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SHORT COMMUNICATION

Stability Criterion for a Finned Spinning Projectile

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ABSTRACT

The state-of-the-art in gun projectile technology has been used for the aerodynamic stabilisation. This approach is acceptable for guided and controlled rockets but the free-flight rockets suffer from unacceptable dispersion. Sabot projectiles with both spin and fins developed during the last decade need careful analysis. In this study, the second method of Liapunov has been used to develop stability criterion for a projectile to be designed with small fins and is made to spin in the flight. This criterion is useful for the designer.

1. INTRODUCTION

The motion of an axis-symmetric projectile in the cross plane moving in the atmosphere[1] is given by

$$\ddot{X} + (-K_1 I + K_2 J)\dot{X} + (K_3 I - K_4 J) X = 0$$

where

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Here, X is non-dimensional velocity in the cross plane, viz., $\begin{pmatrix} v/u \\ w/u \end{pmatrix}$ and K_i ($i =$, 2, 3, 4) are damping, spin, position and magnus parameters, respectively.

The spin-stabilised projectiles require high spin velocity to counterbalance the aerodynamic positional disturbing moments, but the spin in the projectile reduces its penetration capability. The fin-stabilised projectiles require the moments generated by these fins to take care of this disturbing moments. Large finned projectiles cannot give stable design of the gun-projectile system, so the sub-calibre projectiles along with sabot are used these days for short ranges to increase the depth of penetration in the armour. This requires pointed accuracy. Also, the fin size required for stability disturbs the sabot design. Hence, the concept of sabot which is partly spinning (generally of the order of 50 per cent of the spin-stability) and partly finned, both the stability means have been employed. The motion analysis of such bodies has been examined here. For the spinning projectiles the overturning moment $K_3 < 0$, while for the aerodynamically stable projectiles $K_3 > 0$. An attempt has been made to develop a criterion for both partly spinned and partly finned bodies (K_3 is arbitrary).

2. DEVELOPMENT OF STABILITY CRITERION

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The general P-method[2] has been used for the development of stability theory. The choice of arbitrary function in the generating function provides freedom to both the analyst and the designer to look for new designs through the stability constructions.

2.1 Liapunov Approach

For the motion

$$L \equiv \ddot{X} + (-K_1 I + K_2 J)\dot{X} + (K_3 I + K_4 J)X = 0$$

and the generating function

$$N \equiv 2\dot{X} + (P + P_2)X \tag{2}$$

the identity $< L, N > + < L; N > = 0$ leads to a suitable Liapunov function V and its derivative $\dot{V}$ as

$$V = 2\left(\dot{X}' \frac{X'P'}{2}\right)\left(\dot{X} + \frac{PX}{2}\right) + X'\left(2K_3 + 2Q \quad \frac{P'P}{2}\right)X \tag{3}$$

and

$$\dot{V} = \Big[\dot{X}'\sqrt{-(P + P' + 4K_1)} - X'(K_1 I + K_2 J)P + 2K_4 J + 2Q)'\sqrt{-(P + P' + 4K_1)}\Big]$$

$$\Big[\sqrt{-(P + P' + 4K_1)}\dot{X} - \sqrt{-(P + P' + 4K_1)}$$

$$\{(K_1 I + K_2 J)P + 2K_4 J + 2Q\}X\Big] - X'\Big[\{(K_1 I + K_2 J)P + 2K_4 J + 2Q\}'\{-(P + P' + 4K_1)\}^{-1}\{(K_1 I + K_2 J)P + 2K_4 J + 2Q\} + \{(K_3 I + K_4 J)P + P'(K_3 I + K_4 J)'\}\Big]X \tag{4}$$

A quadratic arbitrary matrix Q has been introduced to obtain meaningful criterion. The choice of Q is such that V is a perfect square. Thus

$$Q \quad -K_3 + \frac{P'P}{2} \tag{5}$$

Since $P'P > 0$, Liapunov function is a suitably chosen form. A choice can be made

$$P = -K_1 I + K_2 J \tag{6}$$

Then the asymptotic stability requirement simplifies to

$$K_1^2 K_3 + K_1 K_2 K_4 \quad K_3^2 - K_4^2 > 0 \tag{7}$$

The above inequality can be rewritten as

$$\left(\frac{K_3}{K_1} - \frac{K_1}{2}\right) + \frac{K_1^2}{4} + \frac{K_2 K_4}{4} \quad \left(\frac{K_4}{K_1}\right) > 0$$

The above condition to be independent of K implies

$$\frac{K_1^2}{4} + \frac{K_2 K_4}{4} \quad \left(\frac{K_4}{K_1}\right) > 0 \tag{8}$$

Let it be defined that

$$S = + \frac{\left(\frac{2K_4}{K_1}\right) - K_2}{\sqrt{K_1^2 + K_2^2}}$$

is called modified stability parameter so that the above condition in Eqn (8) simplifies to

$$S \quad S - 2) > 0$$

The design criteria is that S should be bounded from both sides as $0 < S < 2$. For different values of S, the other parameters K_1, K_2 and K_4 can be determined independent of K_3 as shown in Figs (1) and (2).

$$4\left(K_4 - \frac{K_1 K_2}{2} \quad + K_1^2(K_2^2 + 4K_3) - 4K_3^2 > 0\right.$$

It is obvious that $K_2^2 + 4K_3$ has to be positive. This can be achieved easily with a small spin and $|K_3|$ to be small. This inequality can be satisfied for large values of damping parameter K_1 so that the vibrations would damp out in a short travel of

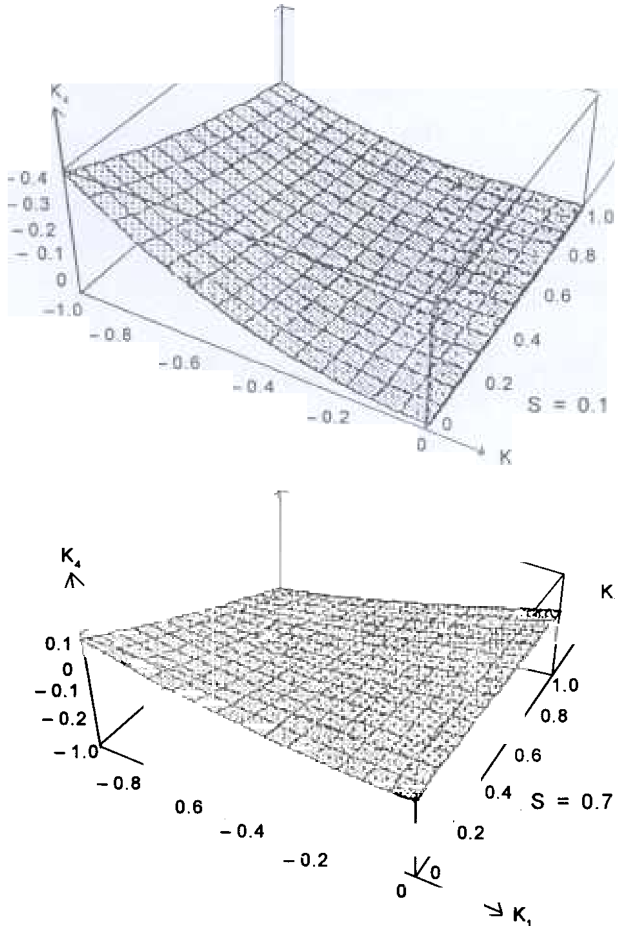

Figure 1. Surfaces in (K_1, K_2, K_4) space for different values of S.

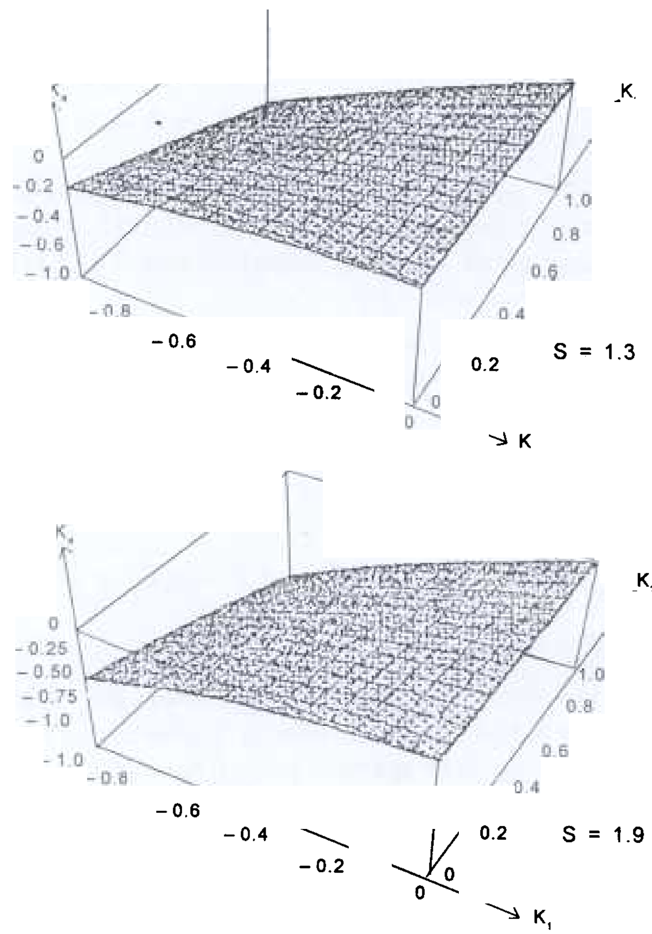

Figure 2. Surfaces in (K_1, K_2, K_4) space for different values of S.

the chosen range. This is one of the essential requirement for a sabot design. This gives the required dynamic stability for the motion of the body which can be met.

2.2 Eigen Value Approach

The Liapunov functions V and $\dot{V}$ can be expressed in matrix form as

$$V = Y^t A_V Y$$

and $\quad \dot{V} = Y^t A_{\dot{V}} Y$

where

$$Y = \begin{pmatrix} \dot{X} \\ X \end{pmatrix} \quad \text{and} \quad A_v = \begin{pmatrix} 2I & P \\ P^t & 2(K_3 + Q) \end{pmatrix} \text{ and}$$

$$A_{\dot{V}} = \begin{pmatrix} P + P^t + 4K_1 & (K_1 I + K_2 J)P + 2K_4 J + 2Q \\ [(K_1 I + K_2 J)P + 2K_4 J + 2Q] & -[(K_3 I + K_4 J)P + P^t(K_3 I - K_4 J)] \end{pmatrix}$$

For the choice of $P = -K_1 I + K_2 J$

and

$$Q = -K_3 I + \frac{P^t P}{2} J$$

A and $A_{\dot{V}}$ are

$$A_{v} = \begin{pmatrix} 2I & -K_1 I + K_2 J \\ -K_1 I + K_2 J) & (K_1^2 + K_2^2) I \end{pmatrix}$$

and

$$A_{\dot{v}} = \begin{pmatrix} 2K_1 I & -2(K_3 I - K_4 J) \\ [-K_3 I - K_3 J) & 2(K_3 K_1 + K_2 K_4) I \end{pmatrix}$$

Both A_v and $A_{\dot{v}}$ will have a pair of repeated eigen values each with multiplicity 2. The calculation of these eigen values is straight-forward and the eigen values are:

$$\lambda_{1,2} = \left[(K_1^2 + K_2^2 + 2) \pm \sqrt{[(K_1^2 + K_2^2)^2 + 4] / 2}\right]$$

for A_v (9)

$$\lambda'_{1,2} = \Big[(K_3 K_1 + K_2 K_4 + K_1 \pm \sqrt{(K_3 K_1 + K_2 K_4 - K_1)^2 + 4(K_3^2 + K_4^2)}\Big]$$

for $A_{\dot{v}}$ (10)

For each matrix A_v and $A_{\dot{v}}$, one gets four linearly independent eigen vectors. The matrix formed by these vectors reduces the given matrices A_v and $A_{\dot{v}}$ to their corresponding diagonal forms.

The asymptotic stability implies that $Re \mid \lambda \mid$ is necessarily positive. For A_v, it is a simple verification within the expression and for $A_{\dot{v}}$, both the eigen values should be negative. The conditions are:

$$\left.\begin{aligned} &K_3 K_1 + K_2 K_4 + K_1 < 0 \text{ and} \\ &K_3 K_1^2 + K_1 K_2 K_4 - K_3^2 - K_4^2 > 0 \end{aligned}\right\} \quad (11)$$

The first condition in Eqn (11) is superfluous as $K_1 < 0$, Eqn (11) can be written as

$$K_1(K_3 K_1 + K_2 K_4 + K_1) - K_1^2 + K_3^2 - K_4^2 > 0$$

If the condition [Eqn (11)] holds, the asymptotic stability is guaranteed and is same as condition in Eqn (7).

3. GENERALISATION OF STABILITY MATRICES

The general form of A_v and $A_{\dot{v}}$ can be generated as

$$M = \begin{pmatrix} A & C \\ C^t & B \end{pmatrix}, \quad A \ \& \ B \text{ are symmetric matrices.}$$

The characteristic equation for M is:

$$\lambda^4 - \sigma_1 \lambda^3 + \sigma_2 \lambda^2 - \sigma_3 \lambda + \sigma_4 = 0$$

$$\sigma \quad = Trace\ M$$

$$\sigma_2 = \sum_{i \neq j} \sigma_{2ij}$$

where

$$\sigma_3 = \sum_{i \neq j \neq k} \sigma_{3ijk} \quad , i, j, k = \ , 2, 3, 4$$

$$\sigma_4 \quad = \text{Det } M$$

Here, σ_i denotes the principle minor of M of order i. The condition for equal eigen values of multiplicity 2 are:

$$\sigma_3^2 = \sigma_1 \sigma_4$$

$$64\sigma_4 = (4\sigma_2 - \sigma_1^2)^2$$

or

$$\sigma_1^3 - 4\sigma_1\sigma_2 + 8\sigma_3 = 0$$

Result 1) If both A and B are diagonal matrices, the form of C is $c_1 I + c_2 J$, such that the matrix M has a pair of repeated eigen values.

Result (2) For C to be either of the type $c_1 I$ or $c_2 J$, A diagonal and B satisfies the condition trace^2 B = 4 det B, such that M has a pair of repeated eigen values.

For the system discussed in Section 2, for V one has:

$$A = 2I, \ B = 2(K + Q) \quad \text{and} \quad C = P$$

Applying *Result* (1), the choice of Q is to be made in such a way that B is diagonal which gives Q also diagonal. Then P is of the form $p_1 I + p_2 J$, which is the selected one. If P is of the type $p_1 I$ or $p_2 J$, by *Result* (2), Q should be selected so as to satisfy the condition trace^2 Q = 4 det Q. A diagonal matrix always satisfies this condition. This has guided to take Q of the form given in Eqn (5).

For $\dot{V}$, $A = P + P^t + 4K_1$

$$B = -[(K_3 I + K_4 J)P + P^t (K_3 I + K_4 J)^t]$$

and

$$C = (K_1 I + K_2 J)P + 2K_4 J + 2Q$$

The condition when A is diagonal implies $P + P^t$ should be diagonal. Hence P is of the form $p_1 I + p_2 J$. This choice of P reduces B to a diagonal matrix. C can be selected to have a form $c_1 I + c_2 J$, by *Result* (1). This can be obtained with the above choice of Q as diagonal.

Generally in a gun system, as A is an inertia matrix, it can be taken as a unit matrix. B comes from cross-force which can be taken to be diagonal. The above choice of $P = p_1 I + p_2 J$ and Q diagonal is well-suited for a gun projectile.

4. CONCLUSIONS

For a generalised motion in a cross plane the form of generating matrix, (P) and parametric matrix, (Q) have been obtained in Section 3 for any projectile that is partly spinned and partly finned to guarantee a proper stable motion if the condition

$$K_1^2 K_3 + K_1 K_2 K_4 - K_3^2 - K_4^2 > 0$$

is met. The results are verified for a particular data. It has been established that the minimum value of spin required is 36 per cent of the spinning projectile.

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Contributor

Dr (Mrs) SD Naik obtained her PhD from the University of Poona, Pune, in 1988. She joined DRDO at the Institute of Armament Technology, Pune, in 1987. Her areas of research include: flight dynamics and stability analysis. She has published seven papers in national/international journals.