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REVIEW PAPER

Aerodynamic Jump for Long Rod Penetrators

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ABSTRACT

Aerodynamic jump for a non-spinning kinetic energy penetrator is neither a discontinuous change in the direction of motion at the origin of free flight, nor is it the converse, i.e. a cumulative redirection over a domain of infinite extent. Rather aerodynamic jump, for such a projectile, is a localised redirection of the centre of gravity motion, caused by the force of lift due to yaw over the relatively short region from entry into free flight until the yaw reaches its first maximum. The primary objective of this paper is to provide answers to the questions like what is aerodynamic jump, what causes it, and what aspects of the flight trajectory does it refer to, or account for.

NOMENCLATURE

A	Projectile cross-sectional area
$\vec{D}$	Aerodynamic drag force
$\vec{L}$	Aerodynamic lift force
cg	Centre of gravity
$\vec{M}$	Moment due to forces L and D about the projectile centre of gravity
$\vec{R}$	Resultant force
Y	Transverse displacement of projectile cg, measured from the swerve axis
AJ	Aerodynamic jump
C_D	Drag coefficient
FF	Free flight
I_t	Transverse moment of inertia of (symmetric) projectile
KE	Kinetic energy
LD	Launch disturbance
C_L	Lift coefficient
C_{l_α}	Derivative of lift coefficient wrt α
C_{m_α}	Derivative of restoring moment coefficient wrt α
d	Projectile diameter
k	Radius of gyration
m	Projectile mass
s	Longitudinal displacement of projectile cg, measured along the swerve axis
t	Time
$\vec{u}$	cg velocity
x	Transverse displacement of projectile cg, measured from the original line of fire, perpendicular to y and z
y	Transverse displacement of projectile cg, measured from the original line of fire, perpendicular to x and z
z	Longitudinal displacement of projectile cg, measured along the original line of fire, perpendicular to x and y
cp	Centre of pressure

Greek symbols

α	Projectile yaw angle
ε	Angle between cg velocity vector and original line of fire
λ	Wavelength of swerve curve

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ρ Air density

Subscripts & superscripts

o Origin of free flight

1 First local maximum (positive or negative) in the swerve curve

→ Vector

– Averaged quantity

^ Unit vector

· Differentiation wrt time

′ Differentiation wrt the trajectory arc length, measured in rod diameters

1. INTRODUCTION

The motion of a projectile can be divided into two general regions: (i) free flight (FF) region, and (ii) launch disturbance (LD) region (prior to FF). For instance, if the projectile is a gun-launched, saboted long rod or kinetic energy (KE) penetrator, then the LD region begins in-bore and extends downrange to the point, where shock waves from the discarding sabot petals no longer interact with the rod. The end of the LD region marks the beginning of the FF region, where the phenomenon known as AJ occurs. The KE penetrator is chosen to facilitate the ensuing discussion and illustrations on the subject of AJ.

Although AJ occurs in the FF region, its magnitude is influenced by events that take place in the LD region. A KE projectile consists of a long rod with an aerodynamically-shaped nose and stabilising tail fins. The high mass-density sub-calibre rod is held centered in the gun bore by a low mass-density full-calibre sabot. The rod can undergo small, lateral, cg displacements and rotations while being propelled longitudinally down the bore. Such in-bore motion permits the projectile cg to exit the barrel with a velocity vector oriented at an ∠CG wrt the instantaneous bore axis. In addition to the rod moving relative to the bore axis, the barrel itself can be moved. Thus, the rod can be launched with the instantaneous pointing angle of the bore axis, ∠PA, different from the original muzzle sight line. Furthermore, the instantaneous bore axis can have a lateral (crossing)

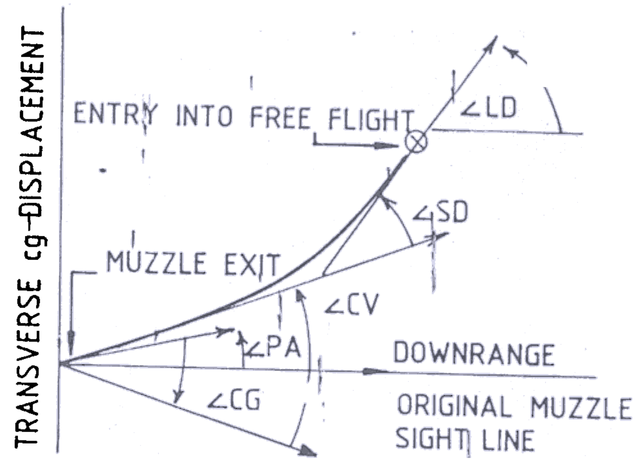


Figure 1. Hypothetical, planar cg motion of a KE rod caused by launch disturbances.

velocity that is transferred to the projectile cg motion. The angular change in the projectile cg velocity due to this barrel crossing motion is denoted by ∠CV. Outside the gun, it is possible for asymmetric sabot discard to create uneven mechanical and aerodynamic forces on the rod that add yet another transverse cg velocity component, and redirection angle, ∠SD. The net effect of these four pre-FF LDs can give the projectile cg a cumulative transverse deflection angle, ∠LD = ∠CG + ∠PA + ∠CV+ ∠SD, at the point where it enters FF (Fig. 1). Techniques to measure these LD components are discussed by Bornstein[1], *et al.*

After transitioning the LD region, side forces can continue to influence the lateral motion of the projectile in the FF region. These side forces are aerodynamic in nature and cause the projectile cg to oscillate (swerve) about a mean FF path (swerve axis), as it travels to the target (Fig. 2). For a typical KE rod (which is statically stable, near-symmetric, and virtually non-rolling), the swerve curve can be approximated by a damped sine wave in both vertical and horizontal directions*. As indicated in Fig. 2, the swerve axis can be, and most often is different from the direction given to the projectile cg as it leaves the LD region. The term AJ, in particular ∠AJ is used to quantify this change in direction.

* The effects of gravity and the Coriolis force on the trajectory are not included in this discussion because they are not aerodynamic in nature; if warranted, their influence can simply be superimposed on the swerve motion.

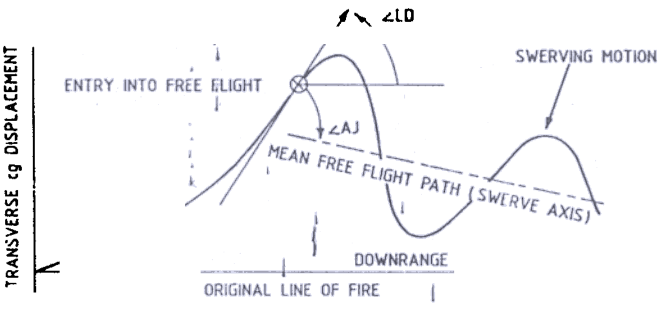


Figure 2. Characterisation of cg transition into FF region

One of the earliest descriptions of AJ was given by Murphy[2] stating that AJ is the angle between the bore sightline and the average trajectory when the other contributors to jump are neglected. Although this definition describes AJ as an angle, it is actually the tangent of the described angle. However, for small AJ angles (typically the case), the angle and its tangent are nearly one and the same. Neglecting other contributors to jump means setting, or assuming, $\angle LD = 0$ in the discussion of Fig. 1. In this case, Fig. 2 would transform into Fig. 3.

Figure 3 depicts that the axis of swerve symmetry is closely aligned with the point of impact on a distant target. In fact, when the FF

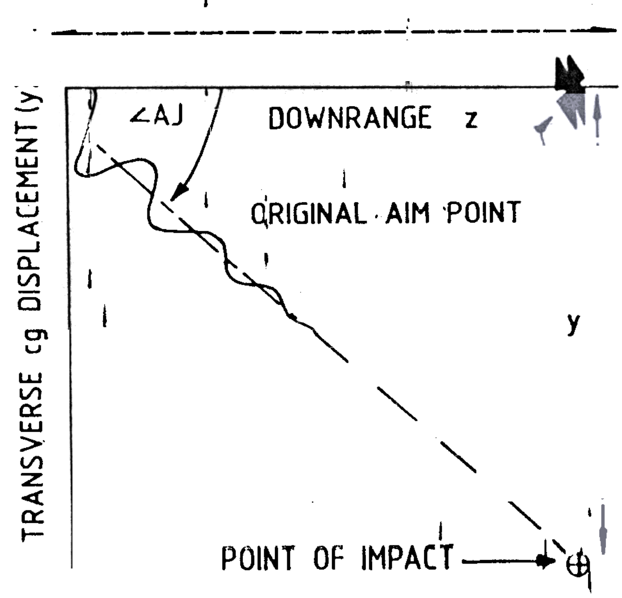


Figure 3. Geometrical view of ∠AJ, neglecting ∠LD

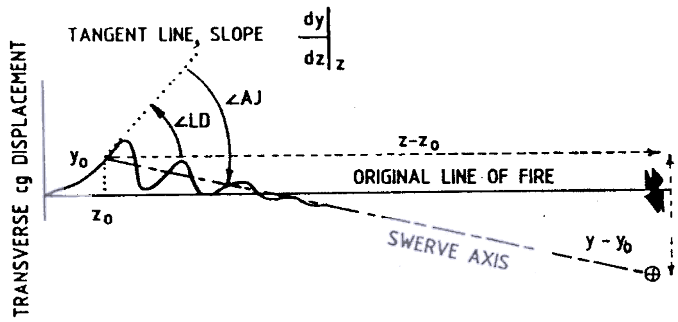


Figure 4. Geometrical rendering of Eqn (2)

trajectory approaches infinity, AJ and ∠AJ are defined by

$$AJ \lim_{z\to\infty}\left[\frac{y}{z}\right]; \ \angle AJ = \tan^{-1}\{AJ\} \underset{AJ,\,\angle AJ}{\approx} AJ$$

where y represents the transverse cg displacement and z represents the longitudinal or downrange displacement**. Both Murphy[2] and Murphy and Bradley[3] begin their discussion of AJ based on Eqn (1). A more detailed expression for AJ, one that does not neglect other contributors to jump, is put forth later by Murphy[4]. This more general definition states#

$$AJ = \lim_{z\to\infty}\left[\frac{y-y_o}{z-z_o}\right] \quad \left.\frac{dy}{dz}\right|_{z_o}$$

$$\angle AJ = \tan \ \{AJ\} \underset{\lim_{z\to\infty}\left[\frac{y-y_o}{z-z_o}\right]\cdot\left.\frac{dy}{dz}\right|_{z_o} \ll 1}{\approx} A \tag{2}$$

where y_o is the transverse cg displacement, and $dy/dz|_{z_o}$ is the tangent to the cg displacement, both at the origin of FF. Figure 4 gives the geometrical interpretation of Eqn (2). Equations (1) and (2) define AJ by calling upon the limit as the trajectory approaches infinity; to some, this may erroneously infer that AJ is an effect that accumulates with downrange distance. An alternative kinematical definition for AJ can be given as one that does not invoke an infinite limit, but rather, attributes AJ to a

** The sign convention for the direction of positive y in Eqn (1) will determine the sign convention for positive AJ.

\# Equation (2) here is actually the single-plane equivalent of combining Murphy's[4] Eqns (9) and (10) with gravitational and Coriol effects neglected.

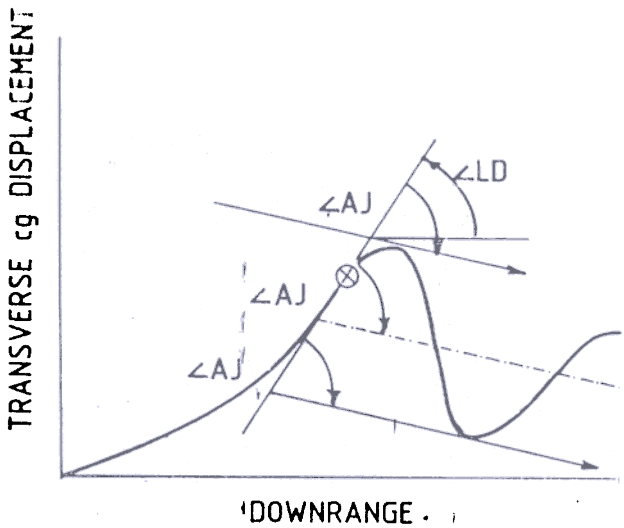


Figure 5. Alternative geometrical definitions for ∠AJ

relatively short segment of the FF trajectory, less than one-half of one swerve oscillation in length.

2. KINEMATICAL DEFINITION FOR AERODYNAMIC JUMP

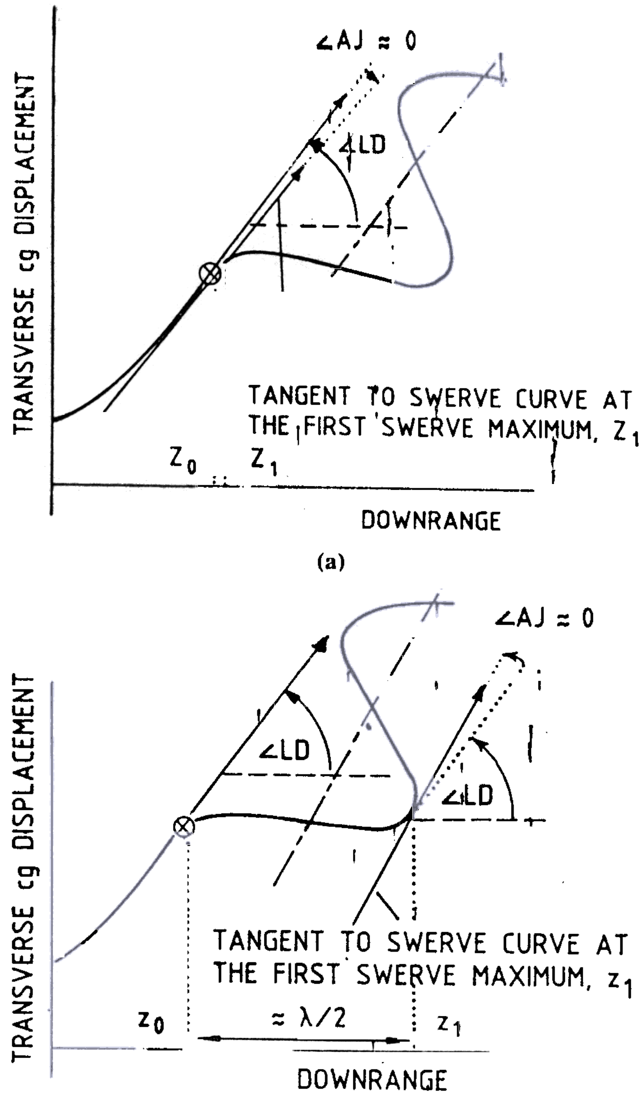
From Eqn (2) and Fig. 4, ∠AJ is the angular change between the tangent to the cg trajectory at the end of LD region and the axis of swerve symmetry. Figure 5 shows that the axis of swerve symmetry runs parallel to the tangent to the swerve curve at any and all of the local swerve maxima[5] (positive or negative wrt the swerve axis).

Hence, ∠AJ can also be defined as the angular difference between the tangent to the swerve curve at the origin of FF and the tangent to the swerve curve at the first (or second, or third, etc.) local maximum in the swerving motion. In equation form, it can be expressed as

$$\angle AJ = \tan^{-1}\left\{ \left.\frac{dy}{dz}\right|_{z_{\text{swerve maxima}}} \right\} - \tan^{-1}\left\{ \left.\frac{dy}{dz}\right|_{z_{\text{origin of free flight}}} \right\}$$

$$\approx \left.\frac{dy}{dz}\right|_{z_{\text{swerve maxima}}} \quad \left.\frac{dy}{dz}\right|_{z_{\text{origin of free flight}}} \tag{3}$$

where the subscripts identify the locations at which the derivatives are to be evaluated. It is noted that unlike Eqns (1) and (2), the definition for ∠AJ given in Eqn (3) does not call upon the limit as the trajectory approaches infinity,



(a)

(b)

Figure 6. Influence of initial (swerve) conditions on AJ: (a) entry into FF at $z_0 \approx z_1$ and (b) entry into FF at $z_0 \approx z_1 - \lambda/2$.

Even though ∠AJ can be defined using the tangent line at any of the local maximum, it is clear that the minimum distance needed to establish the orientation of the swerve axis is the distance to the first swerve maximum, z_1. Thereafter, the cg motion simply oscillates about this axis, albeit with a damped amplitude.

The limitless definition of Eqn (3) facilitates some additional insight into the kinematic relationship between $\angle$AJ and the initial conditions at the origin of FF. Take special cases illustrated in Figs 6(a) and 6(b), unlike Fig. 5, the swerve axes in these two cases are nearly parallel with the LD direction, hence, $\angle$AJ is nearly zero. From Fig. 6(a), for instance, the distance between the origin of FF, z_o, and the first swerve maximum, z_1, is relatively small, at least in comparison to the wavelength of the swerve curve, l. On the other hand, in Fig. 6(b), $z_1 - z_o$ is relatively large, $\approx \lambda/2$. Contrasting the larger $\angle$AJ of Fig. 5 with that of Figs. 6(a) and 6(b), it can be inferred that the largest $\angle$AJ will occur when $z_1 - z_o = \lambda/4$. In fact, if the swerve curve is approximated by a sine wave of the form $y = A \sin(2\pi[z-z_o]/\lambda)$, at least for the first cycle, then, from Eqn (3), the maximum $\angle$AJ would be given by

$$|\angle AJ = \left| \frac{d\left\{ y = A\sin\left(\frac{2\pi[z-z_o]}{\lambda}\right)\right\}}{} \right|_{z_1 = z_o + \lambda/4}$$

$$\left. d\left\{ y = A\sin\left(\frac{2\pi[z-z_o]}{\lambda}\right)\right\} \right|$$

$$\frac{A2\pi}{\lambda}$$

To appreciate the significance of Eqn (4), Fig. 7 illustrates how $\angle AJ_{max}$ varies with A and λ for two cases where y conforms to $A \sin(2\pi[z-z_o]/\lambda)$. From the depiction (and Eqn (4)), a larger A and smaller λ produce a larger $\angle AJ_{max}$. For large calibre guns, A may be of the order of several millimeters, whereas λ is of the order of tens of meters; hence, $\angle AJ_{max}$ from Eqn (4), will be small–of the order of milliradians.

Figures 5–7 illustrate that the axis of swerve symmetry is fixed in space by the time the rod reaches its first swerve maximum, as implied by Eqn (3). These also provide visible examples that support the contention that it is not necessary to take the swerving motion to infinity, as called for in Eqns (1) and (2), in order to establish the direction of the swerve axis.

Based on the kinematical developments discussed here, it is a simple matter to derive a dynamical expression for AJ. However, before such an expression can be formulated, it is beneficial to review some basic aerodynamics.

3. BASIC AERODYNAMIC FORCES & MOMENTS ACTING ON A NON-SPINNING KE PENETRATOR

The force of friction and drag on the projectile are probably the most fundamental of the aerodynamic forces. It is commonly expressed as

$$\vec{D} \quad -\tfrac{1}{2} C_D \rho A |\vec{u}| \vec{u} \qquad (5)$$

and, by virtue of the minus sign, drag is in the direction opposite $\vec{u}$.

The expression for lift is conventionally written as

$$\vec{L} = \tfrac{1}{2} C_L \rho A |\vec{u}|^2 \hat{L}$$

The unit vector direction of the lift force, $\hat{L}$, is perpendicular to the drag force and is in the yaw plane. In this discussion, yaw is the vertical (z–y) plane angle, α, between the projectile's tail-to-nose axis and the tangent to its trajectory (or equally suitable, $\vec{u}$). It is assumed here that a $+\alpha$ means the nose of projectile is above $\vec{u}$.

For small yaw (e.g. $\alpha < 5^o$),

$$C_L = C \qquad (7)$$

Suppose the original direction of fire is defined to be the positive z-axis, with positive y downward, and positive x to the gunner's right. Assume the cg motion is 2-D planar, in particular, assume (for illustrational simplicity) that the motion is confined to the vertical plane, then $\vec{u} = \dot{z}\hat{z} + \dot{y}\hat{y}$ as depicted in Fig. 8.

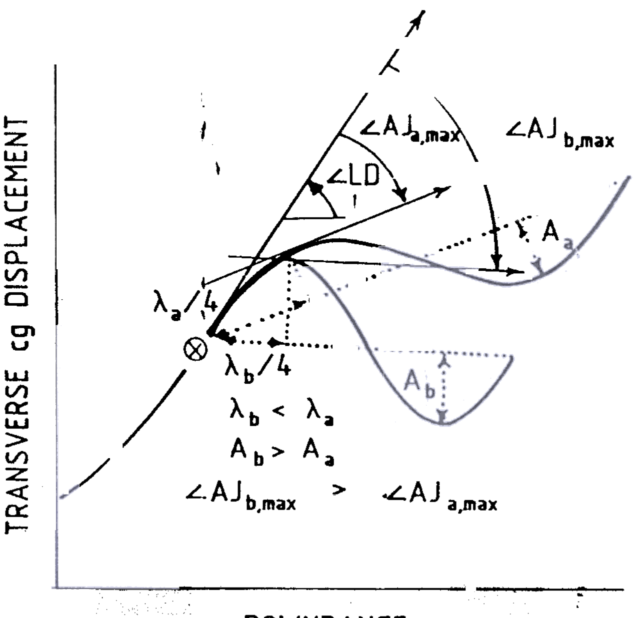


Figure 7. Depiction of variation in $\angle AJ_{max}$ with amplitude and wavelength of the swerve curve.

Suppressing the effects of gravity and the Coriolis force, Newton's second law for linear motion in the $\hat{y}$ direction* dictates that

$$m\frac{d\dot{y}}{dt} = \vec{L}.\hat{y} + \vec{D}.\hat{y} = -\frac{\alpha}{|\alpha|}\left\{|\vec{L}|\cos\varepsilon\right\} + |\vec{D}|\sin\varepsilon \tag{8}$$

where $\alpha/|\alpha|$ accounts for the positive or negative influence of yaw on lift.

The value of ε is always small, and it would expedite the analysis to assume it is zero, however, such an over simplification is not necessary. The cg velocity vector, and hence ε, oscillates about some mean values, $\bar{\vec{u}}$ and $\bar{\varepsilon}$, respectively (in actuality, $\bar{\vec{u}}$ is in the direction of the swerve axis and $\bar{\varepsilon}$ is the angle of the swerve axis wrt the z-axis). If the coordinate axis $\hat{z}$ and $\hat{y}$ are simply rotated by the angle $\bar{\varepsilon}$, and thereafter denoted $\hat{s}$ and $\hat{Y}$, respectively, as shown in Fig. 9, then the equation of motion in the $\hat{Y}$ direction can be written as

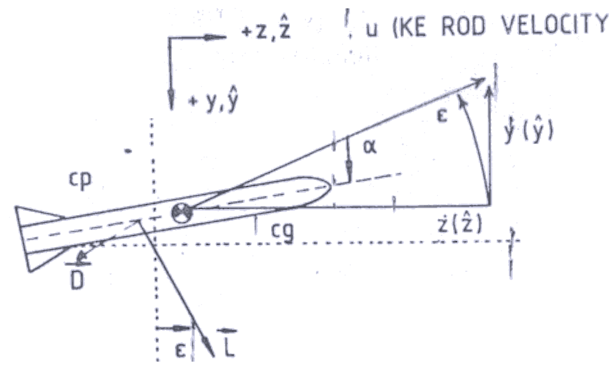


Figure 8. Illustration of the lift and drag force directions, as well as the cg velocity and the cg and cp locations.

$$m\frac{d\dot{Y}(s)}{dt} = \vec{L}.\hat{Y} + \vec{D}.\hat{Y}$$
$$= -\frac{\alpha}{|\alpha|}\left\{|\vec{L}|\cos(\varepsilon - \bar{\varepsilon})\right\} + |\vec{D}|\sin(\varepsilon - \bar{\varepsilon})$$
$$\approx -\frac{\alpha}{|\alpha|}|\vec{L}|$$

where the unit vector $\hat{Y}$ is nearly parallel with the direction of $\vec{L}$ and nearly perpendicular to $\vec{D}$ (even though it may not appear as such in the not-to-scale illustration of Fig. 9, i.e.

$$\varepsilon - \bar{\varepsilon} \approx 0. \text{ Similarly, } \vec{u} \approx \bar{\vec{u}} = \dot{s}\hat{s} \tag{10}$$

Simply stated, Eqn (9) establishes that lift is the primary cause of swerve. From Eqns (6) and (7), lift is proportional to yaw, hence

$$m\frac{d\dot{Y}(s)}{dt} = -\frac{\alpha}{|\alpha|}\left|\frac{1}{2}\rho A|\vec{u}|^2 C_{l_\alpha}\alpha\right|$$
$$= -\frac{1}{2}\rho A|\vec{u}|^2 C_{l_\alpha}\alpha \quad 1)$$

The expression for α must satisfy the torque equation, viz.,

$$mk^2\frac{d\dot{\alpha}}{dt}(\hat{x}) = \vec{M} \tag{12}$$

where k^2 is the radius of gyration of the KE rod about its transverse (x) axis.

* If positive y had been defined as up, rather than down, the signs on the right in Eqn (8) would be reversed.

In FF region, the axis of the projectile will oscillate about its cg trajectory (i.e. the swerve curve). Just as air opposes the forward motion of the projectile, it will also oppose this oscillating motion. Hence, there will be a resisting torque, known as the damping moment, that varies with the yaw rate. As the name implies, the damping moment causes the yaw magnitude to diminish with time of flight.

However, since it has been argued in Eqn (3) and Fig. 5 that $\angle$AJ is established within a relatively short segment of the trajectory, the effect of damping on AJ can be ignored. In this case, the moment $\vec{M}$ about the cg will only be due to the resultant force, $\vec{R}=\vec{L}+\vec{D}$ located at cp (Fig. 8). Thus, for small α

$$\vec{M}=|\mathrm{cg}-\mathrm{cp}|\left(\left|\vec{D}\right|\sin\alpha+\frac{\alpha}{|\alpha|}\left|\vec{L}\right|\cos\alpha\right)(-\hat{x})$$

$$\approx|\mathrm{cg}-\mathrm{cp}|\left(\vec{D}|\alpha+\frac{\alpha}{|\alpha|}\left|\vec{L}\right|(-\hat{x})\right) \tag{13}$$

Using Eqns (5–7) in Eqn (13), yields

$$\vec{M}=\frac{1}{2}\rho A|\vec{u}|^2\,|\mathrm{cg}-\mathrm{cp}|\alpha\left(C_D+C_{l_\alpha}\right)(-\hat{x})$$

$$\frac{1}{2}C_{m_\alpha}\rho A|\vec{u}|^2\,d\alpha\,\hat{x} \tag{14}$$

where $C_{m_\alpha}\left(\equiv-[C_D+C_{l_\alpha}]|\mathrm{cg}-\mathrm{cp}|/d\right)$ is called the derivative of the restoring (overturning, or pitching) moment coefficient wrt α, and d is the rod diameter. By definition, $C_{m\alpha}$ is negative for a statically stable projectile. The coefficients C_D, $C_{l\alpha}$, and $C_{m\alpha}$ can all be determined from wind-tunnel measurements or numerically predicted using computational fluid dynamics.

Substituting Eqn (14) into Eqn (12), one has:

$$mk^2\frac{d\dot{\alpha}}{dt}=\tfrac{1}{2}C_{m_\alpha}\rho A|\vec{u}|^2\,d\alpha \tag{15}$$

Since C_{m_α} is negative for KE projectile, this differential equation for α is of the form $\ddot{\alpha}\propto-\alpha$.

Such an equation has a sinusoidal solution, which means $Y(s)$, from Eqn (11), will have a sinusoidal solution (however, α and Y will be 180° out of phase). It is now proven that this oscillatory motion, coupled with the lift force, can account for AJ in the relatively short region from z_0 to z_1.

4. DYNAMICAL DEFINITION FOR AERODYNAMIC JUMP

From Eqn (3)

$$\angle\mathrm{AJ}=\tan\left\{\left.\frac{dy}{dz}\right|_{z_1}\right\}-\tan\left\{\left.\frac{dy}{dz}\right|_{z_o}\right\}\approx\left.\frac{dy}{dz}\right|_{z_1}-\left.\frac{dy}{dz}\right|_{z_o}$$

$$\left(=\left.\frac{\dot{y}}{\dot{z}}\right|_{z_1}-\left.\frac{\dot{y}}{\dot{z}}\right|_{z_o}\right)\approx\frac{\dot{y}(z_1)-\dot{y}(z_o)}{\dot{z}} \tag{16}$$

Hence, $\angle$AJ can be viewed as a change in slope of the cg trajectory from z_o to z_1 [or, it can be viewed as a change in transverse velocity from z_o to z_1, non-dimensionalised by the longitudinal velocity (approximately constant from z_o to z_1)].

Equations (3) and (16) define $\angle$AJ in terms of dy/dz, and to find its equivalent expression in terms of dY/ds, it is necessary to transform from y and z to Y and s. To that end (with the aid of Fig. 9), it can be shown that

$$y(z)=Y(s)\cos\bar{\varepsilon}+(s-s_o)\sin\bar{\varepsilon}$$

$$z-z_o=(s-s_o)\cos\bar{\varepsilon}-Y(s)\sin\bar{\varepsilon} \tag{17}$$

and

$$\frac{dy(z)}{dz}=\frac{\dfrac{dy(z)}{ds}}{\dfrac{dz}{ds}}=\frac{\dfrac{dY(s)}{ds}\cos\bar{\varepsilon}+\sin\bar{\varepsilon}}{\cos\bar{\varepsilon}-\dfrac{dY(s)}{ds}\sin\bar{\varepsilon}}$$

$$\approx\frac{dY(s)}{ds}+\tan\bar{\varepsilon},\ \text{for}\ \frac{dY(s)}{ds},\bar{\varepsilon}\ll \tag{18}$$

Combining Eqns (16) and (18), one has

$$\angle AJ = \left.\frac{dy}{dz}\right|_{z_1,s_1} - \left.\frac{dy}{dz}\right|_{z_0,s_0} = \left.\frac{dY}{ds}\right|_{s_1,z_1} - \left.\frac{dY}{ds}\right|_{s_0,z_0}$$

$$= \left.\frac{\dot Y}{\dot s}\right|_{s_1} - \left.\frac{\dot Y}{\dot s}\right|_{s_0} \approx \frac{\dot Y(s_1) - \dot Y(s_0)}{|\vec u|} \tag{19}$$

where the subscript notation z_1, s_1 (for example) refers to the point on the swerve curve with coordinate z_1 along the z-axis and s_1 along the s-axis (Fig. 9). In effect, Eqn (19) states that the difference in slopes between the two points on the swerve curve does not change if the coordinate system, used to describe the curve, is rotated through an angle $\bar\varepsilon$.

Combining Eqns (9), (11) and (15), it can be shown that

$$d\dot Y \approx \frac{(\vec L.\hat Y)dt}{m} \approx -\frac{C_{l_\alpha} k^2}{dC_{m_\alpha}} d\dot\alpha \tag{20}$$

Denoting $\dot\alpha_0$ as the yaw rate at entry into FF region, and $\dot\alpha_1$ as yaw rate at the first local maximum in the swerve curve, then integration of Eqn (20) from entry into FF region until the first local maximum in swerve and in yaw yields

$$\dot Y(s_1) - \dot Y(s_0) \approx \frac{1}{m} \int_{t(s_0)}^{t(s_1)} (\vec L.\hat Y)dt \tag{21}$$

$$\approx -\int_{\dot\alpha_0}^{\dot\alpha_1} \frac{C_{l_\alpha} k^2}{dC_{m_\alpha}} d\dot\alpha = -\frac{C_{l_\alpha} k^2}{dC_{m_\alpha}} (\dot\alpha_1 - \dot\alpha_0)$$

Equations (19) and (21) can be combined to show

$$\angle AJ = \left.\frac{dY}{ds}\right|_{s_1} - \left.\frac{dY}{ds}\right|_{s_0,z_0} \approx \frac{1}{m|\vec u|} \int_{t(s_0)}^{t(s_1)} (\vec L.\hat Y)dt$$

$$= -\frac{C_{l_\alpha} k^2}{dC_{m_\alpha}|\vec u|} (\dot\alpha_1 - \dot\alpha_0) \tag{22}$$

Equation (22) reveals that $\angle AJ$ can be viewed as a change in the slope of the cg trajectory from z_0 to z_1, or it can be related to a change in angular rates from z_0 to z_1 (s_0 to s_1). Furthermore, the insertion of the lift correlation in Eqn (20) and its retention in Eqns (21) and (22) underscores the physical explanation that $\angle AJ$ is due to the (integrated) effect of lift, caused by yaw, from z_0 (s_0) to z_1 (s_1).

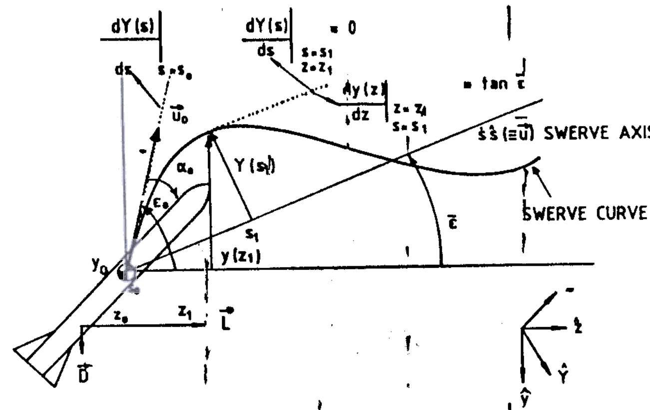


Figure 9. Swerve-fixed (s, y) and earth-fixed ($\hat z, \hat y$) coordinate system.

Equations (19), (21) and (22) could have been simplified by setting $dY/ds|_{s_1,z_1} = 0$ (and therefore $\dot Y(s_1) = 0$), since by definition, $Y(s)$ is at a local maxima at s_1, z_1. (This would also mean [from Eqn (18)] that $dy/dz|_{z_1,s_1} = \tan\bar\varepsilon$, as marked in Fig. 9). Moreover, since α and Y are 180° out of phase, when $\dot Y = 0, \dot\alpha = 0$, hence, Eqn (22) can be simplified to

$$\angle AJ = -\left.\frac{dY}{ds}\right|_{s_0,z_0} \approx \frac{C_{l_\alpha} k^2}{dC_{m_\alpha}|\vec u|} \dot\alpha \tag{23}$$

Also from Eqn (22), it can be seen that $\angle AJ$ will increase if either the integrand (viz., the lift force) or the domain of integration (viz., the lift force action time) increases. In component terms (noting that C_{m_α} will always be negative for a KE rod, and C_{l_α}, k, d, and $|\vec u|$ are all positive), $\angle AJ$ will increase if either (i) C_{l_α} increases (so that the lifting force per degree yaw increases), (ii) k increases (in which case, the rod would rotate slower, and hence the lifting force would act longer), (iii) $\dot\alpha_0$ increases (so that, once again, it would take more time to bring the rod to rest), or (iv) C_{m_α} decreases (so that the overturning moment per degree yaw decreases, again lengthening the action time for the lifting force).

Other equivalent expressions for AJ that can be found in the literature include:

$$\angle AJ = \frac{C_{l_\alpha}}{dC_{m_\alpha}|\vec{u}|}\dot{\alpha}_o \equiv \frac{C_{l_\alpha} I_t}{m|\vec{u}|dC_m}\dot{\alpha}$$

$$\frac{C_{l_\alpha} I_t}{md^2 C_{m_\alpha}}\alpha'_o \qquad (24)$$

where the approximation sign shown in Eqn (23) is discontinued for expediency, I_t (= mk^2) is the moment of inertia of the (symmetric) projectile about its transverse axis, and α'_o is the initial FF rate of change of yaw wrt the trajectory arc length, measured in rod diameters (i.e. $\alpha'_o \equiv d\alpha / d[s/d]$).

Depending upon the coordinate system used, there may or may not be a negative sign on the right hand side in the equalities/identities of Eqn (24). The convention chosen here (which is also the one most often adopted in the field of ballistics), is to define the positive vertical axis (y) as down, and positive yaw (α) as up (up, for α, means its nose is above the cg velocity vector). However, if the positive vertical axis was defined as up, like that of yaw, it would yield a negative sign in the expressions on the right in Eqn (24). The plus sign form of the expression for $\angle$AJ is the most common construction[3,4,6,7]. There is one other sign variation that may appear in the literature, if both the positive vertical axis and positive yaw are defined as down, then the sign is also negative[1,8] in Eqn (24). Regardless of the sign convention for the coordinate system used, it is always the case[3], that jump due to α'_o is in the direction of α'_o.

5. CONCLUSIONS & DISCUSSION

Equation (3) [or Eqn (16)] provides a limitless kinematic definition for $\angle$AJ, which, re-assuringly leads to the traditional dynamic expression for $\angle$AJ, Eqn (23). The origins of possible variations in the sign conventions of Eqn (24) were explored, but the paper's primary objective was to answer the questions: What is AJ, what causes it, and what aspect of the flight trajectory does it refer to, or account for.

For instance, one misconception about AJ can arise from the fact that Eqn (24) only shows a dependence on the initial yaw rate at the origin of FF [concealing the fact that it is actually a difference in rates, Eqn (22), that happens to equal the initial rate, Eqn (23)]. Therefore, some may conclude that AJ is a point-based phenomenon, i.e., it results from (aero) dynamical effects that occur at the origin of FF region. Others, seeking a geometrical explanation for AJ, may forgo the dynamical definition of Eqn (24) and return to its origin in the kinematical definition adopted, for example, by Murphy[2,4]. However, those geometry-based definitions for AJ [viz., Eqns (1) or (2)] call for the cg coordinates to be evaluated in the limit of an infinite trajectory. Thus, there is some risk that those drawing upon these definitions to explain AJ will erroneously assume that it is a transformation that accumulates with downrange distance (not realising that the swerve axis is actually a constant, established long before the trajectory reaches infinity).

The central theme of this paper is to show that AJ is neither a change in direction that takes place at a point, nor is it a curving change that takes place over a domain of infinite extent, rather it is a regional transformation. In particular, using an alternative kinematic definition, it was illustrated geometrically (in terms of the cg trajectory) and proven mathematically (based on Newton's equations of motion) that $\angle$AJ for a (non-spinning) KE penetrator can be accounted for by the change in transverse cg velocity—due to lift—acting for the short period of time and space from entry of the projectile into FF region until it reaches its first local maxima in yaw (or swerve).

REFERENCES

Bornstein, J.; Celmins, I.; Plostins, P. & Schmidt, E. M. Techniques for the measurement of tank cannon jump. US Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, USA, 1988. Report No. BRL-MR-3715.

2. Murphy, C.H. Comments on projectile jump. US Army Ballistic Research Laboratories, Aberdeen Proving Ground, MD, USA, 1990. Report No. BRL-MR-1071; AD-132323.

3 Murphy, C.H. & Bradley, J. W. Jump due to aerodynamic asymmetry of a missile with varying roll rate. US Army Ballistic Research Laboratories, Aberdeen Proving Ground, MD, USA, 1959. Report No. BRL-R-1077; AD-219312.

4. Murphy, C. H. Free flight motion of symmetric missiles. US Army Ballistic Research Laboratories, Aberdeen Proving Ground, MD, USA, 1963. Report No. BRL-R-1216.

5. Bundy, M.L. The regional nature of aerodynamic Jump. US Army Research Laboratory, Aberdeen Proving Ground, MD, USA, 1999. Report No. ARL-TR-1872.

6. Fansler, K. S. & Schmidt, E. M. The influence of muzzle gasdynamics upon the trajectory of fin-stabilized projectiles. US Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, USA, 1975. Report No. BRL-R-1793.

7. Schmidt, E.M. A technique for reduction of launch-induced perturbations. Proceedings of the 8th U.S. Army Symposium on Gun Dynamics, 1996. pp. 12-1–12-7.

8. Lijewski, L.E. Aerodynamic jump prediction for supersonic, high fineness ratio, cruciform finned bodies. *AIAA J. Guid.*, 1982, 5(5), 521-28.

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