Defence Science Journal, Vol. 63, No. 1, January 2013, pp. 101-107, DOI: 10.14429/dsj.63.2641

Effects of Different Meteorological Standards on Projectile Path

R. J. Mukhedkar* and S. D. Naik
Armament Research & Development Establishment, Pune–411 021, India
**E-mail: raj_mukh1960@yahoo.com*

ABSTRACT

Projectile path is dictated by estimated line of fire. Line of fire can be estimated by referring RT as well as trajectory model on computer. RT is prepared under ISA/ICAO meteorological standard conditions without wind effect. Meteorological conditions like density, humidity, pressure, wind, temperature affect the path of the trajectory. Meteorological data plays very important role in trajectory prediction. Trajectory is predicted using RT for a particular weapon where ICAO standard met data is used. Ind Std met data is different from ICAO data. Use of Ind Std data improves the accuracy of trajectory prediction for Indian field deployment. In this paper, comparative study of effect on projectile path under ICAO, Ind Std and actual Indian prevailing met have been carried out and analyzed. From this analysis, a new model has been established that if actual prevailing met is not available then trajectory prediction can be carried out using Ind Std met data with wind data as per date and time. It predicts trajectory very close to actual. Effect of wind has been studied and found that wind effect is very dominant on projectile path. Study of effect of density also has been carried out in this paper. It is observed that Ind Std density values are much close to actual as compared to ICAO. Therefore, if insufficient met is recorded then Ind std density can be used to fill density values for that region.

Keywords: ISA/ICAO standard met, prevailing met, trajectory model, Bernoulli's force, temperature lapse rate, wind effect, air density

Abbreviations

Ind Std	- Indian Standard meteorological data
ISA	- International Standard Atmospheric
ICAO	- International Civil Aviation Organization
met	- Meteorological data
RT	- Range Table
2D	- Two directional
3D	- Three directional
6DOF	- Six degree of freedom
GPS	- Global Positioning System

Notations

S	cross section area of projectile,
v	velocity of projectile,
$\overline{v}$	velocity vector,
$\overline{w}$	wind vector,
ρ	air density at altitude,
ρ_o	air density at sea level,
M	Mach number,
Re	Reynolds number,
$\overline{F}$	Force,
$\overline{H}$	angular velocity,
$\overline{M}$	moment,
F_A	aerodynamic force,
T	temperature in K at altitude,
T_o	temperature at sea level in K,
V	volume,
n	grams per mole,
R	universal gas constant,
Z	altitude,
λ	lapse rate,
p	pressure at altitude,
p_o	pressure at sea level,
g	acceleration due to gravity,
a	sonic velocity,
γ	specific heat ratio of air

1. INTRODUCTION

Meteorological conditions are very complex and different but important phenomenon for researchers who are working in different areas. A number of researchers are working to study effect of atmosphere or meteorological conditions on various entities related to biological, environmental, medical, physical, mechanical, chemical applications.

1.1 Literature Survey

Virk[1], *et al.* discussed about the significant effect of meteorological conditions viz. rain, geometric attenuation and snow on optical wireless communication and free space optical (FSO) link. Du[2], *et al.* have analyzed atmospheric adjustments in the tropical Indian Ocean warming during the 20th century. Bartolucci[3], *et al.* have studied and analyzed atmospheric effect on landsat thermal IR data. Ma[4], *et al.* discussed the influence of meteorological conditions on the occurrence of bacillary dysentery in Lanzhou, China. Shekhar[5] has analyzed effect of temperature on mechanical properties of solid rocket propellants. Chsilp[6,7], *et al.* have developed software with 6DOF trajectory model for compiling of firing table for

Received 4 September 2012, revised 24 December 2012, online published 24 January 2013

artillery projectile. They have not considered the effect of meteorological conditions on projectile performance which is the study carried out in this work.

1.2 Meteorological Data

It consists of information about pressure, temperature, humidity, density, wind direction and wind speed within the medium against altitude. Those meteorological conditions vary with the location and are affecting trajectory of the projectile. Therefore, meteorological conditions are standardized as ISA/ ICAO, Ind Std, Russian Std etc.

1.3 Weapon Dynamics

It is a branch of dynamics related to performance evaluation of weapons. The performance mainly depends on the design parameters, properties of surrounding medium and the launch conditions. The design parameters are mass, shape, size, nose, tail of projectile.

1.4 Range Table Compilation

Range Table for the weapon is compiled using standard met conditions and weapon's design parameters. Normally RTs are compiled using ISA standard met. Ind Std met conditions are different than ISA.

In this paper, effect of met standards as ISA, Ind Std and recorded met on projectile path have been studied and results have been analyzed. Effect of ISA and Ind Std with wind and without wind also has been simulated and analyzed. Wind effect on projectile path has been studied. It is observed that trajectory prediction under Ind Std with recorded wind is close to recorded met. Density effect on projectile has been analyzed and found that density under Ind Std is very close to recorded as compared to ISA.

Accuracy in range and drift has been found better on Indian field if the data for Ind Std met with recorded wind data is used to predict launch parameters.

2. PROJECTILE TRAJECTORY MODELING

The trajectory of a weapon can be estimated with the help of point mass model, modified point mass model or six degrees of freedom model[6,8]. These models give the equations of motion for the weapon. These equations can be developed to define 2D motion or 3D motion. The simplest model given by point mass model defines trajectory in a plane which generally is used to estimate range and maximum altitude. The third direction giving drift is obtained using modified point mass model[6,7,11]. The complexity increases by considering additional forces or moments like those due to rotation of Earth, spinning of projectile and wind. Trajectory is affected by density of the medium and wind in atmosphere[8]. The 6 DOF model is the most general one which can give all aspects of study including stability during flight. The equations being very sensitive to the launch data are rarely used and thus the model is selected depending on the aspect of study. The equations of motion are given by Newton's second law to predict projectile path[8].

$$\frac{d(m(\overline{v}-\overline{w}))}{dt}=\overline{F}, \quad \frac{d\overline{H}}{dt}=\overline{M} \tag{1}$$

$\overline{F}$ consists of gravity force, aerodynamic forces, like drag, lift, Magnus force, forces due to rotation of earth like centrifugal force and $\overline{M}$ defines the associated moments. The Eqn. (1) are resolved in particular directions to obtain corresponding scalar equations. The reference frames are initially defined which give the scalar equations in one of the frames. Aerodynamic forces are expressed in terms of Bernoulli's force in Eqn. (2) as

$$F_A=\frac{1}{2}\rho v^2\, SC(M,\mathrm{Re}) \tag{2}$$

where $\frac{1}{2}\rho v^2$ defines dynamic pressure and C as corresponding Aerodynamic coefficient. The aerodynamic force coefficients are determined using wind tunnel with different flow conditions.

3. ATMOSPHERIC PROPERTIES

The atmospheric properties are related by *p, T* and *V* by Eqn. (3) standard gas equation[8].

$$pV = nRT \tag{3}$$

Temperature varies with altitude (Z), it is obtained in terms of lapse rate (λ) as:

$$T = T_o - \lambda Z \tag{4}$$

Accordingly pressure and density are changed in terms of sea level values and lapse rate as in Eqn. (8).

$$p = p_o(T/T_o)^{-g/R\lambda}, \quad \rho = \frac{p}{RT}, \quad a = \sqrt{\gamma RT} \tag{5}$$

3.1 ISA/ICAO Standard Atmosphere

The sea level meteorological parameters under ISA[9] are defined as follows:

$$\begin{aligned} p_o &= 101325\ N/m^2 \\ \rho_o &= 1.225\ kg/m^3 \\ T_o &= 15^o C \text{ or } 288.15\ K \\ g &= 9.80665\ m/s^2 \\ a &= 340.29\ m/s \\ \gamma &= 1.4 \end{aligned} \tag{6}$$

Temperature lapse rate under ISA standard atmosphere with altitude from 0 to 90 km and above is defined in Table 1.

Table 1. ISA temperature lapse rate

Altitude in km	Lapse rate in K/km
0 to 11	-6.5
11 to 25	0
25 to 47	+3.0
47 to 53	0
53 to 79	-4.5
79 to 90	0
Above 90	4

3.2 Indian Standard Atmosphere

The sea level meteorological parameters under Ind std[10] are defined as follows:

$$\begin{aligned} p_o &= 101000\ N/m^2 \\ \rho_o &= 1.1723\ kg/m^3 \\ T_o &= 27^o C\ or\ 300.15\ \ K \\ g &= 9.78852\ m/s^2 \\ a &= 340.29\ m/s \\ \gamma &= 1.4 \end{aligned} \tag{7}$$

Temperature lapse rate under Indian standard atmosphere from 0 to 79 km altitude is defined in Table 2.

Table 2. ISA temperature lapse rate

Altitude in km	Lapse rate in K/km
0 to 6	-6.0
6 to 16	-6.5
16 to 46	+2.3
46 to 51	0
51 to 74	-3.0
74 to 79	-0.6

3.3 Prevailing/Actual Met Data

Prevailing met data is obtained by launching a balloon. There is a device attached with the balloon which has pressure, temperature, humidity sensors, GPS and trans-receiver. Sensor data (pressure, temperature, humidity) and location of balloon are transmitted to ground station through trans-receiver periodically up to the altitude where balloon communication is aborted from the ground station or balloon is burst due to pressure difference. Ground station derives wind direction and speed from received balloon location data. The derived met data is the actual experiencing recorded met data. This recorded met is used for trajectory prediction. The trajectory computed with this data is considered as actual or experimental trajectory for comparison with different met conditions.

4. EFFECT OF WIND

Wind direction and speed are used to resolve wind in range and line components[12]. This resolved wind data is used in Eqn. (1) to predict projectile path. Range wind affects the range of the projectile and line wind drifts the projectile. There can be head or tail range wind and right to left or left to right cross wind. Head wind under-ranges the projectile, whereas tail wind over-ranges the projectile. Left to right cross wind drifts the projectile towards right to the line of fire and right to left drifts towards left to the line of fire. Wind data significantly affects the range as well as drift of the trajectory and therefore wind data is very important input parameter for trajectory prediction.

5. EFFECT OF AIR DENSITY

Air density is inversely proportional to altitude and drag force is directly proportional to air density. Therefore, object travels at longer distance in less dense air media as compared to heavy dense air media. Therefore high altitude launch point results in to higher range as compared to sea level for the same angle of elevation. Thus it is observed that trajectory is affected by density of the medium and it is one of the non-standard inputs for predicting trajectory path.

Data from Eqns. (6) and (7) shows that initial value of air density under ISA and Ind Std is different at sea level. Density versus altitude values have been estimated under ISA and Ind Std using Eqns. (4), (5), (6) and (7). Density under prevailing recorded met has been estimated for recorded met against altitude. Fig. 1 shows density (kg/m^3) versus altitude (m) graphs under ISA, Ind Std met and prevailing met. It is observed density of Ind Std met and prevailing recorded met (experimental) are very close as compared to ISA. Thus, if recorded met is not available, then density values of Ind Std can be used to apply density effect for trajectory prediction.

Initial values of density at sea level for ISA (6) and Ind Std (7) are different. From Fig. 1, it is seen that correlation between density values under Ind Std and ISA is positive and both values are decreasing as altitude is increasing. Its coefficient of correlation value is 0.999984.

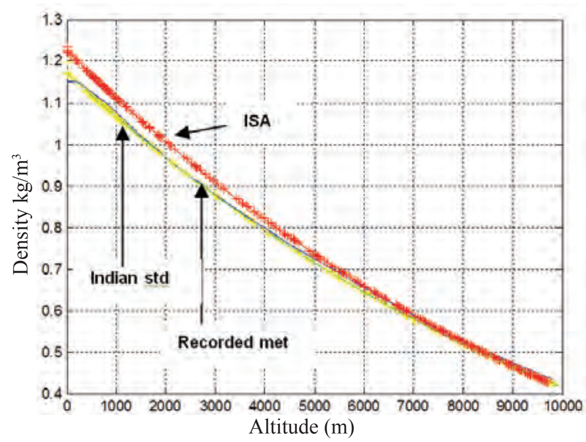

Figure 1. Density (kg/m^3) versus Altitude (m).

6. SIMULATION

Mathematical model for trajectory prediction[6,7] has been developed using Matlab and Simulink. Matlab program is written to initialize model parameters and then to call Simulink model for simulating defined model. Simulink models are developed for predicting trajectory under ISA, Indian standard and prevailing met. Figure 2 shows Simulink model for trajectory prediction under prevailing met. Simulink model is generated by using Simulink tool box blocks such as integrator, Matlab function, XY scope, constant, mathematical function, clock etc. Integrator blocks '1/s' are used in Simulink model to estimate velocity and position of trajectory. Integrator blocks are set to ode45 Dorman Prince Solver with variable time step and tolerance of 1e-3. For each 'Matlab Function block' Matlab function is written, for example 'computermet' function is written to give recorded met parameters as wind direction, wind speed, temperature, pressure, density for given altitude. XY scope blocks are used to display x, y plot, for example 'x vs y' block plots 'range vs altitude'. Constant block is used to define constant value, for example '–g' block is set to -9.80665.

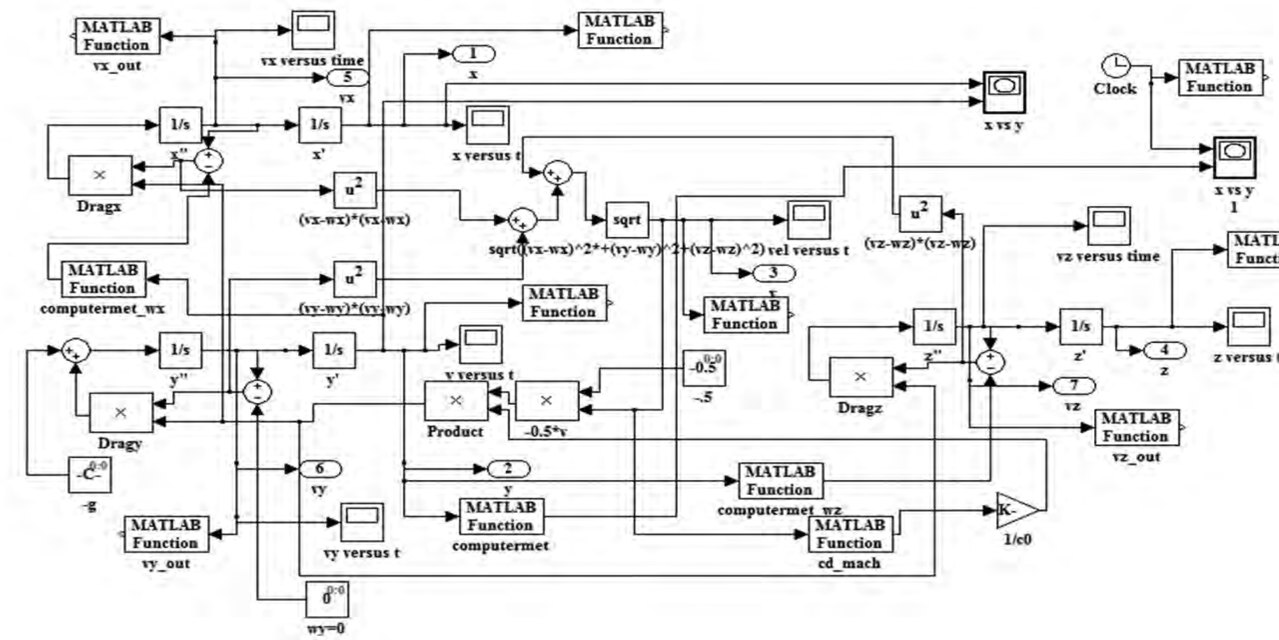

Figure 2. Simulink model for trajectory prediction under prevailing met.

Mathematical function block is used to assign mathematical function for given input, for example 'sqrt' block gives square root value. Clock block outputs the current simulation time.

6.1 Simulation Data

The following data values are used for simulating trajectory under prevailing met, ISA with wind and Indian standard with wind:

- Mass of the projectile 34 kg,
- Launch elevation angle 45 deg,
- Bearing angle 0 deg,
- Muzzle velocity 900 m/s

Wind direction (Fig. 3(a)) and speed (Fig. 3(b)) data values

Wind data is non-standard input data for simulating trajectory path[13]. Fig. 3(a) shows recorded wind direction in degrees against altitude and Fig. 3(b) shows wind speed in m/s against altitude.

7. RESULTS AND DISCUSSIONS

1. Trajectories have been simulated under prevailing met data (actual recorded met) and above simulation input data. Simulated results are shown in Fig 4. Fig. 4(a) shows 3D plot of predicted trajectory, it consists of range, drift and altitude. It shows drifting of trajectory due to wind data of prevailing met. Fig. 4(b) shows 2D plot as altitude versus range. Fig. 4(c) shows 2D plot of drift versus range. Fig. 4(d) shows velocity versus time plot. It shows that velocity is decreasing till vertex height during ascending against gravity and then increases slowly after vertex during descending due to gravity.
2. Trajectories under ISA with wind, ISA w/o wind, Ind Std with wind and Ind Std w/o wind have

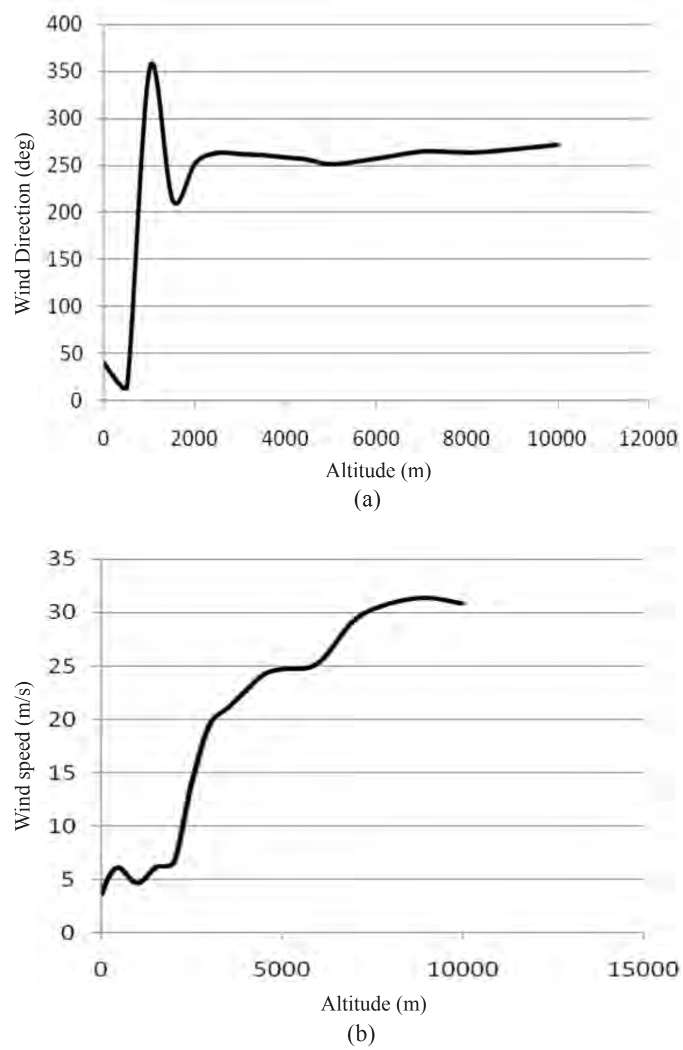

Figure 3. (a) Wind direction (deg) versus Altitude (m) and (b) Wind speed (m/s) versus Altitude (m).

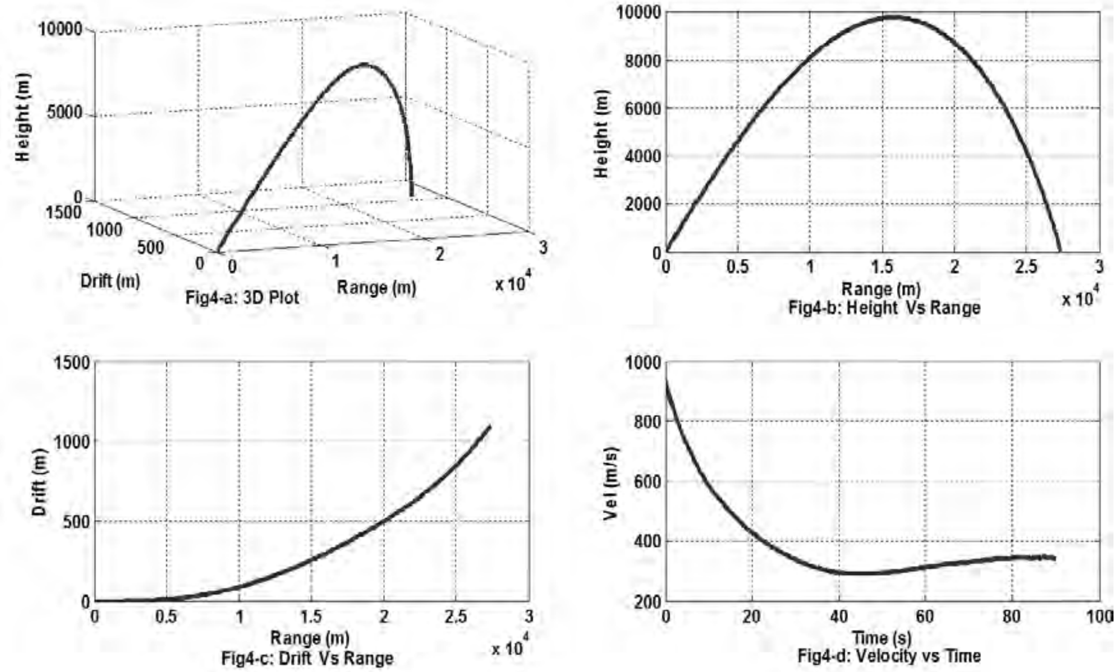

Figure 4. Simulated results under prevailing met data.

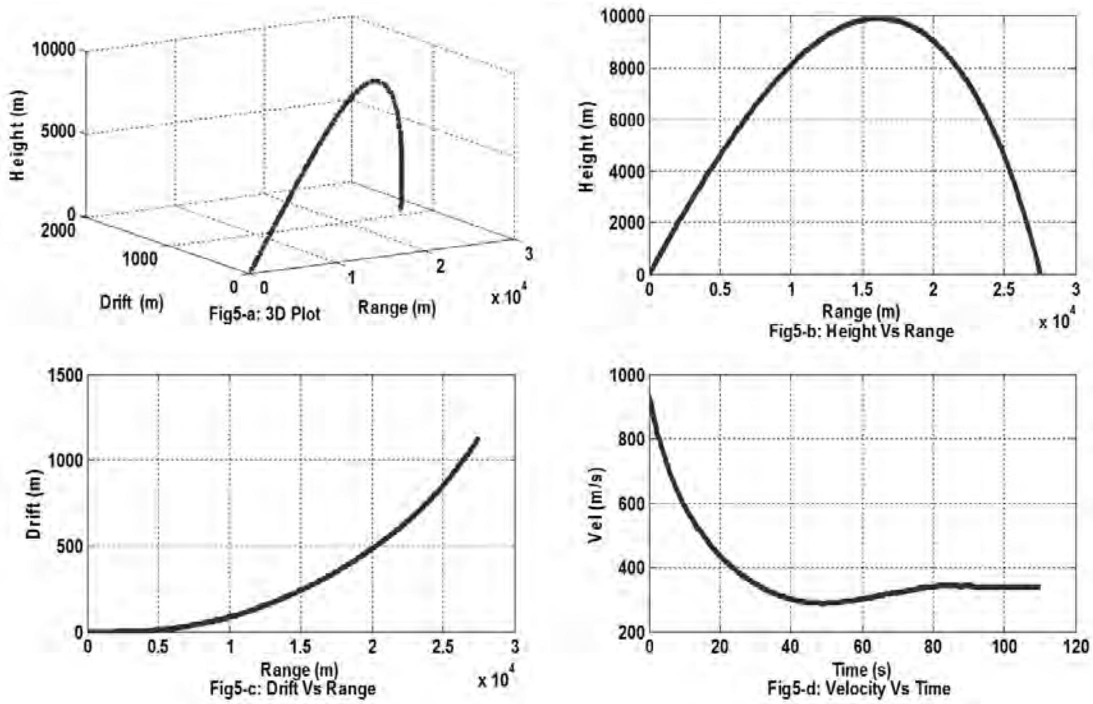

Figure 5. Simulated results under Ind std using recorded wind data.

Table 3. Simulated trajectory data with and without wind

	1 ISA w/o wind	2 ISA with wind	3 (2 – 1) Difference between ISA with & w/o wind	4 Ind Std w/o wind	5 Ind Std with wind	6 (5 – 4) Difference between Ind Std with & without wind
Range (m)	27042	27136	94	27454	27565	111
Drift (m)	0	1094	1094	0	1096	1096
Altitude (m)	9707	9712	5	9940	9836	-104

Table 4. Comparison of simulated trajectories

	1 ISA with wind	2 Ind Std with recorded wind	3 Prevailing met	4 (3-1) Difference prevailing met and ISA with wind & % deviation	5 (3-2) Difference prevailing met and Ind Std with wind & % deviation
Range(m)	27136	27565	27404	268 (0.977)	161 (0.587)
Drift(m)	1094	1096	1095	1 (0.091)	-1 (0.091)
Altitude(m)	9712	9836	9795	83 (0.847)	-41 (0.418)

been simulated and results are tabulated in Table 3. From Table 3, it is seen that difference in trajectory predicted values of with and without wind under ISA (column 3) and Ind Std (column 6) are at higher side. This shows the significant effect of wind on trajectory parameters.

3. Trajectories under Ind std Atmosphere with wind (Fig. 5) have been simulated. Fig. 5 (a) shows 3D-plot of predicted trajectory, it consists of range, drift and altitude. It shows drifting of trajectory due to recorded wind data. Fig. 5(b) shows 2D plot as altitude versus range. Fig. 5(c) shows 2D plot of drift versus range. Fig. 5 (d) shows velocity versus time plot. It shows that velocity is decreasing till vertex height during ascending against gravity and then increases slowly after vertex during descending due to gravity. The results of Ind Std with recorded wind, ISA with recorded wind are compared with actual trajectory (prevailing met) and are tabulated in Table 4.

From Table 4, it is seen that difference in trajectory predicted values of prevailing met and ISA with wind (column 4) is more than that of prevailing met and Ind Std with wind (column 5). Predicted trajectory under Ind Std atmosphere data is closer to the actual met as compared to ISA.

8. CONCLUSIONS

For Indian field, if the recorded met data is not available then Ind Std met data can be used instead of ISA for predicting trajectory with wind data which is function of date and time. Ind Std air density values can be used at zones/altitudes where recorded density data is insufficient. This study would be valid to Russian Standard also with historical wind to predict projectile path for Russian field. In general it can be concluded that our model can be used for predicting trajectory under standard met conditions of required region instead of ISA and wind data which is function of date and time of that region.

REFERENCES

1. Virk Kaur A.; Malhotra, J. & Pahuja, S. Link margin optimization of free space optical link under the impact of varying meteorological conditions. *Int. J. Eng. Sci. Technol.,* 2012, **4**(3), 1120-1125.
2. Du, Y. & Xie, S. Role of atmospheric adjustments in the tropical Indian Ocean warming during the 20th century in climate models. *Geophysical Research Letters*, 2008 **35**(8). doi:10.1029/2008GL033631
3. Bartolucci, L.; Chang, M.; Anuta, P. & M Graves, Atmospheric effects on Landsat TM Thermal IR Data. *IEEE Trans. Geosci. Remote Sensing*, 1988, **26**(2), 171-176.
4. Ma, Y.; Wang, S. & Shang, K. Effect of Meteorological Conditions on occurrence of intestinal epidemic in Lanzhou, China, 978-1-4244-4713-8/ 2010 IEEE
5. Shekhar, H. Effect of temperature on mechanical properties of solid rocket propellants. *Def. Sci. J.* 2011, **61**(6), 529-533.
6. Chusilp, P.; Charubhun, W. & Ridluan, A. Developing firing table software for artillery projectile using iterative search and 6-DOF trajectory model. *In* the Second TSME International Conference on Mechanical Engineering, Krabi, 19-21 October 2011.
7. Chusilp, P.; Charubhun, W. & Nuktumhang, N. Investigating and iterative method to compute firing angles for artillery projectiles. *In* the 2012 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Kaohsiung, Taiwan, 11-14, July 2012, pp 940-945.
8. Textbook of Ballistics & Gunnery (TBBG), Her Majesty's Stationary Office (HMSO), London, **1**,1987
9. Dr Derek Bray's Fundamental concepts on External Ballistics site, International Standard Atmosphere (ISA) www.dcmt.cranfield.ac.uk/aeroxtra/exb1b.htm, (Accessed in 2001)
10. Ananthasayanam, M.R. & Narasimha, R. Standards for the tropical Indian atmosphere. *Space Research J.*, 1980, **20**, 25-28.
11. Norton, D.J. External Ballistic Trajectory Computations for Direct/Indirect Fire Weapon Systems. *In* the 22nd International Symposium on Ballistics, Vancouver, Canada, 14-18 Nov 2005, pp 136-143.
12. Rossouw, W.J. Wind induced yaw, drag and lift, *In* the 17th International Symposium on Ballistics, Midrand, South Africa, 23–27 Mar 1998, pp 180-185.
13. Dullum, O. Wind sensitivity modelling of unguided rocket artillery. *In* the 15th International Symposium on Ballistics, Jerusalem, Israel, 21-24 May 1995, pp.373-380.

Contributors

Mr RJ Mukhedkar received his MSc (Computer Science) and MTech (Modeling and Simulation) from Pune University. He is currently pursuing his doctoral studies at Defence Institute of Advanced Technology (DIAT) Deemed University (DU), Pune. He is currently Scientist at ARDE, Pune. His research interests include: Advanced modelling techniques for Military applications, Weapon Target Allocation Problem, Trajectory Correction System, Network Centric Warfare, complex warfare systems modelling and simulation.

Dr (Mrs) SD Naik received MSc (Mathematics) and PhD (Applied Mathematics) from Pune University. She is currently Scientist at ARDE, Pune. Her research interests include: Flight Dynamics and allied problems related to Defence. She has over 50 publications in Peer reviewed International Journals/ conferences. She has guided 18 M.Tech and 6 PhD students. She is a member of Indian Women Scientist's Association, Indian Mathematical Society, Indian Academy of Industrial and Applicable Mathematics, Indian Society of Industrial and Applied Mathematics. She was awarded the Laboratory Award– Scientist of the year in 2003.