

Generalised Model for Aircraft Vulnerability by Different Weapon Systems

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ABSTRACT

The authors' earlier model for the vulnerability of aircraft where aircraft was considered as a combination of cylinder, cones and wedges has been extended to the case when structural data of aircraft as well as its vital parts are given in the form of three-dimensional curvilinear triangles. In the case of VT-fused ammunition, spherical normal distribution has been used to estimate the landing probability of the shell in a cylindrical vicinity region around the aircraft. Kill criteria of vital parts have been redefined.

1 INTRODUCTION

Study of vulnerability of a combat aircraft against a ground-based air defence system is of utmost importance for the design and development of aircraft as well as weapon systems. A number of models have been reported in literature. A simplistic model, where areas of vulnerable parts projected on a given plane are given as inputs, was reported by Ball¹. A dynamic model of aircraft vulnerability, where a typical aircraft is assumed to be moving along an arbitrary profile, was studied². In the study it was assumed that the aircraft comprises various sections modelled as cones, cylinders, wedges, etc. Damage to the aircraft due to explosive charge as well as fragments, when warhead/ammunition explodes in its near vicinity, has been considered. Based on the total energy requirements, kill criterion has been taken as the minimum number of fragments required to penetrate and kill a particular part. In the case of blast waves, it is assumed that the probability of kill is 1, depending upon the impulse transmitted to the structure. A typical aircraft and a typical air defence gun with DA/VT-fused ammunition

have been considered for validation of the model. The above study has now been extended by giving structural data for the aircraft as well as its vital parts in the form of triangular elements. These triangular elements are obtained by dividing the surface of the whole aircraft and its vital parts into small three-dimensional triangles. For this purpose, one has to go to the drawings of the aircraft and obtain the (x,y,z) coordinates of apexes of all the triangles. Kill criterion due to fragment hits has been modified so as to be based on fragment energy concept (Section 5.1). A three-dimensional model for single shot hit probabilities has been presented in Section 5.2. The effect of redundant vulnerable parts on the overall kill probability of the aircraft has also been studied.

2. MATHEMATICAL MODEL

It has been assumed in the model that the aircraft is approaching a friendly vulnerable target (which is also the location of the air defence gun) in a level flight. The direction cosines (DCs) of the aircraft wind and body axes wrt the fixed frame

of reference GXYZ with origin at the gun/missile position (origin G) are given by aircraft flight profile equations. Once the aircraft enters friendly territory, it is first detected by the surveillance radar and is then tracked by the tracking radar in order to get its profile. A ground-based AD gun fires at the future position of the aircraft subject to the condition that the shell and the aircraft reach that point simultaneously. In the present paper, it has been assumed that the aircraft has five vital parts. A part is defined as a vital part if its failure results in immediate failure of the aircraft to continue its flight mission. These parts are avionics, pilot 1, pilot 2, two fuel tanks and two engines. Fuel tanks are of two types; one is the fuselage fuel tank and the other is situated in the two wings. It is assumed that if one of the vital parts is killed, the aircraft is killed.

Effective probability of kill of a typical vital² part (say, j^{th}) of an aircraft by a round is defined as

$$\bar{P}_k^j = P_d, P_h, P_f P_{k/hf}^j$$

where $P_d, P_h, P_f P_{k/hf}^j$ are the probabilities of detection of the aircraft, single shot hit on a vital part, fuse functioning and kill subject to being hit and fuse functioning, respectively. P_h is hit on the part in the case of DA-fused ammunition and landing in the vicinity zone in the case of proximity-fused ammunition. In the present model, fuel tanks are taken as redundant vital parts. If a typical vital part consists of r redundant parts which can function independently, then the vital part may be treated as killed if all of its redundant parts are killed¹. It is known that redundancy of vital parts greatly reduces vulnerability of the aircraft. Kill criterion of an aircraft, for DA and VT-fused ammunition, has been discussed in Sections 4 and 5, respectively. Two fuel tanks located in the two wings are treated as one part, as they are interconnected. Similarly, the two engines are taken as redundant vital parts. Pilot 1 and pilot 2 are non-redundant. The probability of kill of an aircraft as well as its vital parts has been discussed here for two types of weapon systems, viz., AD guns with DA and proximity-fused ammunition.

3. PROBABILITY OF DETECTION

The probability of detection of an aircraft is an important parameter for the assessment of its survivability/vulnerability, and for a typical air defence radar³ is given as

$$\left. \begin{aligned} P_d(X) &= 1.0 && \text{for } 0 \leq X < 0.15 \\ P_d(X) &= 31.470 X^3 - 33.7136 X^2 \\ &\quad + 8.57498 X + 0.33782 && \text{for } 0.15 \leq X \leq 0.42 \\ P_d(X) &= -8.2691 X^3 + 18.5793 X^2 \\ &\quad - 13.9667 X + 3.51733 && \text{for } 0.42 < X \leq 0.75 \\ P_d(X) &= 0 && \text{for } X > 0.75 \end{aligned} \right\} \quad (2)$$

where $X=(R/R_0)$, R being the distance of aircraft from the radar and for the typical radar, $R_0 = 65.5$ km. R_0 involves height of the target and other radar specifications.

4. VULNERABILITY OF AIRCRAFT DUE TO DA-FUSED AMMUNITION

To evaluate single shot hit probability P_h on a typical vital part of the aircraft, its projection is obtained on a plane normal to the line of shot (N-plane) by simple geometric transformations (Fig. 1). Probability of hit on vital part is the same as that on its projection on N-plane² is given as

$$P_h = \frac{1}{2\pi\sigma_s\sigma_t} \iint_{F_1} \exp \left\{ -\frac{1}{2} \left(\frac{s^2}{\sigma_s^2} + \frac{t^2}{\sigma_t^2} \right) \right\} ds dt \quad (3)$$

where F_1 is the projected region of the component triangles over N-plane; s, t is the two-dimensional coordinate system on N-plane and σ_s, σ_t are standard deviations of impact points from the origin along S and T axes due to in-built system errors. In this paper, it has been assumed that $\sigma_s = \sigma_t$.

To determine the damage caused by the AD guns to a given aircraft, the following methodology has been adopted in the model:

- Considering the actual terminal velocity, mass and calibre of the shell, penetration in the vital as well as the non-vital parts has been

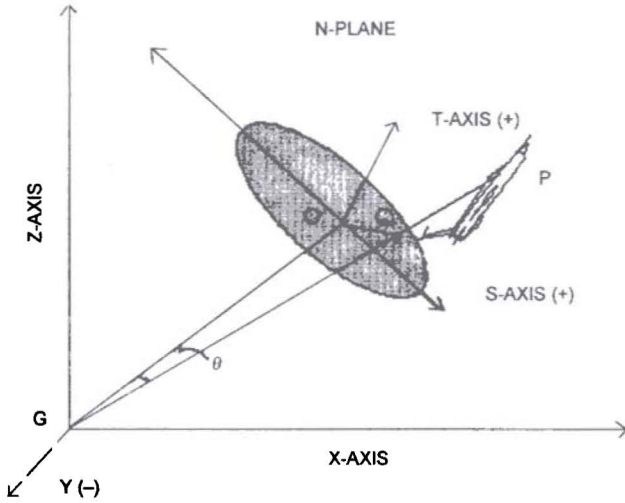


Figure 1. Projection of a typical point P of the aircraft, as a point Q on N-plane.

calculated using Eqn (4). Equivalent thickness of various parts of the aircraft are given in Table 1 (see Moss⁴, *et al.*), whereas probability of kill of the aircraft, for different levels of penetration in different vital parts has been given in Table 2. Table 2 gives data for two different cases, i.e. external and internal bursts. When the shell after impact does not penetrate the outer skin of a vital part [i.e. $V_r \leq 0$ in Eqn (4)], and explodes, it is taken as an external burst, otherwise it is an internal burst. These probabilities of kill are for K -type of kill by a specific gun on a typical target only. For other types of kill, data has to be evaluated by experiments.

- (b) For a non-exploding projectile, the probability of kill is given by Eqn (7), where critical and

uncritical energies for the kill of various vulnerable parts are given in Table 1. The remaining energy (E_r) is the energy left over after penetrating the thickness (Table 2) of the vital part. While analysing penetration, overlapping of one triangular element by the other is taken into account (Appendix 1).

- (c) Thus, knowing the probabilities of kill of different vital parts, the probability of kill of the aircraft is calculated using Eqn (15).

The remaining velocity (V_r) after penetrating⁵ the vulnerable part is given by

$$V_r = (V - \pi \rho D_0 R^2 V \sin \alpha / m) \cos \theta \quad (4)$$

where V is the normal striking velocity; V_r , the remaining velocity; ρ , the density of the target; α , the nose cone angle; θ , the striking angle of the projectile; D_0 , the thickness of the target; R , the radius of the projectile; and m , the mass of the projectile.

In the present model, it has been assumed that if the shell hits at any of the vital or non-vital (rest of the aircraft body) parts, its probability of kill depends on whether the shell penetrates the aircraft body or not. The probabilities of kill of the aircraft's components (P_{kh}^j), are computed for the K -kill of the aircraft, when a small calibre DA-fused high explosive projectile (23 mm) hits it. In the case of a higher calibre projectile, these probabilities of kill are multiplied by the factor:

$$E_f = 2.0 [1 - 0.5 \exp(-(Q - Q_1)/Q_1)] \quad (5)$$

Table 1. Equivalent thickness of various vital and non-vital parts

Component	Critical energy required to kill vital part (uncritical energy is given in brackets) (J)	Equivalent thickness of dural (mm)
Avionics	339 (81.4)	27
Pilot section	678 (81.4)	5
Engines	1356 (135.6)	20
Fuel tanks	339 (81.4)	20
Remaining parts	1350 (135.0)	20

Table 2. Variation of F_{kh}^j for different levels of bursts

Component	Probability of K -kill (F_{kh}^j)	
	Internal burst (%)	External burst (%)
Avionics	50	10
Pilot sec	50	10
Engines	30	25
Fuel tanks	75	30
Remaining parts	25	05

where Q_1 is the total energy (explosive energy + kinetic energy) released by a typical 23 mm shell with 20 g HE, and Q is the total energy of the given shell. This has been done to take into account the effect of higher explosive energies of different shells on the kill of the aircraft. It is well known that damage to a target is an exponential function of the explosive energy. Thus the probability of kill of the j^{th} component ($P_{k/h}^j$) is given as

$$P_{k/h}^j = P_{k/hf}^j \cdot F_{k/h}^j E_f \quad (6)$$

where $P_{k/h}^j$ is the probability of kill given by Eqn (7), and factor $F_{k/h}^j$ is given in Table 2. It is to be noted that factor E_f is multiplied with the total kill probability, only in the case of DA-fused ammunition. In Eqn (6), j is not the dummy index, but indicates the j^{th} vital part.

5. VULNERABILITY OF AIRCRAFT DUE TO PROXIMITY-FUSED AMMUNITION

In this section, the case of vulnerability of the aircraft, when attacked by a shell fitted with proximity-fused ammunition is discussed. A vital component is treated as killed if the remaining energy of the fragment after penetrating the vital part is more than the critical energy (Table 2) required to kill it.

5.1 Energy Criterion for Kill

In case of exploding ammunition, viz., shells and non-exploding projectiles, viz., fragments, the probability of kill of a vital part depends on E_r of the fragment after penetration and is given as

$$P_{k/hf} = \begin{cases} 1 & E_r > E_c \\ \frac{E_r - E_u}{E_c - E_u} & E_u < E_r < E_c \\ 0 & E_r > E_u \end{cases} \quad (7)$$

where E_r is the kinetic energy of a fragment after penetrating the outer structure; E_u is the uncritical energy so that if the energy imparted to the aircraft component is $< E_u$, no damage is caused; E_c is the critical energy required to kill the component, so

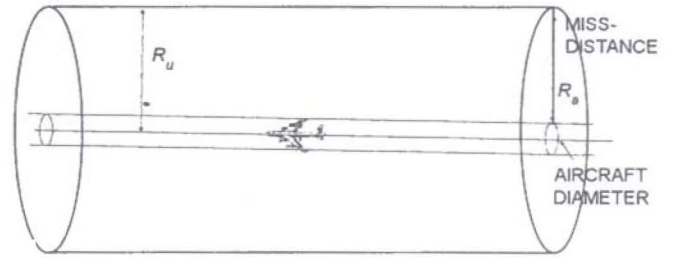


Figure 2. Vicinity region of the aircraft

that if the energy imparted to the aircraft is $> E_c$, complete damage is caused.

5.2 Probability of Landing in the Case of VT-Fused Shell

A proximity-fused ammunition shell can land anywhere in the vicinity of the aircraft. Vicinity is defined as the volume around the aircraft body in which if the shell lands, the fuse will function with probabilities (P_f); elsewhere it will not function² ($P_f = 0$). For the missile warhead, the probability of fuse functioning is assumed to be¹. Let R_a and l_o be the radius and half length of fuselage of the aircraft, respectively and m_s , the miss-distance from the surface of the aircraft. Distance m_s is the distance from the aircraft surface within which the fuse will function. A coaxial cylinder surrounding the aircraft is drawn, so that the axis of the cylinder is u -axis, with centre O , length $2(l_o + m_s)$ and radius R_u , such that $R_u = R_a + m_s$. Volume of the cylinder is nothing but the vicinity region. Vicinity layers for simplicity are assumed to be coaxial cylinders with centre at the centre of the aircraft.

Probability of landing and fuse functioning of the projectile in terms of fixed coordinate system around the aircraft is:

$$P_{Lf} = \frac{1}{(\sqrt{2\pi}\sigma)^3} P_f (r - R_a) \exp \left\{ -(\bar{x}^2 + \bar{y}^2 + \bar{z}^2) / 2\sigma^2 \right\} d\bar{x}d\bar{y}d\bar{z} \quad (8)$$

where

$$\begin{aligned} \bar{x} &= x - x_0 \\ \bar{y} &= y - y_0 \\ \bar{z} &= z - z_0 \end{aligned}$$

and $P_f(r-R_a)$ is the probability of the fuse functioning. R_a is the value of r at the surface of the aircraft, r being the radial distance from the u -axis. Equation (8) is a simple extension of two-dimensional Gaussian distribution to a three-dimensional case. By converting $(\bar{x}, \bar{y}, \bar{z})$ coordinates to moving coordinates (u, v, w) with the help of linear transformations, one gets:

$$P_{Lf} = \frac{1}{(\sqrt{2\pi}\sigma)^3} P_f(r-R_a) \cdot \exp \left\{ -\frac{(u^2 + v^2 + w^2)}{2\sigma^2} \right\} J_1 \left(\frac{\bar{x}, \bar{y}, \bar{z}}{u, v, w} \right) du dv dw$$

where

$$J_1 \left(\frac{\bar{x}, \bar{y}, \bar{z}}{u, v, w} \right) = \begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix}$$

is the Jacobian used for transformation of coordinates from one set of coordinate systems to another set of coordinate systems. Transforming the above equation to cylindrical coordinates (r, θ, ζ) , one gets:

$$P_{Lf} = \frac{1}{(\sqrt{2\pi}\sigma^2)^3} P_f(r-R_a) \exp \left\{ -1/2 \frac{r^2 + \zeta^2}{\sigma^2} \right\} J_1 \cdot r d\zeta \cdot dr \cdot d\theta \quad (9)$$

The probability of kill of the j^{th} vital part due to the shell landing at a typical point $P(r, \theta, \zeta)$ of the vicinity region is:

$$dP_k^j = P_{Lf} P_{k/h}^j \quad (10)$$

where $P_{k/h}^j$ is the probability of kill of the j^{th} part subject to a hit on it. Since the shell can land anywhere in the vicinity region, the cumulative kill probability $P_{k/i}^j$ of the j^{th} part, due to the i^{th} (say) shell landing anywhere in the vicinity of the aircraft is obtained by integrating Eqn (10) over the whole vicinity region, i.e.:

$$P_k^j = \left(\int_{R_a}^{R_L} + \int_{R_a}^{R_V} \right) \int_0^{2\pi} \int_{-(l_o+m_s)}^{(l_o+m_s)} P_{Lf} \cdot P_{k/h}^j \cdot r dr \cdot d\theta \cdot d\zeta$$

The expression for $P_{k/h}^j$ has been explained in Section 4 [Eqn (6)]. Evaluation of $P_{k/h}^j$ depends on the kill criteria. In integral (Eqn 11), limit R_L is a typical distance from the aircraft, such that if the shell explodes between R_V and R_L , damage is due to the blast as well as the fragments; otherwise, it is only due to the fragments.

It has been assumed here that the kinetic energy of a projectile hitting the j^{th} vital part from a particular elemental point in the vicinity region² is constant. While calculating the impact velocity of the fragment, attenuation of fragment velocity due to drag has been taken into account⁶.

6. PENETRATION LAWS

If the component is inside the structure, the fragments before hitting it penetrate the aircraft structure. During penetration in the outer skin, some of the energy is lost. The remaining energy, after penetration, is responsible for the damage to the component. V_r of the fragment is governed by the laws of terminal ballistics⁷ and is given by the empirical relation:

$$V_r = V_s \quad K t^{a_1} m^{a_2} (\sec \theta)^{a_3} V_s^{a_4} \quad (2)$$

where values of a_i for duraluminium are given in Table 3. In Eqn (12), m_s is the striking mass in grains; V_s , striking velocity in ft/s; θ , the impact angle; t , thickness in inches; and K is a constant, depending on the shape of the projectile.

Table 3. Coefficients of Thor equations of penetration for duraluminium

Coefficients	Duraluminium
a_1	1.029
a_2	-0.386
a_3	1.251
a_4	-0.139
K	9.0843×10^{-4}

7. CUMULATIVE KILL PROBABILITY

As the aircraft is considered to have been divided into y parts, let $P_{k/i}^j$ be the single shot kill probability of the j^{th} vital part due to the i^{th} fire, each fire

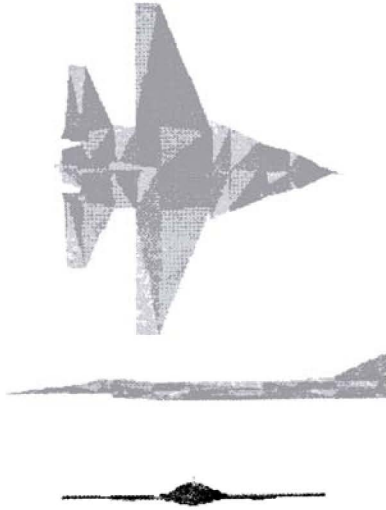


Figure 3. Graphic output of top and side view of the aircraft

having n rounds. The cumulative kill probability $\bar{P}_k^j$ of the j^{th} vital part in N number of fires can be given as

$$\bar{P}_k^j = 1 - \prod_{i=1}^N [1 - P_{k/i}^j] \quad (13)$$

Further, the aircraft can be treated as killed if at least one of its vital parts, is killed. Thus, in this case, the cumulative kill probability for the aircraft as a whole can be given as

$$\bar{P}_k = 1 - \prod_{j=1}^y [1 - \bar{P}_k^j] \quad (14)$$

In case the j^{th} part of the aircraft has y_i redundant parts, $\bar{P}_k$ is given by:

$$\bar{P}_k = 1 - \prod_{j=1}^y \left[1 - \prod_{r=1}^{y_i} [\bar{P}_{k/r}^j] \right] \quad (15)$$

where $P_{k/r}^j$ denote the kill probability of the r^{th} redundant part of the j^{th} vital part.

8. DATA USED

8.1 Target Aircraft

Structural data of the aircraft as well as its vital parts have been discussed earlier in terms of triangular elements.

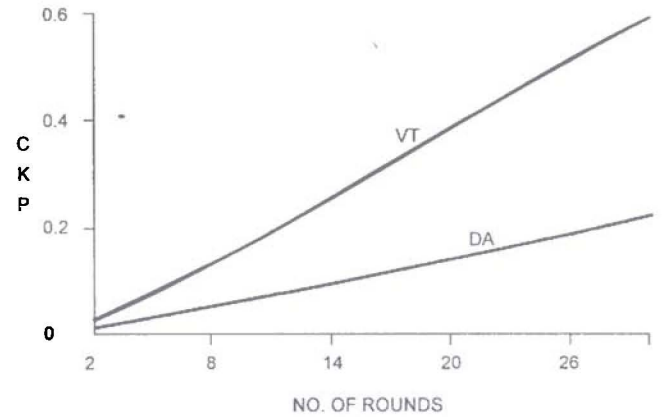


Figure 4. Variation of CKP vs number of rounds (engagement range 2000 m).

An air defence twin barrel gun with DA/VT-fused ammunition is considered for this study, the data for which is:

Parameter	Value
System error	3 mrad
Number of barrels	2
Firing rate	5 rounds/gun barrel
Probability of DA-fuse functioning	0.99
Probability of proximity fuse functioning	Fig 3 (Ref. 2)
Time of continuous firing of gun	3 s
Maximum range of gun	5000 m
Minimum range of gun	500 m
Maximum detecting range	10000 m

9. RESULTS & DISCUSSION

Data on variation of probability of kill of the aircraft and its various vital parts vs number of rounds for DA-fused ammunition are given in Table 4. Various parts of the aircraft have been modelled as a collection of three-dimensional triangular elements (Fig. 1). Although the kill criteria and vital parts of the aircraft considered in the earlier model² are not exactly the same, yet the cumulative kill probability of the aircraft vs number of rounds for the range 2000 m has been shown in Tables 4 and 5 in order to have a feel of comparison.

It has been assumed that the aircraft is coming from a large distance along an arbitrary path and is being tracked by the tracking radar. The aircraft has been considered coming across the gun position at an altitude of 100 m and at speed 240 m/s. The twin barrel gun located at the origin of fixed coordinate axes starts engaging the target aircraft from range R for a period of 3s.

Variation of probability of kill of the aircraft vs number of rounds (Fig. 4) and engagement range (Fig. 5) for DA and proximity-fused ammunitions has been evaluated. It is observed (Fig. 4) that the

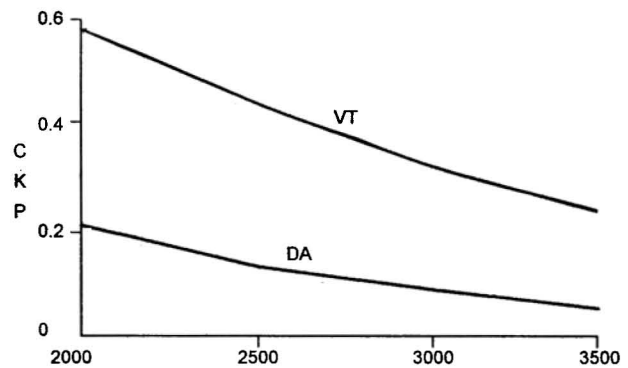


Figure 5. Variation of CKP vs range (30 rounds)

Table 4. Variation of probability of kill of the aircraft and its five vital parts due to DA-fused ammunition (engagement range = 2000 m)

Rounds	Aircraft	Avionics	Pilot 1	Pilot 2	Fuselage fuel tank	Wing fuel tanks	Engine 1	Engine 2	Others
6	0.0270	0.0008	0.0129	0.0020	0.0015	0.0196	0.0008	0.0008	0.0115
	0.0543	0.0016	0.0261	0.0040	0.0032	0.0396	0.0017	0.0017	0.0234
	0.0820	0.0024	0.0397	0.0061	0.0049	0.0600	0.0027	0.0027	0.0355
	0.1099	0.0032	0.0536	0.0083	0.0068	0.0807	0.0036	0.0036	0.0480
	0.1381	0.0041	0.0679	0.0106	0.0087	0.1018	0.0047	0.0047	0.0607
14	0.1665	0.0051	0.0825	0.0130	0.0108	0.1233	0.0058	0.0058	0.0737
	0.1952	0.0061	0.0974	0.0154	0.0130	0.1451	0.0070	0.0070	0.0871
	0.2242	0.0071	0.1126	0.0180	0.0153	0.1673	0.0083	0.0083	0.1009
18	0.2535	0.0082	0.1283	0.0207	0.0179	0.1898	0.0096	0.0096	0.1152
20	0.2829	0.0094	0.1444	0.0235	0.0205	0.2127	0.0111	0.0111	0.1297
22	0.3127	0.0106	0.1610	0.0264	0.0234	0.2360	0.0126	0.0126	0.1447
	0.3426	0.0119	0.1781	0.0295	0.0265	0.2597	0.0142	0.0142	0.1601
26	0.3726	0.0132	0.1955	0.0327	0.0298	0.2838	0.0160	0.0160	0.1758
28	0.4027	0.0145	0.2133	0.0361	0.0334	0.3081	0.0179	0.0179	0.1921
30	0.4329	0.0162	0.2317	0.0396	0.0373	0.3328	0.0200	0.0200	0.2087

Table 5. Variation of probability of kill of the aircraft and its five vital parts due to VT-fused ammunition (engagement range = 2000 m)

Rounds	Aircraft	Avionics	Pilot	Pilot 2	Fuselage fuel tank	Wing fuel tanks	Engine	Engine 2	Others
2	0.0352	0.0122	0.0049	0.0047	0.0526	0.0585	0.0302	0.0314	0.0883
4	0.0727	0.0243	0.0100	0.0095	0.1035	0.1148	0.0602	0.0625	0.1702
6	0.1119	0.0363	0.0151	0.0143	0.1527	0.1688	0.0901	0.0935	0.2461
8	0.1520	0.0482	0.0203	0.0193	0.2003	0.2206	0.1197	0.1242	0.3163
10	0.1926	0.0601	0.0256	0.0244	0.2462	0.2703	0.1492	0.1547	0.3811
12	0.2332	0.0718	0.0311	0.0296	0.2904	0.3178	0.1784	0.1849	0.4407
14	0.2735	0.0835	0.0367	0.0349	0.3329	0.3632	0.2073	0.2148	0.4955
16	0.3131	0.0950	0.0423	0.0403	0.3738	0.4064	0.2360	0.2444	0.5459
18	0.3518	0.1065	0.0481	0.0459	0.4130	0.4476	0.2644	0.2736	0.5919
20	0.3894	0.1178	0.0540	0.0516	0.4505	0.4867	0.2924	0.3024	0.6340
22	0.4257	0.1291	0.0601	0.0574	0.4864	0.5238	0.3201	0.3309	0.6724
24	0.4607	0.1402	0.0662	0.0633	0.5207	0.5590	0.3474	0.3589	0.7073
26	0.4942	0.1512	0.0725	0.0694	0.5534	0.5923	0.3743	0.3864	0.7390
28	0.5261	0.1620	0.0789	0.0756	0.5845	0.6236	0.4007	0.4135	0.7678
30	0.5564	0.1728	0.0855	0.0819	0.6140	0.6532	0.4268	0.4401	0.7937

cumulative probability of kill increases as the number of rounds fired increases in all cases, but it decreases with the engagement range (Fig. 5). It is observed that in the present study, the cumulative kill probability due to DA charge is higher compared to that in the earlier study². It is due to the higher dimensions of the vital parts, specially for pilot¹. Even if the shell hits the outer structure of a part, it is being treated as killed. The second reason is increase in the number of vital parts. But in the case of proximity-fused ammunition, the vital part can be treated as killed only when the fragment penetrates the outer structure and hits the component at an internal location with the required energy to kill the vital part. This is the reason why the cumulative kill probability of the aircraft, in the case of proximity-fused ammunition is lower compared to the earlier model². Another reason is redundancy of some of the vital parts.

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Projection of Triangular Faces over N-Plane and their Overlapping by Each Other

The projection of a triangular face over a plane normal to the line joining the centre of aircraft and the gun position (N-plane, in Ref. 2 it is called D-plane) will be a triangle or a straight line. Computations of the vertices of the projected triangles on N-plane have been discussed in details².

For the determination of hit probability of a triangle or solid angle subtended, it is important to know, whether a particular triangular element is on the side of aircraft facing the source point of the projectile, or is on the other sides. This can be decided by considering the angle between the line joining the source point to the geometric centre of the triangular element and the normal to the triangular element at its geometric centre, as given below.

In the following paragraphs source point of projectile means the gun point while finding hit probabilities. But while estimation of solid angle, the source point means the point where shell explodes, i.e. source point of fragments.

Let (u_i, v_i, w_i) for $i=1,3$, (u_g, v_g, w_g) and (u_c, v_c, w_c) , be the coordinates of the vertices of a triangular face, the source point G of the projectile, and geometric centre C of the triangular face, respectively, wrt 2nd frame of reference (moving frame with origin as the centre of the aircraft². Then

$$u_c = \frac{1}{3} \sum u_i$$

$$v_c = \frac{1}{3} \sum v_i$$

$$w_c = \frac{1}{3} \sum w_i$$

The direction cosines (DCs) of the line joining the points G and C are:

$$l = (u_c - u_g) / D_s$$

$$m = (v_c - v_g) / D_s$$

$$n = (w_c - w_g) / D_s$$

$$D_s = \sqrt{\sum (u_c - u_g)^2}$$

DCs (a, b, c) of the normal to a triangular surface, the coordinates of whose apexes are (u_i, v_i, w_i) $i = 1, 2, 3$ can be obtained as

$$a = \frac{A}{\sqrt{A^2 + B^2 + C^2}}$$

$$b = \frac{B}{\sqrt{A^2 + B^2 + C^2}}$$

$$c = \frac{C}{\sqrt{A^2 + B^2 + C^2}}$$

where

$$A = \begin{vmatrix} v_1 & w_1 & 1 \\ v_2 & w_2 & 1 \\ v_3 & w_3 & 1 \end{vmatrix}$$

$$B = \begin{vmatrix} u_1 & w_1 & 1 \\ u_2 & w_2 & 1 \\ u_3 & w_3 & 1 \end{vmatrix}$$

$$C = \begin{vmatrix} u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \\ u_3 & v_3 & 1 \end{vmatrix}$$

Let α is the angle between line $\vec{GC}$ and normal to the triangular face then

$$\alpha = \cos^{-1}(al + bm + cn)$$

$\alpha \leq 90^\circ \Rightarrow$ Triangle is facing opposite to the source point G

$\alpha > 90^\circ \Rightarrow$ Triangle is facing towards the source point G and can get impact provided it is not being overlapped by some other triangular face of the aircraft.

To know that the i^{th} triangle is overlapped wholly or partially by another j^{th} triangle, the following method is to be adopted:

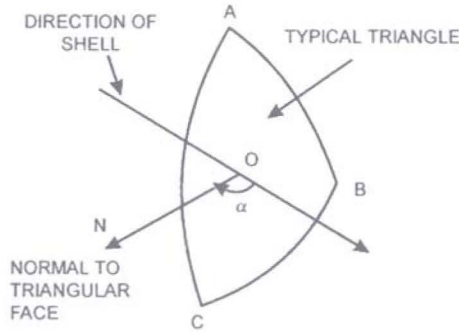


Figure 6. Triangle facing the path of fragment/projectile

Let (s_{ik}, t_{ik}) , (s_{jk}, t_{jk}) , $k = 1, 3$, are respectively the coordinates of the corners of projection of i^{th} and j^{th} triangle of the aircraft over N-plane wrt point G. These triangles are further subdivided into smaller rectangular meshes in order to assess the overlapped area.

Let (s_o, t_o) be the central point of a typical rectangle of the i^{th} triangle. If this point falls on in the j^{th} triangle formed by the vertices (s_{jk}, t_{jk}) , $k = 1, 3$ then it implies that this rectangle is overlapped by the j^{th} triangular face. If it is not covered by j^{th} triangle, other triangles are tested. Similar test is applied to all the rectangular elements of the i^{th} triangle. Thus if all the rectangular elements are not covered by any other triangle, it implies that this triangle is not being overlapped by any of the triangles and can be considered to find solid angle or hit probability.

Let a rectangle with centre $c_p(s_o, t_o)$ is overlapped by j^{th} triangle. In that case it has to be checked whether the j^{th} triangular face is near to the source point of the projectile or the i^{th} triangular face is nearer. Whichever triangle is nearer, it will be overlapping other. It can be done in the following steps:

- (a) Let a 3rd coordinate system, OST, be defined with origin at O and S-T plane being normal to line joining projectile and centre of the aircraft (N-plane). Direction cosines of OS and OT axis wrt 1st frame of reference is a fixed coordinate frame with origin fixed on earth surface and x-axis along the flight path of the aircraft) are given by

$$\left(\frac{m_o}{\sqrt{1-n_o^2}}, \frac{-l_o}{\sqrt{1-n_o^2}}, 0 \right) \left(\frac{-n_o l_o}{\sqrt{1-n_o^2}}, \frac{m_o n_o}{\sqrt{1-n_o^2}}, \sqrt{1-n_o^2} \right)$$

where l_o , m_o , n_o are DCs of line GO.

- (b) Let DCs of the line, joining points $c_p(s_o, t_o)$ and source point of the projectile G wrt 3rd frame are (l'_3, m'_3, n'_3) .

- (c) Let (l'_1, m'_1, n'_1) be the DCs of $\vec{GC}_p$ with reference to 1st frame. Thus

$$\begin{aligned} l'_1 &= l'_3 l_s + m'_3 l_t + n'_3 l_r \\ m'_1 &= l'_3 m_s + m'_3 m_t + n'_3 m_r \\ n'_1 &= l'_3 n_s + m'_3 n_t + n'_3 n_r \end{aligned}$$

where (l_s, m_s, n_s) ; (l_t, m_t, n_t) and (l_r, m_r, n_r) are the DCs of the axes of 3rd frame wrt 1st frame and the DCs of the line GC_p wrt 2nd frame say (l'_2, m'_2, n'_2) .

$$\begin{aligned} l'_2 &= l'_1 l_1 + m'_1 m_1 + n'_1 n_1 \\ m'_2 &= l'_1 l_2 + m'_1 m_2 + n'_1 n_2 \\ n'_2 &= l'_1 l_3 + m'_1 m_3 + n'_1 n_3 \end{aligned}$$

- (d) Finally find the equation of the line wrt 2nd frame as the line passes through some point whose coordinate wrt 2nd frame are known.

- (e) Let the line meet the i^{th} triangular face at point O_i with coordinates $O_i(uo_i, vo_i, wo_i)$

$$\begin{aligned} uo_i &= u_g + l'_2 R_o \\ vo_i &= v_g + m'_2 R_o \\ wo_i &= w_g + n'_2 R_o \end{aligned}$$

$$R_o = - \frac{(au_g + bv_g + cw_g + d_i)}{(al'_2 + bm'_2 + cn'_2)}$$

$$d_i = au - bv - cw$$

- (f) Find the intersection of the line with j^{th} triangular face $uj_i(uo_i, vo_i, wo_i)$ as explained above.
- (g) Find the distances of the line GO_i and GO_j if $GO_i > GO_j$ implies that this rectangle is being overlapped by j^{th} triangular face and need not

be considered to find hit probability or solid angle, $GO_i < GO_j$ means that the rectangle is not being overlapped by j^{th} triangular face.

(h) Same methodology can be used to check overlapping by other triangular faces, i.e. for all j 's.

(i) The same method is to be repeated for all the rectangles of the i^{th} triangle on N-plane.