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Free Convection Flow of a Non-Newtonian Fluid in a Vertical Channel

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ABSTRACT

The steady two-dimensional free convection flow of a Walters fluid (model B′) in a vertical channel one of whose walls is wavy, has been investigated analytically. The governing equations of the fluid and the heat transfer have been solved subject to the relevant boundary conditions by assuming that the solution consists of two parts: a mean part and disturbance or perturbed part. To obtain the perturbed part of the solution, the long wave approximation has been used and to solve the mean part, a well-known approximation used by Ostrach has been utilised. The relevant flow and the heat transfer characteristics, namely the skin-friction and the rate of heat transfer at both the walls have been discussed in detail.

1. INTRODUCTION

Viscous fluid flow over a wavy wall has attracted the attention of relatively few researchers, although the analysis of such flows finds application in different areas, such as transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporisation in combustion chambers. Lekoudis, Nayfeh and Saric[1] presented a linear analysis of compressible boundary layer flows over a wavy wall. Sankar and Sinha[2] studied in detail the Rayleigh problem for a wavy wall. Lessen and Gangwani[3] made a very interesting analysis of the effect of small amplitude wall waviness upon the stability of the laminar boundary layer. In all these problems, the authors have taken the wavy walls to be horizontal. Vajravelu and Sastri[4] made an analysis of the free convection heat transfer in viscous incompressible fluid between a long vertical wavy wall and a parallel flat wall. Das and Ahmed[5] extended this problem to magneto-hydrodynamic case. Das and Deka[6] discussed a numerical approach of this problem.

Non-Newtonian fluids are of increasing importance in modern technology due to its growing use in many activities, such as molten plastic, paints, drilling, and petroleum and polymer solutions. The Walters fluid is one of such fluids. The constitutive equation for Walters fluid (model B′) is:

$$\sigma^{ik} = -pg_{ik} + \sigma'_{ik}$$
$$\sigma'_{ik} = 2\eta_0 e^{ik} - 2K_0 e'^{ik}$$

where σ^{ik} is the stress tensor; p, an isotropic pressure; g_{ik}, the metric tensor of a fixed coordinate system; x^i, υ^i, the velocity vector; $\acute{e}^{ik}$, in the contravariant form is:

$$'^{ik} = \frac{\partial e^{ik}}{\partial t} + \upsilon^j e^{ik}{}_{,j} - \upsilon \quad j e^{ij} - \upsilon^i{}_{,j} e^{jk} \qquad (2)$$

It is the convected derivative of the deformation rate tensor (e^{ik}) defined by

$$2e_{ik} = \upsilon_{i,k} + \upsilon_{k,} \qquad (3)$$

Here, η_0 is the limiting viscosity at small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \quad and \quad k_0 = \int_0^\infty \tau N(\tau)d\tau \qquad (4)$$

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$N(\tau)$ being the relaxation spectrum as introduced by Walters[7,8]. This idealised model is a valid approximation of Walters fluid (model B') taking very short memories into account so that terms involving

$$\int_0^{\infty} \tau^n n(\tau)\, d\tau, n \geq 2 \quad (5)$$

are neglected

In this paper, the steady free-convective flow and heat transfer in a Walters fluid between a long vertical wavy wall and a parallel flat wall has been studied. The problem has been solved by a linearisation technique, wherein the solution is made up of two parts: a mean or zero-order part corresponding to the fully developed mean flow and disturbed part. To obtain the solution of the perturbed part, long wave approximation has been applied and to solve the mean part, the well-known approximation used by Ostrach[9] has been utilised. Expressions for the zero-order[9] and first-order velocity, temperature, skin-friction and heat transfer at the walls are obtained.

2. GOVERNING EQUATION OF MOTION

The steady two-dimensional laminar free-convective Walters fluid flow along the vertical channel has been considered as shown in Fig.1. The X-axis is taken vertically upwards and parallel to the flat wall, while the Y-axis is taken perpendicular to it in such a way that the wavy wall is represented by $Y = \in^* \cos kX$ and the flat wall by $Y = d$. The wavy and flat walls are maintained at constant temperature T_ω and T_1, respectively.

The following assumptions are made:

(a) All the fluid properties except the density in the buoyancy force are constant.

(b) The dissipative effects and the work of deformation are neglected in the energy equation.

(c) The volumetric heat source/sink term in the energy equation is constant.

(d) The wavelength of the wavy wall is large compared with the breadth d of the channel.

The boundary conditions relevant to the problem are taken as

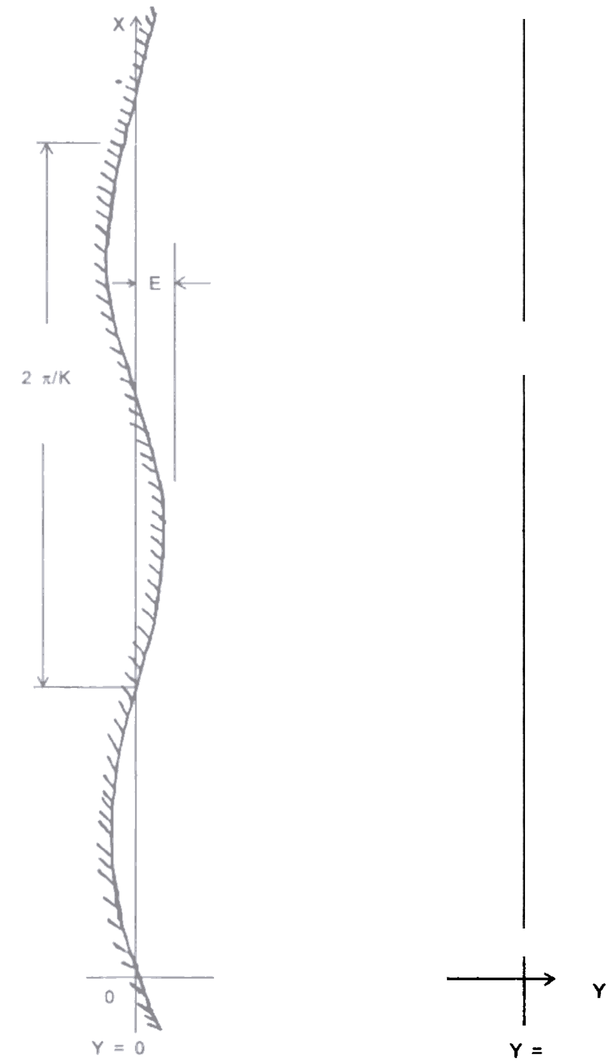

Figure Flow Configuration

$$\left.\begin{aligned} U=V=0,\ T=T_w \ \ on \ \ Y=\in^* \cos kX \\ V=0,\ T=T_1 \ \ on \ \ Y=d \end{aligned}\right\} \quad (6)$$

Introducing the following non-dimensional variables in the governing equations for velocity and temperature

$$x = \frac{X}{d},\ y = \frac{Y}{d},\ u = \frac{Ud}{v},\ v = \frac{Vd}{v}$$

$\theta = (T-T_s)/(T_\omega - T_s)$, T_s is the fluid temperature in static condition.

$$\bar{p} = p^*/\rho(\upsilon/d)^2 \quad (7)$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters[7,8]. This idealised model is a valid approximation of Walters fluid (model B') taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n n(\tau)\, d\tau, n \geq 2 \qquad (5)$$

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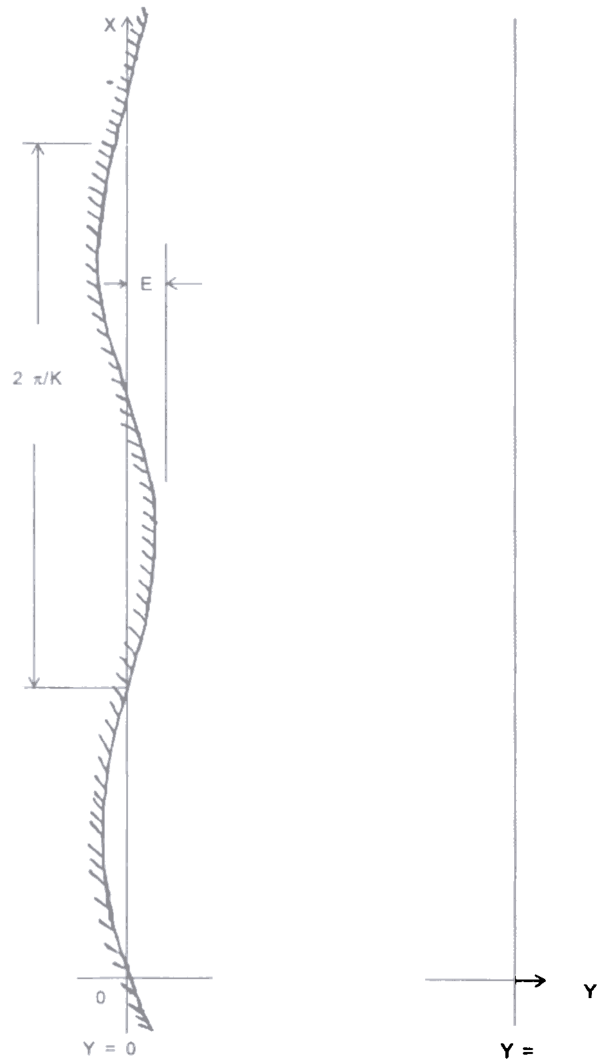

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$$x=\frac{X}{d},\ y=\frac{Y}{d},\ u=\frac{Ud}{\nu},\ v=\frac{Vd}{\nu}$$

$\theta = (T-T_s)/(T_\omega-T_s)$, T_s is the fluid temperature in static condition.

$$\bar{p} = p^*/\rho(\upsilon/d)^2 \qquad (7)$$

Table 1. Skin-friction for case I (m=1)

K	0		0.15		0.25	
α	σ_w	σ_1	σ_w	σ_1	σ_w	σ_1
-5	1.524625	-1.524767	1.524402	-1.524849	1.524253	-1.524937
-2	2.215911	-2.216038	2.215452	-2.216128	2.215146	-2.216210
0	2.676766	-2.676861	2.676103	-2.676961	2.675661	-2.677061
2	3.137619	-3.137664	3.136715	-3.137679	3.136112	-3.137689
5	3.838895	-3.828833	3.827559	-3.828878	3.826668	-3.828908
8	4.520168	-4.519958	4.518316	-4.520041	4.517082	-4.520097
10	4.981013	-4.980684	4.978771	-4.980797	4.977276	-4.980873

Table 2. Skin-friction for case II (m=1)

K	0		0.15		0.25	
α	σ_w	σ_1	σ_w	σ_1	σ_w	σ_1
-5	1.524625	-1.524911	1.524179	-1.525201	1.523881	-1.525350
-2	2.215906	-2.216160	2.214988	-2.216440	2.214376	-2.216527
0	2.676755	-2.676945	2.675429	-2.677246	2.674545	-2.677347
2	3.137600	-3.137691	3.135792	-3.137721	3.134586	-3.137741
5	3.828861	-3.828737	3.826189	-3.828825	3.824408	-3.828885
8	4.520115	-4.519694	4.516411	-4.519857	4.513942	-4.519965
10	4.980945	-4.980283	4.976461	-4.980502	4.973472	-4.980653

One obtains the equation of continuity as

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{8}$$

the momentum equation becomes:

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-K\Bigg[2u\frac{\partial^3 u}{\partial x^3}+2\upsilon\frac{\partial^3 u}{\partial x^2\partial y}$$

$$+u\frac{\partial^3 u}{\partial x\partial y^2}+u\frac{\partial^3 \upsilon}{\partial x^2\partial y}+\upsilon\frac{\partial^3 u}{\partial y^3}+\upsilon\frac{\partial^3 \upsilon}{\partial x\partial y^2}$$

$$-6\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{\partial u\partial^2 v}{\partial y\partial x^2}-4\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}-2\frac{\partial \upsilon}{\partial y}\frac{\partial^2 u}{\partial y^2}$$

$$-3\frac{\partial^2 u}{\partial x\partial y}\frac{\partial \upsilon}{\partial x}-3\frac{\partial u}{\partial x}\frac{\partial^2 \upsilon}{\partial x\partial y}-3\frac{\partial u}{\partial y}\frac{\partial^2 \upsilon}{\partial y^2}$$

$$\frac{\partial \upsilon}{\partial x}\frac{\partial^2 \upsilon}{\partial y^2}-\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial x} \qquad \frac{\rho g_x d^3}{\rho \upsilon^2} \tag{9}$$

$$u\frac{\partial v}{\partial x}+\upsilon\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 \upsilon}{\partial y^2}-K\Bigg[u\frac{\partial^3 u}{\partial x^2\partial y}+u\frac{\partial^3 \upsilon}{\partial x^3}$$

$$+\upsilon\frac{\partial^3 u}{\partial x\partial y^2}+\upsilon\frac{\partial^3 \upsilon}{\partial x^2\partial y}+2u\frac{\partial^3 \upsilon}{\partial x\partial y^2}+2\upsilon\frac{\partial^3 \upsilon}{\partial y^3}$$

$$-2\frac{\partial u}{\partial x}\frac{\partial^2 \upsilon}{\partial x^2}-2\frac{\partial \upsilon}{\partial x}\frac{\partial^2 u}{\partial y^2}-3\frac{\partial^2 u}{\partial x^2}\frac{\partial \upsilon}{\partial x}$$

$$-3\frac{\partial^2 u}{\partial x\partial y}\frac{\partial \upsilon}{\partial y}\; 3\frac{\partial u}{\partial x}\frac{\partial^2 \upsilon}{\text{[illegible]}}-\frac{\partial^2 \upsilon}{\text{[illegible]}}\frac{\partial \upsilon}{\partial x}$$

$$-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2}-6\frac{\partial \upsilon}{\partial y}\frac{\partial^2 \upsilon}{\partial y^2}-4\frac{\partial \upsilon}{\partial x}\frac{\partial^2 \upsilon}{\partial x\partial y}$$

and the energy equation as

$$P\left(u\frac{\partial \theta}{\partial x}+\upsilon\frac{\partial \theta}{\partial y}\right)=\frac{\partial^2 \theta}{\text{[illegible]}}+\frac{\partial^2 \theta}{\text{[illegible]}}+\alpha$$

subject to boundary conditions:

Table 3. Skin-friction for case III (m=1)

	0		0.15		0.25	
α	σ_w	σ_1	σ_w	σ_1	σ_w	σ_1
-5	3.04925	-3.049820	3.048357	-3.050020	3.047762	-3.050499
-2	4.431813	-4.432320	4.429976	-4.432580	4.428752	-4.432753
0	5.353511	-5.353890	5.350859	-5.353902	5.349091	-5.354051
2	6.275202	-6.275382	6.271585	-6.275443	6.269174	-6.275483
5	7.657725	-7.657475	7.652380	-7.657651	7.648818	-7.657771
8	9.040231	-9.039389	9.032824	-9.039715	9.027888	-9.039936
10	9.961892	-9.960568	9.952925	-9.961008	9.946945	-9.961308

Table 4. Skin-friction for case IV (m=1)

K	0.		0.15		0.25	
α	σ_w	σ_1	σ_w	σ_1	σ_w	σ_1
-5	3.049324	-3.049164	3.048442	-3.049294	3.047854	-3.050346
-2	4.431832	-4.431897	4.429999	-4.431957	4.428778	-4.432030
0	5.353449	-5.353900	5.350797	-5.354002	5.349029	-5.354204
2	6.275026	-6.276040	6.271417	-6.276105	6.269011	-6.276149
5	7.657319	-7.659492	7.652046	-7.659699	7.648532	-7.659839
8	9.039532	-9.043214	9.032383	-9.043634	9.027614	-9.043919
10	9.960966	-9.965834	9.952486	-9.966446	9.946832	-9.966860

$$u=0,\ \upsilon=0,\ \theta=1 \quad on\ y=\varepsilon\cos\lambda x$$
$$u=0,\ \upsilon=0,\ \theta=m \quad on\ y=1 \tag{12}$$

where

α = $Qd^2/k(T_\omega-T_s)$, the non-dimensional heat source/sink parameter

p = $\eta_0 c_p/k$, the Prandtl number

ε = ε^*/d, the non-dimensional amplitude parameter

λ = kd, the non-dimensional frequency parameter

m = $(T_1-T_s)/(T_\omega-T_s)$, the wall temperature ratio

K = $2K_0/(\rho d^2)$

and ρg_x is the buoyancy term in X-direction, where the subscript s denotes quantities in the static fluid condition. Now introducing the non-dimensional quantity as:

$$G = d^3 g_x \beta(T_w-T_s)/\upsilon^2$$

the Grashof number and using the equation of state, one has

$$\rho = \rho_s[1-\beta(T-T_s)]$$

and also adopting the perturbation scheme

$$u(x,y) = u_0(y)+\varepsilon u_1(x,y),\ \upsilon(x,y) = \varepsilon\upsilon_1(x,y)$$
$$\bar{p}(x,y) = p_0(x)+\varepsilon p_1(x,y),\ \theta(x,y) = \theta_0(y)+\varepsilon\theta_1(x,y) \tag{13}$$

where the perturbations u_1, υ_1, p_1 and θ_1 are small compared with the mean or zero-order quantities, Eqns (8) to (11) yield the following non-dimensional equations:

$$\frac{d^2u_0}{dy^2}+G\theta_0=0,\ \frac{d^2\theta_0}{dy^2}=-\alpha \tag{14}$$

to the zero-order and

$$\frac{\partial u_1}{\partial x}+\frac{\partial \upsilon_1}{\partial y}=0 \tag{15}$$

$$u_0\frac{\partial u_1}{\partial x}+\upsilon_1\frac{\partial u_0}{\partial y}= -\frac{\partial P_1}{\partial x}+\frac{\partial^2 u_1}{\partial x^2}-K\left[u_0\frac{\partial^3 u_1}{\partial x^3}+u_0\frac{\partial^3 u_1}{\partial x\partial y^2}\right.$$

Table 5. Skin-friction for case I ($m = -1$)

K	0		0.15		0.25	
α	σ_w	σ_1	σ_w	σ_1	σ_w	σ_1
-5	-0.3482883	1.9555300	-0.3482771	1.9554200	-0.3482697	1.9553790
-2	0.3430082	1.2645570	0.3429959	1.2644910	0.3429876	1.2644130
0	0.8038702	0.8039326	0.8037955	0.8039041	0.8037456	0.8038786
2	1.2647300	0.3433274	1.2645560	0.3432728	1.2644400	0.3477632
5	1.9560170	-0.3475443	1.9556230	-0.3476755	1.9553600	-0.3477632
8	2.6472990	-1.0383720	2.6466020	-1.0385890	2.6461370	-1.0387340
10	3.1081520	-1.4989000	3.1072050	-1.4991780	3.1065740	-1.4993640

Table 6. Skin-friction for case II ($m = -1$)

K	0		0.15		0.25	
α	σ_w	σ_1	σ_w	σ_1	σ_w	σ_1
-5	-0.3482946	1.9550360	-0.3482723	1.9548170	-0.3482575	1.9545370
-2	0.3430069	1.2643820	0.3429822	1.2642500	0.3429657	1.2641950
0	0.8038698	0.8039941	0.8037204	0.8038772	0.8036208	0.8038160
2	1.2647290	0.3436446	1.2643800	0.3435357	1.2641480	0.3434630
5	1.9560110	-0.3468069	1.9552230	-0.3470676	1.9541480	-0.3472422
8	2.6472840	-1.0371710	2.6458890	-1.0375990	2.6449590	-1.0378860
10	3.1081290	-1.4973640	3.1062360	-1.4979120	3.1049740	-1.4982810

$$+\upsilon_1 \frac{\partial^3 u_0}{\partial y^3} + \frac{\partial u_0}{\partial y}\frac{\partial^2 \upsilon_1}{\partial x^2} - \frac{\partial u_0}{\partial y}\frac{\partial^2 u_1}{\partial x \partial y} - \frac{\partial \upsilon_1}{\partial y}\frac{\partial^2 u_0}{\partial y^2}\Bigg] \quad (16)$$

$$u_0 \frac{\partial \upsilon_1}{\partial x} = -\frac{\partial P_1}{\partial y} + \frac{\partial^2 \upsilon_1}{\partial x^2} + \frac{\partial^2 \upsilon_1}{\partial y^2} - K\Bigg[u_0 \frac{\partial^3 \upsilon_1}{\partial x \partial y^2} + u_0 \frac{\partial^3 \upsilon_1}{\partial x^3} - 2\frac{\partial^2 u_0}{\partial y^2}\frac{\partial \upsilon_1}{\partial x} - 2\frac{\partial u_0}{\partial y}\frac{\partial^2 \upsilon_1}{\partial x \partial y} \quad (17)$$

and

$$P\left(u_0 \frac{\partial \theta_1}{\partial x} + \upsilon_1 \frac{\partial \theta_0}{\partial y}\right) = \frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2}$$

to the first-order. In deriving the first equation in Eqn (14), the constant pressure gradient term $\partial/\partial x(p_0 - p_s)$ has been taken equal to zero following Ostrach[2]. In view of Eqn (13), the boundary condition in Eqn (12) can be split up into the following two parts:

$$\left.\begin{aligned} u_0 = 0,\ \theta_0 = 1 \quad & on\ y = 0 \\ u_0 = 0,\ \theta_0 = m \quad & on\ y = 1 \end{aligned}\right\} \quad (19)$$

$$\left.\begin{aligned} u_1 = -Re\left(u_0' e^{i\lambda x}\right),\ \upsilon_1 = 0,\ \theta = -Re\left(\theta_0' e^{i\lambda x}\right) \quad & on\ y = 0 \\ u_1 = 0,\ \upsilon_1 = 0,\ \theta_0 = 0 \quad & on\ y = 1 \end{aligned}\right\} \quad (20)$$

where the prime denotes differentiation wrt y.

3. METHOD OF SOLUTION

The solution for the zero-order velocity (u_0) and the zero-order temperature (θ_0) satisfying the differential Eqn (14) and the boundary conditions (19) are given by

$$u_0 = \frac{G}{12}\left[\left(H_1 + 2H + 6\right)y - H_1 y^4 - 2Hy^3 - 6y^2\right]$$

$$\theta_0 = 1 + Hy + H_1 y^2$$

Table 5. Skin-friction for case I ($m= -1$)

K	0		0.15		0.25	
α	σ_w	σ_1	σ_W	σ_1	σ_W	σ_1
-5	-0.3482883	1.9555300	-0.3482771	1.9554200	-0.3482697	1.9553790
-2	0.3430082	1.2645570	0.3429959	1.2644910	0.3429876	1.2644130
0	0.8038702	0.8039326	0.8037955	0.8039041	0.8037456	0.8038786
2	1.2647300	0.3433274	1.2645560	0.3432728	1.2644400	0.3477632
5	1.9560170	-0.3475443	1.9556230	-0.3476755	1.9553600	-0.3477632
8	2.6472990	-1.0383720	2.6466020	-1.0385890	2.6461370	-1.0387340
10	3.1081520	-1.4989000	3.1072050	-1.4991780	3.1065740	-1.4993640

Table 6. Skin-friction for case II ($m= -1$)

K	0		0.15		0.25	
α	σ_W	σ_1	σ_W	σ_1	σ_W	σ_1
-5	-0.3482946	1.9550360	-0.3482723	1.9548170	-0.3482575	1.9545370
-2	0.3430069	1.2643820	0.3429822	1.2642500	0.3429657	1.2641950
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10	3.1081290	-1.4973640	3.1062360	-1.4979120	3.1049740	-1.4982810

$$+\upsilon_1\frac{\partial^3 u_0}{\partial y^3}+\frac{\partial u_0}{\partial y}\frac{\partial^2\upsilon_1}{\partial x^2}-\frac{\partial u_0}{\partial y}\frac{\partial^2 u_1}{\partial x\partial y}$$

$$-\frac{\partial\upsilon_1}{\partial y}\frac{\partial^2 u_0}{\partial y^2}\Bigg] \quad (16)$$

$$u_0\frac{\partial\upsilon_1}{\partial x}=\frac{\partial P_1}{\partial y}+\frac{\partial^2\upsilon_1}{\partial x^2}+\frac{\partial^2\upsilon_1}{\partial y^2}-K\Bigg[u_0\frac{\partial^3\upsilon_1}{\partial x\partial y^2}\quad u_0\frac{\partial^3\upsilon_1}{\partial x^3}$$

$$-2\frac{\partial^2 u_0}{\partial y^2}\frac{\partial\upsilon_1}{\partial x}-2\frac{\partial u_0}{\partial y}\frac{\partial^2\upsilon_1}{\partial x\partial y}\Bigg] \quad (17)$$

and

$$P\left(u_0\frac{\partial\theta_1}{\partial x}+\upsilon_1\frac{\partial\theta_0}{\partial y}\right)=\frac{\partial^2\theta_1}{\partial x^2}+\frac{\partial^2\theta_1}{\partial y^2} \quad (18)$$

to the first-order. In deriving the first equation in Eqn (14), the constant pressure gradient term $\partial/\partial x(p_0-p_s)$ has been taken equal to zero following Ostrach[2]. In view of Eqn (13), the boundary condition in Eqn (12) can be split up into the following two parts:

$$\left.\begin{array}{ll} u_0=0,\ \theta_0=1 & on\ y=0 \\ u_0=0,\ \theta_0=m & on\ y=1 \end{array}\right\}$$

$$\left.\begin{array}{ll} u_1=-Re\left(u_0' e^{i\lambda x}\right),\ \upsilon_1=0,\ \theta=-Re\left(\theta_0' e^{i\lambda x}\right) & on\ y=0 \\ u_1=0,\ \upsilon_1=0,\ \theta_0=0 & on\ y=1 \end{array}\right\}$$

where the prime denotes differentiation wrt y.

3. METHOD OF SOLUTION

The solution for the zero-order velocity (u_0) and the zero-order temperature (θ_0) satisfying the differential Eqn (14) and the boundary conditions (19) are given by

$$u_0=\frac{G}{12}\left[(H_1+2H+6)y-H_1y^4-2Hy^3-6y^2\right]$$

$$\theta_0=1+Hy+H_1y^2$$

Table 7. Skin-friction for case III ($m = -1$)

K	0		0.15		0.25	
	σ_w	σ_t	σ_w	σ_t	σ_w	σ_t
	-0.6965906		-0.6965460	3.9098360	-0.6965162	3.9096760
	0.6860130		0.6859635	2.5285020	0.6859305	2.5284720
0	1.6077390		1.6074400	1.6077560	1.6072410	1.6076330
2	2.5294580		2.5287610	0.6870721	2.5282950	0.6869266
5	3.9120220		3.9104470	-0.6941354	3.9093960	-0.6944846
8	5.2945700		5.2917800	-2.0751980	5.2899200	-2.0757740
10	6.2162590		6.2124730	-2.9958250	6.2099490	-2.9965630

where $H_1 = \frac{\alpha}{2}, H = m + \frac{\alpha}{2} + 1$

In order to solve Eqns(15) to (18) for the first-order quantities, it is convenient to introduce the stream function $\overline{\psi}_1$ denoted by

$$u_1 = -\frac{\partial \overline{\psi}}{\partial y} \quad \upsilon_1 = \frac{\partial \overline{\psi}}{\partial x}$$

$$\overline{\psi}(x,y) = e^{i\lambda x}\psi(y), \; \theta_1(x,y) = e^{i\lambda x}t(y)$$

these equations can be reduced to the ordinary differential equations:

$$\psi^{iv} - \psi''\left[2\lambda^2 + i\lambda u_0\right] + \psi\left[\lambda^4 + i\lambda u_0'' + iu_0\lambda^3\right] + Ki\left[-\lambda u_0\psi + 2u_0\lambda^3\psi'' - 2\lambda^3 u_0'\psi' - 3\lambda^3 u_0''\psi + \lambda u_0^{iv}\psi - \lambda^5 u_0\psi\right] = Gt' \quad (22)$$

and

$$t'' - \lambda^2 t = Pi\lambda\left(u_0 t + \psi\theta_0'\right) \quad (23)$$

subject to boundary conditions

$$\left.\begin{matrix} \psi = 0, & t = -\theta_0' & on\, y = 0 \\ \psi = 0, & t = -\theta_0' & on\, y = 1 \end{matrix}\right\} \quad (24)$$

If one considers only small values of λ (or $k <<$ then substituting

$$\psi(\lambda, y) = \sum_{j=0}^{2} \lambda^j \psi_j \quad t(\lambda, y) = \sum_{j=0}^{2} \lambda^j t_j$$

into Eqns(22), (23) and (24), to the order of λ^2, the following sets of ordinary differential equations and corresponding boundary conditions are obtained:

$$\psi_0^{iv} = Gt_0' \quad t_0'' = 0$$

$$\psi_1^{iv} = iu_0\psi_0'' - iu_0''\psi_0 + ik\left[u_0\psi_0^{iv} - u_0^{iv}\psi_0\right] + Gt_1'$$
$$t_1'' = Pi\left[u_0 t_0 + \psi_1\theta_0'\right] + t_0$$

$$\psi_2^{iv} = 2\psi_0'' + iu_0\psi_1'' - iu_0''\psi_1 + Ki\left[u_0\psi_v^{iv}\right.$$
$$t_2'' = Pi\left[u_0 t_1 + \psi_1\theta_0'\right] + t_0$$

(27)

and

$$\left.\begin{matrix} \psi_0' = u_0', \; \psi_0 = 0, \; t_0 = -\theta_0' & on\, y = 0 \\ \psi_0' = 0, \; \psi_0 = 0, \; t_0 = 0 & on\, y = 1 \end{matrix}\right\}$$

$$\left.\begin{matrix} \psi_1' = 0, \; \psi_1 = 0, \; t_1 = 0 & on\, y = 0 \\ \psi_1' = 0, \; \psi_1 = 0, \; t_1 = 0 & on\, y = \end{matrix}\right\}$$

$$\left.\begin{matrix} \psi_2' = 0, \; \psi_2 = 0, \; t_2 = 0 & on\, y = 0 \\ \psi_2' = 0, \; \psi_2 = 0, \; t_2 = 0 & on\, y = 1 \end{matrix}\right\}$$

Solution for Eqns (25) to (27) consistent with the boundary conditions, (28) to (30) have been obtained but not presented here for the sake of brevity. From these solutions, the first-order velocity components are given by

Table 8. Skin-friction for case IV ($m= -1$)

K	0		0.15		0.25	
α	σ_w	σ_1	σ_w	σ_1	σ_w	σ_1
-5	-0.6967614	3.9090670	-0.6967188	3.9087300	-0.6966904	3.9085710
-2	0.6859352	2.5283730	0.6858828	2.5280100	0.6858478	2.5278000
0	1.6076720	1.6077120	1.6073710	1.6074790	1.6071700	1.6073570
2	2.5293620	0.6869006	2.5286620	0.6866832	2.5281960	0.6865380
5	3.9118590	-0.6945852	2.9102420	-0.6951095	3.9091940	-0.6944846
8	5.2941730	-2.0763730	5.2914240	-2.0772470	5.2895910	-2.0778340
10	6.2156980	-2.9977215	6.2120160	-2.9988580	6.2095620	-2.9996220

$$\left.\begin{aligned} u_1 &= \psi_i' \sin\lambda x - \psi_r' \cos\lambda x \\ \upsilon_1 &= -\lambda\psi_r \sin\lambda x - \lambda\psi_i \cos\lambda x \end{aligned}\right\} \quad (31)$$

where $\psi_r = \psi_0 + \lambda^2\psi_2$, $\psi_i = -\lambda G\psi_3$ where $\psi_3 = -i\psi_1/G$. The first-order temperature is given by

$$\theta_1 = \left(t_0 + \lambda^2 t_2\right)\cos\lambda x - Pt_3 \sin\lambda x,\ t_3 = -^{it_1} \quad (32)$$

The velocity components (u,υ) of the non-Newtonian fluid are as follows:

$$u = \frac{G}{12}\left[(H_1 + 2H + 6)y - H_1 y^4 - 2Hy - 6y^2 - \varepsilon\left[\psi_r' \cos\lambda x + \psi_i \sin\lambda x\right]\right. \quad (33)$$

$$\upsilon = -\varepsilon\lambda\left[\psi_r \sin\lambda x - \psi_i \cos\lambda x\right] \quad (34)$$

The temperature field for the flow is given by

$$\theta = 1 + Hy + H_1 y^2 + \varepsilon\left[\left(t_0 + \lambda^2 t_2\right)\cos\lambda x - Pt_3 \sin\lambda x\right]$$

4. RESULTS & DISCUSSION

The shearing stress σ_{xy} at any point in the fluid is given in non-dimensional form by

$$\sigma_{xy} = \frac{d^2\bar{\sigma}_{xy}}{\rho\upsilon^2} = u_0'(y) + \varepsilon e^{i\lambda x}\bar{u}_1'(y) + i\varepsilon\lambda e^{i\lambda x}\bar{\upsilon}_1(y)$$
$$+K\varepsilon\left[3u_0' e^{i\lambda x}\bar{\upsilon}_1'(y) + u_0'(y)(i\lambda)e^{i\lambda x}\bar{u}_1(y) - u_0''(y)e^{i\lambda x}\bar{\upsilon}_1(y) + u_0(y)\lambda^2 e^{i\lambda x}\bar{\upsilon}_1(y) - u_0(y)(i\lambda)e^{i\lambda x}\bar{u}_1'(y)\right] \quad (36)$$

At the wavy wall, $y = \varepsilon\cos\lambda x$ and at the flat wall $y = 1$, σ_{rv} becomes

$$\sigma_w = \sigma_0^0 + \varepsilon\left[u_0''(0)\cos\lambda x - \psi_0''(0)\cos\lambda x + \lambda G\psi_3''(0)\sin\lambda x - \lambda^2\psi_2''(0)\cos\lambda x\right] - 2\varepsilon\lambda\sigma_0^0 K\left[\psi_0'(0)\sin\lambda x + \lambda G\psi_3'(0)\cos\lambda x\right] \quad (37)$$

and

$$\sigma_1 = \sigma_1^0 + \varepsilon\left[\lambda G\psi_3''(1)\sin\lambda x - \psi_0''(1)\cos\lambda x - \lambda^2\psi_2''(1)\cos\lambda x\right] + \lambda\varepsilon K\left[\lambda G u_0''(1)\psi_3(1)\cos\lambda x - 2\sigma_1^0\{\psi_1'(1)\sin\lambda x\ \lambda G\psi_3'(1)\cos\lambda x\}\right] \quad (38)$$

respectively, where $\sigma_0^0 = u_0'(0)$ and $\sigma_1^0 = u_0'(0)$ are the zero-order skin-friction at the walls, and $\bar{u}_1(y)$ and $\bar{\upsilon}_1(y)$ are given by

$$u_1(x,y) = e^{i\lambda x}\bar{u}_1(y),\quad \upsilon_1(x,y) = e^{i\lambda x}\bar{\upsilon}_1(y) \quad (39)$$

The non-dimensional heat transfer coefficient known as Nusselt number (N_u) is given by

$$N_u = \frac{\partial\theta}{\partial y} = \theta_0'(y) + \varepsilon e^{i\lambda x}t'(y) \quad (40)$$

At the wavy wall, $y = \varepsilon\cos\lambda x$ and at the flat wall, $y = 1$, N_u takes the form

$$N_{uw} = N_{u_0}^0 + \varepsilon[\theta''_0(0)\cos\lambda x + t'_0(0)\cos\lambda x + \lambda^2\cos\lambda x t'_2(0) - Pt'_3(0)\sin\lambda x] \quad (41)$$

and

$$N_{u_1} = N^{o}_{u_1} + \varepsilon\left[t_0'(1)\cos\lambda x + t_2'(1)\lambda^2\cos\lambda x - Pt_3'(1)\sin\lambda x\right] \quad (42)$$

respectively, when $N \;\; = \theta_0'(0),\; N^{0}_{u_1} = \theta_0'(1)$.

The purpose of this study is to bring out the effects of non-Newtonian parameter on the flow and heat transfer characteristics as the effects of other parameters have been discussed in detail by Vajravelu and Sastri[4]. The non-Newtonian effect is exhibited through the non-dimensional parameter (K). All the corresponding results for Newtonian fluid are obtained by setting $K = 0$.

It was noticed from differential Eqn (14) that the non-dimensional (zero-order) temperature of the fluid is affected only by the parameter α and the wall temperature ratio (m) and that the non-dimensional velocity of the fluid is affected by the free convection parameter (G) in addition to the parameters α and m but not by the non-Newtonian parameter K. It has been observed from the expressions in Eqns (41) and (42) that the heat transfer coefficients are not significantly affected by the parameter K. The skin friction at $y = 0$, in general, is an increasing function of G, while that at $y=1$ decreases with an increase in G, this behaviour holding for any value of m. To observe the non-Newtonian effect, the skin-friction coefficient is presented for various combinations of the parameters as follows:

Case			III	V
	5.00	5.00	10.00	0.00
	0.01	0.02	0.01	0.01
P	0.71	0.71	0.71	7

Tables (1) to (8) show the behaviours of the skin-friction at the channel walls for different cases when $m = 1$ or -1. It is found from the tables that the skin-friction at the wavy wall σ_w is an increasing function of G, P, λ, α while the reverse behaviour occurs at the flat wall σ_1 for $K = 0, 0.15, 0.25$. Again in case of equal wall temperature ($m = 1$), the skin-friction at both the walls decreases for increasing α and K but when the average of the temperatures of the two walls is equal to that of the static fluid ($m = -1$), both $|\sigma_w|$ and σ_1 decrease with increase of α and K.

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