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AMERICAN METHOD AND COMPARISON OF THE DIFFERENT METHODS OF INTERNAL BALLISTICS.*

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The empirical system of Internal Ballistics initiated by Le Duc is widely used in America. The following are the symbols and working formulas in the Le Duc's system.

Symbols	Unit	Definition
x	ft.	Travel of projectile in bore to any point.
X	ft.	Total travel of projectile to the muzzle.
v	ft./sec.	Velocity of projectile in bore at any point x.
V	ft./sec.	Muzzle velocity of projectile.
P_g	tons/in²	Pressure in gun when projectile is at any point x.
P_p	tons/in²	Portion of P_g producing translational velocity of projectile.
$P_{g\,(max)}$	tons/in²	Maximum pressure attained in gun during travel of projectile.
w	lb.	Weight of powder.
W	lb.	Weight of projectile.
S	Cu. in.	Volume of chamber.
A	Sq. in.	Area of cross section of bore.
Δ		Loading density $= 27{\cdot}68$ w/S
δ		Specific gravity of powder.
γ		Ratio of the specific heats of product gases.
β		Powder constant depending on form and dimensions of grain, % of volatiles, etc.
a		Constant depending on nature and wt. of powder, wt. of projectile and loading density.
b		Constant depending on powder, chamber volume, loading density and weight of projectile.

WORKING FORMULAE

$$v = \frac{ax}{b+x} \qquad \ldots\ldots (1)$$

$$V = \frac{aX}{b+X} \qquad \ldots\ldots (1a)$$

*Paper read at a meeting of the Defence Science Organisation in April 1951.

$$\left.\begin{aligned} a &= 6823\ \Delta^{\frac{1}{12}} \left(\frac{w}{W}\right)^{\frac{1}{2}} \\ &= \frac{6823\ (27\cdot 68)^{\frac{1}{12}}\ (w)^{\frac{7}{12}}}{S^{\frac{1}{12}} \cdot W^{\frac{1}{2}}} \end{aligned}\right\}\ldots\ldots(2)$$

$$b = \beta\left(1 - \frac{\Delta}{\delta}\right)\left(\frac{S}{W}\right)^{\frac{2}{3}} \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\left.\begin{aligned} b &= \frac{186\cdot 53}{AS^{\frac{1}{6}}}\ \frac{w^{\frac{7}{6}}}{P_{g(max)}} \\ &= \frac{Q\ w^{\frac{7}{6}}}{P_{g(max)}} \end{aligned}\right\}\ldots\ldots(4)$$

$$\begin{aligned} P_g &= 1\cdot 12 \cdot P_p \\ &= \frac{1\cdot 12}{2240} \cdot \frac{wa^2 b}{gA} \cdot \frac{x}{(b+x)^3} \ldots\ldots\ldots\ldots\ldots\ldots(5) \end{aligned}$$

$$P_{g(max)} = \frac{1\cdot 12}{15120}\ \frac{wa^2}{g\ A\ b} \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

From (5) and (6)

$$\left.\begin{aligned} P_g &= \frac{27}{4}\ b^2 P_{g(max)} \cdot \frac{x}{(b+x)^3} \\ &= R_g\ \frac{x}{(b+x)^3} \end{aligned}\right\}\ldots(7)$$

EXPLANATORY NOTE ON THE WORKING FORMULAE.

(*i*) Eq. (1) is the fundamental empirical assumption in Le Duc's system in which the velocity-space curve of the projectile in the bore is assumed to be a hyperbola.

(*ii*) From eq. (*1*) it is seen that when $x=\infty$, $v=a$. Hence 'a' represents the muzzle velocity of the projectile if the barrel were infinitely long, *i.e.*, when the muzzle energy represents all the work the powder charge is capable of doing. This idea enables 'a' to be expressed by the equation.

$$a=\sqrt{2E}\ \Delta^{\frac{\gamma-1}{2}}\left(\frac{w}{W}\right)^{\frac{1}{2}} \qquad \text{.....................(8)}$$

where E is the work done by one pound of gas in adiabatically expanding from an initial specific gravity of unity to a final specific gravity of zero (i.e. from an initial volume of 27·68 cu. in. to a final volume of ∞). For most American powders, E=653 ft. tons per pound and $\gamma=^{7}$.
Hence we get

$$a=9706\ \Delta^{\frac{1}{12}}\left(\frac{w}{W}\right)^{\frac{1}{2}} \qquad \text{.............................(9)}$$

But due to energy losses in heating and expanding the gun, forcing pressure friction etc., none of which has been taken into account in eq. (8), eq. (9) does not agree with experiment. From a large number of experiments, an empirical value 6823 is used in place of 9706 in eq. (9) and hence we get eq. (2).

For British propellants the value of 'E' is different. Moreover, γ is about 5/4 for most British propellants. Hence the empirical value to be used for British propellants in eq. (2) would differ from 6823. The actual value can only be obtained by analysing a large number of firing results with British propellants by Le Duc's formulae.

(*iii*) Eq. (5) is deduced directly from eq. (1) except for the empirical factor 1·12. This factor arises because the pressure in the gun, besides accelerating the projectile, does various items of non-useful work such as heating and expanding the gun, over-coming shot-start pressure, accelerating the powder gases, imparting recoil energy to the gun, overcoming friction, etc. The actual value 1·12 is again chosen from a large number of observed firing results.

From eq. (5) it follows that the pressure in the gun is maximum when $x = \frac{b}{2}$. 'b' therefore represents twice the travel of the projectile to the point of maximum pressure. Substituting this value for X in eq. (5) we get eq. (6). Substituting the value of 'a' from eq. (2) in eq. (6), we get eq. (4) which expresses 'b' in terms of the observed maximum pressure, weight of the powder and the gun data.

By semi-empirical arguments, 'b' can also be expressed by the equation

$$b = \beta\left(1 - \frac{\Delta}{\delta}\right)\frac{S^{x}}{w^{y}} \qquad \text{.............................(10)}$$

where β is a powder constant depending only on the nature of the powder used. It represents the relative quickness of the powder and hence depends on web thickness, grain size, % of remaining volatiles, etc. The slower the powder, the larger the value of β. (This β should not be confused with the 'β' used in the G. M. II method.) From experiments it is found that $x=y=\frac{2}{3}$. Eq. (3) therefore follows).

(*iv*) Eq. (5) or eq. (7) may be used to determine the pressure-space curve.

Applications to problems.

(1) *Peak Pressure, Muzzle Velocity and* β :—If we know β and S for the powder, eq. (3) enables us to calculate 'b' and hence $P_{g(max.)}$ can be calculated from eq. (4). If β is not known, then, since '*a*' is known from eq. (2) 'b' can be calculated from the experimentally observed muzzle velocity by using eq. (la). 'b' can also be calculated from eq. (4), if the peak pressue is known. If 'b' is known by either of these two methods, 'β' for the powder can be calculated from eq. (3). It is therefore seen that of the three quantities, other two as well as the pressure-space curve [by using eq. (5) or eq. (7)] can be easily calculated by using other easily determinable gun and powder data.

(2) *Pressure Problem.*—The problem in this case is to decide whether a particular powder, a definite weight of which produces a known muzzle velocity in a given gun (with peak-pressure within safe limits) is suitable to be used in the gun. The gun strengths at various points in the gun are supplied and the American practice is to see that the factor of safety, viz., the ratio of the gun strength at any point to the maximum pressure which the point would be subjected to, is 1·4 or more. As already mentioned, the value of 'b' can be calculated. Knowing b, 'b'/2 gives the point where the projectile is, when the peak pressure is reached. It should be remembered that the entire barrel upto this point (and a little beyond this point to take account of inaccuracies in calculation) is subjected to the peak pressure, maximum pressures to which points situated beyond are subjected, are then calculated by using eq. (5) or eq. (7) and factors of safety are calculated for various representative points from the known gun strengths at these points. If the factor of safety is everywhere 1·4 or more the powder is suitable ; otherwise not.

(3) *Reduced Velocity Problem, Type I.*—If the muzzle velocity produced in a given gun by a given weight of a particular powder is known the problem in this case is to calculate the muzzle velocity for a given reduced weight of the same powder in the same gun.

Here the value of 'b' can be calculated for the first case as already indicated. Then from eq. (3), we get β. The reduced charge weight gives the new value of loading density to be used. The new values of '*a*' and 'b' and hence the required reduced muzzle velocity can therefore be calculated.

(4) *Reduced Velocity Problem, Type II.*—If the muzzle velocity produced in a given gun by a given weight of a particular powder is known the problem is to calculate the reduced weight of the same powder that will produce a given reduced muzzle velocity. The problem can be worked in two ways. The first is the method of successive approximations utilising a useful though approximate relation, *viz.*,

$$\left(\frac{w_2}{w_1}\right)^{0\cdot7} = \frac{V_2}{V_1} \quad \text{.......................(11)}$$

(It is to be noted that for British propellants, the index to be used in eq. (11) is somewhat different.) From the given data, we can calculate the appropriate value of 'b', say b_1, as already indicated and hence β for the powder. Using eq. (11), an approximate value of w_2 is calculated. Using this value of w_2, and the calculated value of β, the new values of 'a' and 'b' say a_2 and b_2 are calculated and from eq. (1a) the muzzle velocity to be expected for the charge w_2 is calculated. If this does not agree with the desired reduced muzzle velocity, a new value of w_2 is chosen again using eq. (11). This process is repeated until two charge weights are obtained giving muzzle velocities one greater and the other less than the desired reduced muzzle velocity but both near enough to it. A simple inter-polation then gives the required reduced charge.

The second method consists in first finding out β for the powder from the given data. Then supposing that a loading density $\triangle_2$ will produce the required reduced muzzle velocity, the new values of 'a' and 'b' can be expressed in terms of $\triangle_2$. Substituting in eq. (1a) (with V= the desired reduced muzzle velocity) we get an equation which can be solved for $\triangle_2$ by a graphical method. From this $\triangle_2$, we can calculate the reduced charge necessary to produce the required reduced muzzle velocity.

(5) *Transfer of Powder Problem, Type I.*—If the muzzle velocity produced in a given gun by a given weight of a particular powder is known the problem is to calculate the muzzle velocity to be expected in another gun of known characteristics by a prescribed weight of the same powder. In this case, from the data given for the first gun, the value of β for the powder is calculated and using this value of β, the values of 'a' and 'b' are calculated for the second gun and therefrom the muzzle velocity to be expected by a prescribed charge weight.

(6) *Transfer of Powder Problem, Type II.*—If the muzzle velocity produced in a given gun by a given weight of a particular powder is known, the problem is to calculate the weight of the same powder which can safely be used in another gun of known characteristics, the maximum pressure which the latter can safely stand being given. In this case, first of all β for the powder is calculated from the data given for the first gun. From equations (3) and (4) we have

$$\frac{186 \cdot 53 w^{\frac{7}{6}}}{A S^{\frac{1}{6}} P_{g(max)}} = \beta \left(1 - \frac{\triangle}{\delta}\right) \left(\frac{S}{W}\right)^{\frac{2}{3}}$$

$$= \beta \left(1 - \frac{27.68w}{S\delta}\right) \left(\frac{S}{W}\right)^{\frac{2}{3}}$$

Since for the 2nd gun, all quantities except w are known, the above gives an equation in w. This equation can be solved for w by a graphical method and we thus get the required weight of the powder to be used. By using eq. (5) or eq. (7), the maximum pressure to which any point in the gun will be subjected is then calculated for a few representative points and the safety factor calculated as mentioned in the 'Pressure Problem'. If the calculated pressure anywhere exceeds the safe pressure the charge weight is reduced until the calculated pressures with the new charge weight are safe all along the bore.

(7) *Transfer of Powder Problem, Type III.*—The muzzle velocity produced in a given gun by a given weight of a particular powder being known, the problem is to calculate what weight of the same powder will produce a pre-assigned muzzle velocity in another gun of known characteristics. In this case value of β for the powder is first calculated from the data given for the first gun. Using this value of β, the values of 'a' and 'b' for the second gun are expressed in terms of the loading density for the second gun. In eq. (la) putting V equal to the muzzle velocity required for the second gun, we therefore get an equation which can be solved for $\triangle$ by a graphical method as already mentioned. From this value of $\triangle$, we can calculate the charge weight necessary to produce the required muzzle velocity in the second gun.

(8) *Other Problems.*—The above list is not exhaustive, but illustrates the types of problems that can be tackled by using Le Duc's formulas. Certain other problems like the one of calculating the change in muzzle velocity corresponding to a given change in the weight of the projectile can also be solved by using Le Duc's formulas.

The system of Internal Ballistics initiated by Hirschfelder et al.

This is another American method but unlike Le Duc's method, which is empirical, is based on a theoretical foundation. The details of the method will not be discussed here. The fundamental basis of the method is the same as that of the British method (G. M. II) which has already been discussed. The difference lies in the different sets of parameters used in solving the equations and the different approximations used to take account of the correction factors. For instance, in the G. M. II method, the heat loss is taken account of by increasing the projectile weight by 2% while in the method of Hirschfelder *et al*, the same is taken account of by introducing a pseudo-ratio of specific heats of the product gases, $\bar{\gamma}$, given by the relation $\bar{\gamma}-1=(\gamma-1)\ (1+\beta)$, where γ is the true ratio of the sp. heats and β is equal to the ratio of the total heat loss to the kinetic energy of the projectile.

Comparison of the different methods.

Besides the two American methods already mentioned there is another American method known as Bennett's method. The British methods in use besides the G. M. II method, are the 'RD 38' method and Goldie's method which is almost identical with the G.M. II method but uses different parameters for the solution of the fundamental equations. Of all these methods Le Duc's is almost entirely empirical; all the rest have a more or less theoretical foundation. A comparison of the latter methods brings out the following points. The kinetic energy of the shot is separately taken account of in all of them except in the 'RD 38' method where it is accounted for by using a constant gas temperature smaller than the temperature of explosion. The covolume correction term is included in all except in the 'RD 38' method. The rate of burning of the propellant is assumed to be proportional to the pressure p in all except in the Bennett's method where it is taken proportional to $p^{\frac{2}{3}}$. The allowance for band engraving is made (*i*) in the 'RD 38 method by assuming a reduced propellant size, (*ii*) in the G. M. II and Goldie's methods by assuming a varying shot-start pressure, (*iii*), in Hirschfelder's method by assuming a shot start pressure equal to $(2500/d^{\frac{1}{3}})$ lb./ in.$\frac{1}{3}$, where 'd' is the calibre of the gun in inches and (*iv*) in Bennett's method by taking a shot start pressure equal to 2500 lb./in.^{2} The empirical allowance for heat loss is not included in Goldie's and Bennett's methods and the empirical allowance for friction is included only in the G. M. II and Hirschfelder's methods. By a numerical internal ballistic analysis of a set of firing results in a number of American naval guns with multitude propellant, A.W. Goldie has made a comparison between five different internal ballistic methods—two British ones, the 'RD 38' and the shot start theory (Goldie's) and three American ones, viz., Le Duc's, Hirschfelder's and Bennett's methods. Based on this analysis, the following comments were offered by him. The British shot start theory (Goldie's) method gave good results apart from one large error in pressure and was very accurate for velocity estimation. The 'RD 38' method gave one large error in velocity. It also gave one large error in pressure in the same gun in which Goldie's method gave a large error in pressure, from which it appears that the particular gun possessed rather unusual shot start conditions. Le Duc's methods gave good accuracy in almost all cases. Hirschfelder's method gave very erratic results and the accuracy of estimation of pressures was poor on the whole. Bennett's method gave paradoxical results. It was the most accurate for pressure estimation and the least accurate for velocity estimation. Goldie concluded that only a slight trend towards greater accuracy of estimation was revealed with better theoretical methods and on the whole the results of the test suggested that the use of a single empirical procedure for the comparison of different ballistic systems is unsatisfactory. A better comparison should result if the methods were used in conjunction with the particular empirical procedures employed with them in practice.

It is interesting to note that Le Duc's method gave good results in almost all cases. But in all cases American ammunition was used in American guns and it is doubtful if the method would give equally good results when applied to British guns or British propellants. The advantage of Le Duc's method, however, in its simplicity compared with the other theoretical methods and the quickness with which results can be calculated by this method.

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