$, Eqn (23). The origins of possible variations in the sign conventions of Eqn (24) were explored, but the paper's primary objective was to answer the questions: What is $AJ$, what causes it, and what aspect of the flight trajectory does it refer to, or account for.

For instance, one misconception about $AJ$ can arise from the fact that Eqn (24) only shows a dependence on the initial yaw rate at the origin of FF [concealing the fact that it is actually a difference in rates, Eqn (22), that happens to equal the initial rate, Eqn (23)]. Therefore, some may conclude that $AJ$ is a point-based phenomenon, i.e., it results from (aero) dynamical effects that occur at the origin of FF region. Others, seeking a geometrical explanation for $AJ$, may forgo the dynamical definition of Eqn (24) and return to its origin in the kinematical definition adopted, for example, by Murphy\(^2,4\). However, those geometry-based definitions for $AJ$ [viz., Eqns (1) or (2)] call for the cg coordinates to be evaluated in the limit of an infinite trajectory. Thus, there is some risk that those drawing upon these definitions to explain $AJ$ will erroneously assume that it is a transformation that accumulates with downrange distance (not realising that the swerve axis is actually a constant, established long before the trajectory reaches infinity).

The central theme of this paper is to show that $AJ$ is neither a change in direction that takes place at a point, nor is it a curving change that takes place over a domain of infinite extent, rather it is a regional transformation. In particular, using an alternative kinematic definition, it was illustrated geometrically (in terms of the cg trajectory) and proven mathematically (based on Newton's equations of motion) that $\angle AJ$ for a (non-spinning) KE penetrator can be accounted for by the change in transverse cg velocity—due to lift—acting for the short period of time and space from entry of the projectile into FF region until it reaches its first local maxima in yaw (or swerve).

REFERENCES


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