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REVIEW PAPER

Multiple-input Multiple-output Radar Waveform Design Methodologies

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ABSTRACT

Multiple-input multiple-output (MIMO) radar is currently an active area of research. The MIMO techniques have been well studied for communications applications where they offer benefits in multipath fading environments. Partly inspired by these benefits, MIMO techniques are applied to radar and they offer a number of advantages such as improved resolution and sensitivity. It allows the use of transmitting multiple simultaneous waveforms from different phase centers. The employed radar waveform plays a key role in determining the accuracy, resolution, and ambiguity in performing tasks such as determining the target range, velocity, shape, and so on. The excellent performance promised by MIMO radar can be unleashed only by proper waveform design. In this article, a survey on MIMO radar waveform design is presented. The goal of this paper is to elucidate the key concepts of waveform design to encourage further research on this emerging technology.

Keywords: Multiple-input multiple-output, MIMO systems, radars

1. INTRODUCTION

Multiple-input Multiple-output (MIMO) systems have the potential to dramatically improve the performance of radar systems over single antenna systems. The MlMO extensions to radar have been introduced in 2003[1]. The notion of MIMO radar is that there are multiple radiating & receiving sites and the collected information is processed together. MIMO radars can be considered as a generalization of multistatic radar concepts. Figure 1 shows the basic structure of MIMO radar. Recognizing the connection between MIMO wireless communication and MIMO radar could help motivate new radar concepts. Some traditional radar examples that could be described within MlMO context are synthetic-aperture radar (SAR) and fully polarimetric radar.

The MIMO radar is introduced as a concept[2] that capitalizes on the radar cross section (RCS) scintillations with respect to the target aspect in order to improve the radar's performance. Establishing the direction of the received signal in order to locate the target, known as direction finding (DF), is one of the important tasks of MIMO radar. The MIMO mode of multifunction digital array radars (DARs)[3] are well-suited to functions involving broad searches. The MIMO radar processing techniques that utilize multiple space-time coded waveforms with multiple receive phase centers improves the surveillance performance[4].

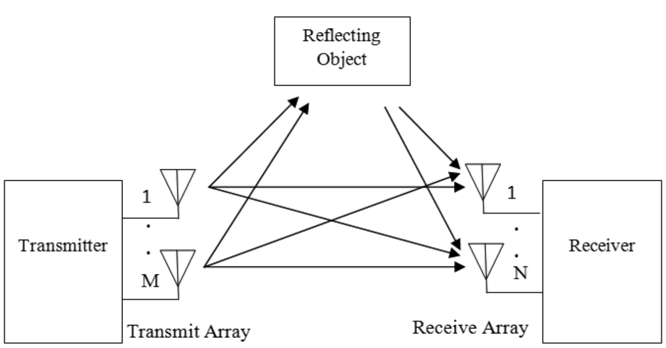

Figure 1. Illustration of basic MIMO radar.

The MIMO radar offers diversity gain in improving the detection/estimation performance[5], and spatial resolution gain in enhancing the resolution performance[6]. The performance of MIMO radar in detecting slow moving target is illustrated by an application to airborne ground moving-target indication (GMTI) radar[7]. The potential resolution improvement of MIMO radar is related to an increase in the virtual array size. Adaptive techniques are known to have much better resolution and interference rejection capability than their data-independent counterparts. MIMO radar makes it possible to use adaptive localization and detection techniques directly, unlike phased-array radar[8-10]. The combination of the longer illumination and the larger aperture of the MIMO radar provides for the possibility of improved minimum detectable velocity for GMTI systems. The self-interference mitigation property of MIMO radar makes it applicable in medical field also[11].

Fundamentally, there are two basic categories of MIMO radar according to the antenna configuration[12] namely bistatic MIMO radar[13] and colocated MIMO radar[14]. Bistatic MIMO radar uses transmit antennas separated far from each other.

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Given differences in observing angles on a particular target, the spatial transmit diversity gain can be obtained. According to their receive antennas, this class can be further divided into two kinds of configurations. The first kind is with the conventional receive array, like the phased array, performs direction finding[15]. This is called mixed MIMO setup[16]. The second is with the widely-separated antennas, leading to the spatial receive diversity gain. In this case, improved parameter identifiability and estimation accuracy can be obtained. This configuration is known as statistical MIMO. In co-located MIMO radar, the transmit antennas are closely spaced such that the RCS observed by the transmitting antenna elements are identical. Radars in this class usually transmit spatially orthogonal signals to achieve the spatial diversity gain, leading to significantly improved detection performance including virtual aperture extension, spatial coverage extension, beam pattern improvement and increase of the limit on the number of targets, compared with its phased array counterparts.

MIMO radar is suited for applications related to static and moving-target detection. MIMO radar outperforms the single-input multiple-output (SIMO) phased array radar in the Neyman-Pearson sense[17] for probabilities of detection greater than 0.8. The number of targets that can be uniquely identified by the radar is one of the basic aspects of MIMO radar called as parameter identifiability. The waveform diversity offered by MIMO radar enables a much improved parameter identifiability and it is observed that the maximum number of targets that could be identified by the MIMO radar set-up is M times that of its phased-array counterpart, where M is the number of transmit antennas[18]. Non parametric adaptive techniques like Capon, amplitude and phase estimation (APES), Capon and APES (CAPES) and capon and approximate maximum likelihood (CAML) and parametric techniques like maximum-likelihood (ML), Bayesian information criterion (BIC), approximate cyclic optimization (ACO) and exact cyclic optimization (ECO) for parameter estimation are applied to MIMO radar[12]. Tabrikian[19,20] has derived the Cramer-Rao bound (CRB) and the Barankin bound for target localization. It is shown that the orthogonal signals have better performance compared to coherent signals. For static MIMO radar angular estimation Cramer-Rao bound has been described[21-23].

For skywave HF over-the-horizon radar (OTHR), MIMO space time adaptive processing (STAP) is presented with the goal of mitigating radar clutter[24]. Novel forms of MIMO radar system like spatial MIMO system, coherent netter radar (NR) system, rephased coherent netted radar (RPNR) and decentralized radar network (DRN) have been examined[25]. The processing approaches exhibit considerable variability in performance with respect to False alarm rate (FAR), detection, jamming tolerance and coverage. Space-time coding (STC) is recognized as the key ingredient to achieve full diversity, and design criteria for both the transmitter and the receiver[26,27] of the MIMO radar.

The MIMO radar system can choose freely the probing signals transmitted via its antennas. But standard phased-array radar transmits scaled versions of a single waveform. Hence waveform design is a critical component that determines the MIMO radar performance. Radar can employ different waveforms so that the performance does not degrade in the presence of different environments of targets[28]. In the last few years, research in the area of MIMO radar waveform design experienced an expansion.

This paper provides an overview of research, development, and the application of various computational methods for MIMO radar waveform design, based on an extensive number of published papers.

2. SIGNAL MODEL

The signal model for different MIMO configurations is discussed in this section. Consider a MIMO radar system with M transmits antennas and N receive antennas. Let the discrete-time base-band signal transmitted by the m^{th} antenna be $x_m(k)$. Let θ denote the location parameter of a generic target, for example, its azimuth angle and its range. The transmitted signal vector from all M transmit antennas and the transmitter steering vector are given by[29],

$$\mathbf{x}(k) = \left[x_1(k), x_2(k), \ldots, x_M(k)\right]^T \text{ and}$$

$$a(\theta) = \left[e^{-j2\pi f_0 \tau_1(\theta)}, e^{-j2\pi f_0 \tau_2(\theta)}, \ldots, e^{-j2\pi f_0 \tau_M(\theta)}\right]^T$$

Under the assumption that the transmitted probing signals are narrow-band and that the propagation is nondispersive, the base-band signal at the target location can be described by the expression

$$\sum_{m=1}^{M} e^{-j2\pi f_0 \tau m(\theta)} x_m(k) = \mathbf{a}^*(\theta)\mathbf{x}(k), \quad k = 1, 2, \ldots, K \tag{1}$$

where, f_0 is the carrier frequency of the radar, τ_m is the time taken by the signal emitted via the m^{th} transmit antenna to arrive at the target, $(\bullet)^*$ denotes the conjugate transpose, K denotes the number of samples of each transmitted signal pulse.

2.1 Colocated MIMO Radar

Data collected by MIMO radar with identically located transmit and receive antennas under the simplifying assumption of point targets can be described by the equation[29]

$$y(k) = \beta \mathbf{a}^c(\theta)\mathbf{a}^*(\theta)\mathbf{x}(k) + \mathbf{n}(k) \tag{2}$$

where β is the complex amplitudes proportional to the radar-cross section, $n(k)$ denotes the interference-plus-noise term, $(\bullet)^c$ and denotes the complex conjugate.

The received signal for separately located transmit and receive antennas[30]

$$\mathbf{y}(t) = \beta \mathbf{a}_T^*(\theta)\mathbf{x}(t)\mathbf{a}_R(\theta) + \mathbf{n}(t) \tag{3}$$

where $\mathbf{a}_T(\theta)$ and $\mathbf{a}_R(\theta)$ are the actual transmit and actual receive steering vectors associated with θ, $x(t)$ refers to the waveform vector $\mathbf{x}(t) = \left[x_1(t), \ldots x_M(t)\right]^T$ and t is the time index. The returns due to the mth transmitted waveform can be recovered by match filtering the received data to $x(t)$ as,

$$y_m = \int_{T_0} y(t) x_m^*(t)\,dt$$

After matched filtering the received data with each transmitted waveform, the M N*1 virtual data vector can be written as,

$$\mathbf{y} = \beta \mathbf{a}_T(\theta) \otimes \mathbf{a}_R(\theta) + \mathbf{n} \tag{4}$$

where $\otimes$ is the kronecker product and **n** accounts for the noise component.

2.2 Statistical MIMO Radar

The received signal for a widely separated transmit and receive antennas with extended target assumption[31],

$$y_n(k)=\sum_{i=1}^{M} h_{in}s_i(k)+\xi_n(k),\quad k=1,2,...,K \tag{5}$$

where $y_n(k)$ is the received waveform at the n^{th} receive at the k^{th} time instant. The target is assumed to be point-like between each pair of transmit and receive antennas and h_{in} is the target impulse response from the i^{th} transmit antenna to the n^{th} receive antenna. Collecting the received waveforms from all the N receive elements, the received signal in matrix form can be written as,

$$\boldsymbol{Y}=\boldsymbol{SH}+\xi \tag{6}$$

where $\mathbf{Y}=\left[\mathbf{y}_1^T,\mathbf{y}_2^T,...,\mathbf{y}_N^T\right]$ is Gaussian distributed with zero mean and covariance $\left(\mathbf{SRS}^H+\sigma_\xi^2\mathbf{I}_k\right)$ and $\boldsymbol{y}_n$ is the received signal at the nth receive element obtained by stacking the $\boldsymbol{K}$ samples for an observation time of $\boldsymbol{T}$ seconds in a row is given by $\mathbf{y}_n=\mathbf{h}_n^T\mathbf{S}^T+\xi_n$. The transmitted signal $\boldsymbol{S}=[\boldsymbol{s}(1),\boldsymbol{s}(2),...,\boldsymbol{s}(K)]^T$ where $\mathbf{s}(k)=\left[s_1(k),s_2(k),...,s_M(k)\right]^T$ The columns of $\mathbf{H}=\left[\mathbf{h}_1,\mathbf{h}_2,...\mathbf{h}_N\right]$ are independent and identically distributed (i.i.d.) with distribution $CN(0,\boldsymbol{R_H})$ where $\mathbf{h}_n=\left[h_{1n},h_{2n},...,h_{Mn}\right]^T$. The columns of $\xi=\left[\xi_1^T,\xi_2^T,...,\xi_N^T\right]$ are i.i.d. with zero mean and covariance matrix $\sigma_\xi^2\mathbf{I}_k$.

3. TARGET MODEL

Radar transmits a well-defined, controlled signal. But the signal measured at the receiver output is superposition of different components like target, clutter, noise and jamming. When the electromagnetic wave is incident upon the point target some of the incident power is reradiated toward the radar. The reradiated power is a function of RCS which refers to an effective area that intercepts the transmitted radar power and then scatters that power isotropically back to the radar receiver. Several authors have considered various models for target reflection in MIMO radar systems. Targets are often modeled as point scatterers[28]. This is applicable for closely spaced sensors and for the case having large range between target and the array. However, as the resolution of radar systems increases, a better model is that of an extended target[5] or distributed source model which is spread in range, azimuth, and Doppler. In statistical MIMO radar, the spacing between the array elements is large. Due to the target's complex shape and the distance between the array elements, every element observes a different aspect of the target. Therefore, the point source model is not adequate for describing the received signal in statistical MIMO radar, and a more detailed model must be developed. The target model can be deterministic or statistical[32]: the former assumes that the target characteristics are fixed and known (possibly up to some unknown parameters which can be estimated), while the latter treats the target as a random variable and attempts to characterize its statistics. Similarly, different models can be used for the interference environment like clutter, jamming and noise[33,34].

4. MIMO RADAR WAVEFORM DESIGN APPROACHES

In general there are two views of waveform design[12]. In the first view, the design of the signal waveform matrix is considered. In the second view, the details of the waveform matrix are not considered directly. Rather, only the intertransmitter signal correlation matrix is optimized. Given the correlation matrix, the problem becomes that of determining a signal waveform matrix.

MIMO radar waveform design takes into account numerous performance parameters and technical constraints. The performance measures include the mean square error (MSE) in estimating the target impulse response, normalized MSE (NMSE), the mutual information (MI) between the received signal and the target impulse response and signal to interference noise ratio (SINR). Sum power constraint, uniform elemental power constraint, average power ratio constraint, constant energy constraint, structure constraint, semi-definite rank constraint and norm constraint are the technical constraints used for waveform design. The research in the relevant literature deals with each one of these parameters separately, or concerns the overall waveform optimization. Radar waveforms are designed either to optimize target detection or to extract target information when the radar targets are modeled as extended target[35].

4.1 Information Theory Based Approaches

Among the criterions for waveform optimization, information theoretic criterion plays an important part and has been proposed for radar waveform design for many years. The application of information theory to radar can be traced to the early 1950s by Woodward and Davies when they examined the use of information-theoretic principles to obtain a posteriori radar receiver[36]. But the connection between information theory and radar waveform design[26] was given by Bell in 1993.

Yang and Blum[37] considered waveform design for MIMO radar problem in terms of both information theoretic and estimation theoretic criteria under the total transmit power constraint. In information theoretic point of view the waveform is designed to maximize the mutual information (MI) between the random target impulse response and the reflected waveforms given the knowledge of transmitted waveforms.

Waveform design with white noise[37] is extended to include colored noise[31]. Maximizing the relative entropy is used as a measure for waveform design and it is observed that larger relative entropy results in better detection performance. It has been shown in this paper that to maximize the MI and relative entropy, the optimal waveform should 'match' with the target and colored noise. It is also shown that the power allocation methods for the singular values of the optimal transmitted waveform for maximizing the MI and relative entropy are different from each other.

4.2 Estimation Theory Based Approaches

Minimizing MSE or NMSE in estimating the target impulse response is the criteria in estimation theoretic view point. MI, MMSE and NMSE criteria are used for waveform design with colored noise in a mixed MIMO set up[16]. The colocated antennas in the receiver allow combining the received signals coherently to obtain a processing gain of *N*, where *N* is the number of receive antennas. The results show that the equivalence between the MI and MMSE criteria does not hold when the noise is colored. For the case of statistical MIMO radar with widely separated transmit and receive antennas it is observed that to obtain minimum MMSE/NMSE the eigenvectors of target and noise should be carefully paired[38].

4.3 Other Waveform Design Concepts

A procedure to design the optimal waveform which maximizes the signal-to-interference plus-noise ratio (SINR) at the output of the detector using gradient descent algorithm, given the knowledge of target and clutter statistics is developed for co-located MIMO radar[39]. These waveforms take on arbitrary shapes and so while optimizing the specified criterion, they are not guaranteed to have good temporal characteristics nor good spatial characteristics. But if space-time constraints are imposed on the transmitted waveforms it would provide better control of the spatial and temporal characteristics of the radar illumination[40].

Signal design for MIMO radar could be illustrated under five other groups, viz., adaptive waveform design[41-45], beamforming[29,30,46-52], waveform synthesis[53-56], waveform optimization[21, 23, 57-60], and code design[61-63].

5. WAVEFORM DESIGN AND POWER ALLOCATION

It is observed that to extract information about the target, it might be advantageous to distribute the available energy among the various modes of the extended target[35]. Viewing the characteristics of these waveforms in terms of the distribution of energy among target scattering modes gives both physical and information-theoretic insights into the design and performance of these waveforms. Most of the waveform design problems take into account only the total transmitted power.

If the waveform design takes into account only the total transmitted power the resulting waveforms, while optimizing the specified criterion, are not guaranteed to have good temporal or spatial characteristics. So space time constraints are introduced in the waveform design[40].

5.1 Sum Power Constraint Based Approaches

In most of the approaches the power is allocated to transmit antennas based on the sum power constraint at the transmitter.

Mutual Information

Consider a MIMO radar equipped with *M* transmitting elements and *N* receiving elements. The MI between **Y** and **H,** given the knowledge of **S** is given by[37],

$$\mathbf{I}(\mathbf{Y};\mathbf{H}/\mathbf{S}) = \log\left[\det\left(\mathbf{I}_K + \sigma_\xi^{-2}\mathbf{S}\mathbf{R}_H\mathbf{S}^H\right)\right] \tag{7}$$

where $\det\{\bullet\}$ refers to determinant of a matrix. Using the determinant property,

$$\det\left(\mathbf{I}_p + \mathbf{AB}\right) = \det\left(\mathbf{I}_q + \mathbf{BA}\right)$$

MI in Eqn (7) can be written as,

$$\mathbf{I}(\mathbf{Y};\mathbf{H}/\mathbf{S}) = \log\left[\det\left(\mathbf{I}_M + \mathbf{R}_H\mathbf{S}^H\mathbf{S}\right)\right]$$

where the variance of noise $\left(\sigma_\xi^2\right)$ is assumed to be unity. The problem of waveform design based on MI is expressed as,

$$\max_{\mathbf{S}} \log\left[\det\left(\mathbf{I}_M + \mathbf{R}_H\mathbf{S}^H\mathbf{S}\right)\right] \tag{8}$$

$$\left[tr\left(\mathbf{S}^H\mathbf{S}\right)\right] \le \beta$$

where β is the sum of the average transmit powers.

Minimum Mean Square Error

In estimating the target impulse response MMSE is given by,

$$MMSE = tr\left\{\left(\sigma_\xi^{-2}\mathbf{S}^H\mathbf{S} + \mathbf{R}_H^{-1}\right)^{-1}\right\} \tag{9}$$

The problem of waveform design based on MMSE is expressed as

$$\min_S tr\left\{\left(\sigma_\xi^{-2}\mathbf{S}^H\mathbf{S} + \mathbf{R}_H^{-1}\right)^{-1}\right\} \tag{10}$$

$$\left[tr\left(\mathbf{S}^H\mathbf{S}\right)\right] \le \beta$$

The constrained optimization problem in Eqns (8) and (10) are solved using the method of Lagrange multipliers and the optimum waveform is given by[37],

$$\mathbf{S} = \psi\left(diag\left[\left(\mu - \frac{\sigma_\xi^2}{\lambda_1}\right)^+, \ldots, \left(\mu - \frac{\sigma_\xi^2}{\lambda_M}\right)^+\right]\right)^{1/2}\mathbf{U}^H \tag{11}$$

where the columns of ψ are orthonormal, **U** is obtained from the singular value decomposition (SVD) on $\mathbf{R}_H$, as $\mathbf{R}_H = \mathbf{U}\Lambda U^H$ and $\Lambda = diag\left(\lambda_1, \lambda_2, \ldots, \lambda_M\right)$ where λ_i is the eigen value of the covariance matrix of the target impulse response. The scalar constant μ satisfies

$$\sum_{i=1}^{M}\left(\mu - \frac{\sigma_\xi^2}{\lambda_i}\right)^+ = \beta \tag{12}$$

Under total power constraint the two criteria lead to the same solution. The solution employs water-filling in spatial mode. Waterfilling over the eigen modes of the spatio-temporal channel matrix results in emphasis of strong targets. Waterfilling with respect to spatial modes is not desirable especially in tracking scenarios[64]. It is assumed that the exact characterization of the target power spectral density (PSD) is available, but in practice it is very difficult to obtain perfect knowledge of target PSD. So the authors in a later work[65] assumed that the actual PSD is only known to lie in some class of possible PSDs. Based on this formulation a minimax robust scheme is developed under both the MI and the MMSE criteria. The results indicate that the MI and MMSE criteria lead to different minimax robust waveforms, which is in stark contrast to their earlier findings[37], where the MI and MMSE lead to the same result. It was observed that solution offered in the previous works[37,65] was not in the ultimate form of the transmit waveforms. So the desired form of the waveform is achieved by using the method of Kronecker structured matrix estimation, e.g., ML estimation and separable least squares framework estimation[66]. Yang and Blum proposed an iterative optimization algorithm based on the alternating projection method to determine waveform solutions that can simultaneously satisfy a structure constraint and optimize the design criteria where operations like the Procrustean transformation and the Kronecker product approximation are used to provide closed-form solutions[67]. The waveform solutions obtained through the proposed algorithm attains virtually indistinguishable performance when compared to that predicted in earlier works[37,65].

5.2 Per Antenna Power Constraint Based Approach

Conventionally, the power is allocated to transmit antennas based on the sum power constraint at the transmitter. But the wide power variations across the transmit antennas poses a severe constraint on the dynamic range and peak power of the power amplifier at each antenna. The recently proposed p-norm constraint jointly meets both the average per-antenna power constraint and the average sum power constraint to bound the dynamic range of the power amplifier at each transmit antenna[68]. The optimal power allocation using the concept of waterfilling, based on the sum power constraint is the special case of p =1. Instead of considering each constraint in a separate way, a unified framework is developed to obtain the optimal solution using Karush Kuhn Tucker (KKT) approach.

Mutual Information

For the signal model shown in Eqn (5) for widely separated antenna case, the problem statement for MI criterion with p-norm constraint[69],

$$\max \log[\det (\mathbf{I}_M + \mathbf{R}_H \mathbf{S}^H \mathbf{S})] \tag{13}$$

$$s.t. \left[tr\left(\mathbf{S}^H \mathbf{S}\right)^p \right]^{1/p} \le J$$

where J is a constant. Let α be the constraint on the power of individual antenna elements and β is the constraint on the sum of the average transmit powers. If, p = 1, then the constant J will be equal to the sum power constraint β. For values of p within the interval, $1 < p < \infty$, with J = α satisfies both the maximum power constraint and sum power constraint.The transmitted signal matrix is given by,

$$\mathbf{S} = \psi\left(diag\left(d_1, d_2, ..., d_M\right)\right)^{1/2} \mathbf{U}^H \tag{14}$$

where d_i is the power allocation value on the transmit antenna, is the diagonal element of $\mathbf{D} = \mathbf{X}^H \mathbf{X}$ and $\mathbf{X} = \mathbf{SU}$. To find the values of d_i the problem statement in Eqn (14) is written as,

$$\max \sum_{i=1}^{M} \log\left(1 + \lambda_i d_i\right) \tag{15}$$

$$s.t. \sum_{i=1}^{M} d_i^p \le J^p \text{ and } d_i \ge 0$$

The Lagrangian for Eqn (15) can be written as,

$$L(\mathbf{d}, \eta, h) = \sum_{i=1}^{M} \log\left(1 + \lambda_i d_i\right) + \sum_{i=1}^{M} d_i \eta_i + h\left(J^p - \sum_{i=1}^{M} d_i^p\right) \tag{16}$$

where $\mathbf{d} = (d_1, d_2, ...d_M)$, $\eta = (\eta_1, \eta_2, ..., \eta_M)$, η and h are Lagrangian multipliers. Using KKT optimality conditions the following solutions are obtained. It is shown that for the limiting case p = 1 it holds that,

$$d_i = \left(\mu - \frac{1}{\lambda_i}\right)^+ \tag{17}$$

μ such that $\sum_{i=1}^{M} d_i = J$

For $p = \infty$, $\|\mathbf{D}\|_p = \max\left(|d_1|, |d_2|, ..., |d_M|\right)$ and for the general case $1 < p < \infty$, d_i can be determined by numerically solving the equation $d_i^p + \frac{1}{\lambda_i} d_i^{p-1} = \mu$ using a quadratically convergent algorithm such as nested Newton algorithm. The update equation for d_i and μ are given by,

$$d_{i,k+1}^{(n)} = d_{i,k}^{(n)} - \frac{\left(d_{i,k}^{(n)}\right)^p + \left(d_{i,k}^{(n)}\right)^{p-1} \big/ \lambda_i - \mu^{(n)}}{p\left(d_{i,k}^{(n)}\right)^{p-1} + (p-1)\left(d_{i,k}^{(n)}\right)^{p-2} \big/ \lambda_i}, \quad 1 \le i \le M \tag{18}$$

$$\mu^{(n+1)} = \mu^{(n)} - \frac{L\left(\mu^{(n)}\right)}{L'\left(\mu^{(n)}\right)}$$

respectively, where $L(\mu) = \sum_i \left[q_i^{-1}(\mu)\right]^p - J^p$, $\mu \ge 0$, $1 \le i \le M$ and

$$L'(\mu) = \sum_i p\left[q_i^{-1}[\mu]\right]^{p-1} q_i^{-1'}(\mu) = \sum_i \frac{1}{1 + \frac{p-1}{p\lambda_i}\left[q_i^{-1}(\mu)\right]^{-1}} \tag{19}$$

Minimum Mean Square Error

The problem statement for MMSE criterion is given by [70]

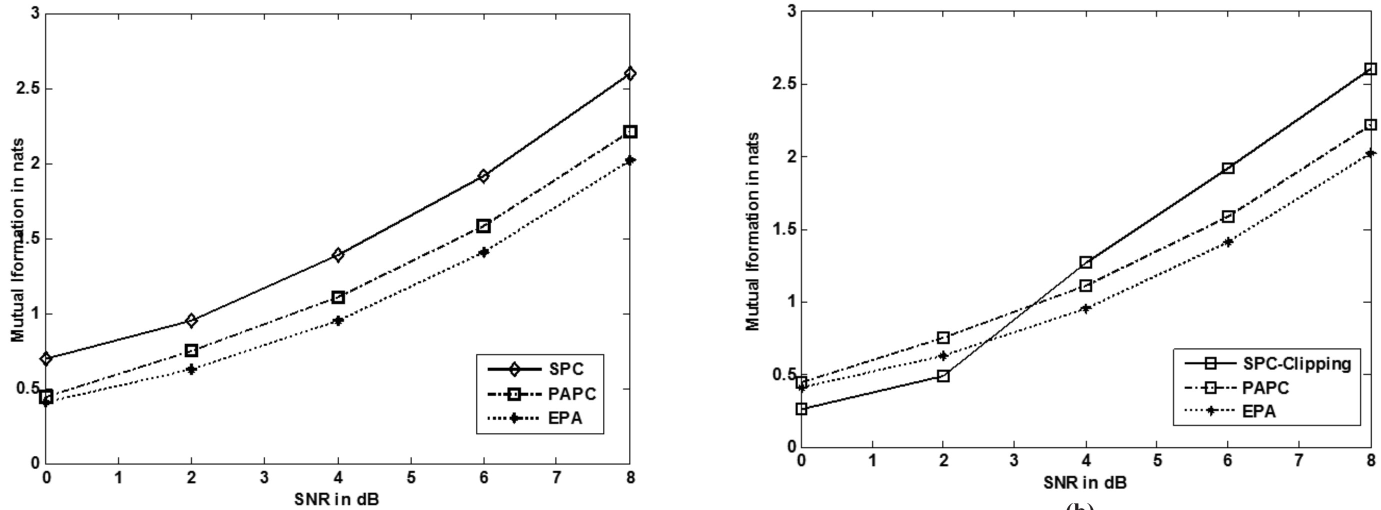
$$\min \sum_{i=1}^{M} \left(\frac{\lambda_i}{d_i \lambda_i + 1}\right)$$

$$s.t. \sum_{i=1}^{M} d_i^p \le J^p \text{ and } d_i \ge 0 \tag{20}$$

Forming the Lagrangian for Eqn (20) as in the case of MI and solving using KKT optimality conditions, the solution is same as that of MI criterion for the case of p=1 and $p = \infty$. For the case $1 < p < \infty$, using the nested Newton algorithm, the update equation for d_i for the system of equation $d_i^{p+1} + \frac{2}{\lambda_i} d_i^p + \frac{1}{\lambda_i^2} d_i^{p-1} = \mu$ is

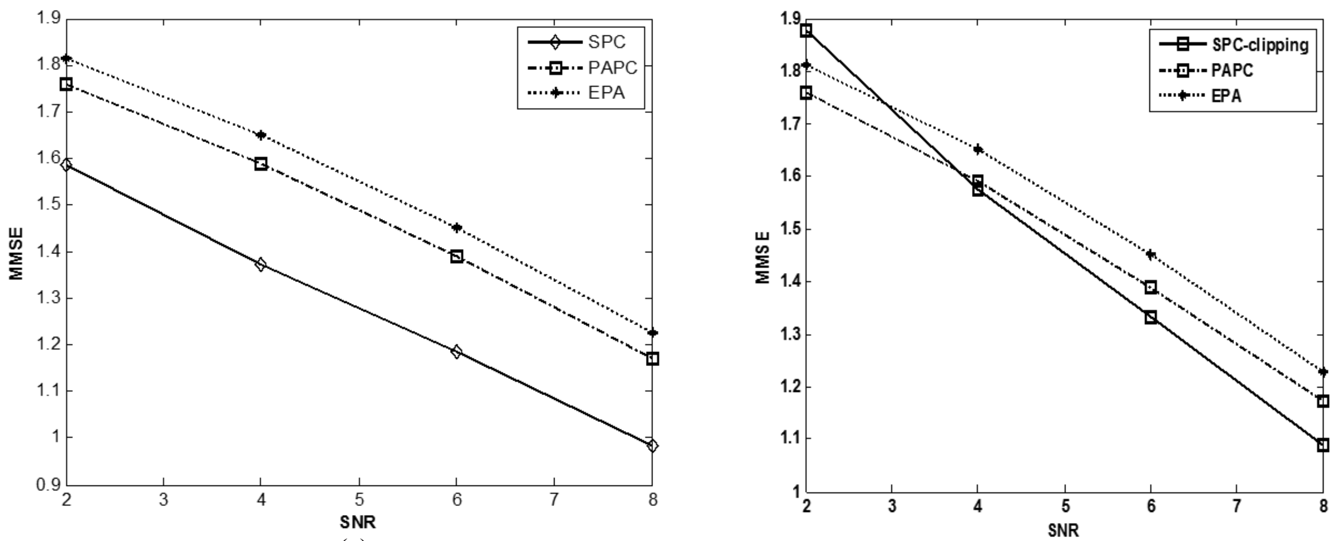
$$d_{i,k+1}^{(n)} = d_{i,k}^{(n)} - \frac{\frac{\left(d_{i,k}^{(n)}\right)^{p-1}}{\lambda_i^2} + \frac{2\left(d_{i,k}^{(n)}\right)^p}{\lambda_i} + \left(d_{i,k}^{(n)}\right)^{p+1} - \mu^{(n)}}{(p-1)\frac{\left(d_{i,k}^{(n)}\right)^{p-2}}{\lambda_i^2} + \frac{2p\left(d_{i,k}^{(n)}\right)^{p-1}}{\lambda_i} + (p+1)\left(d_{i,k}^{(n)}\right)^p}, \quad 1 \le i \le M \tag{21}$$

A MIMO radar system with $\boldsymbol{M}$ = 5 transmit and $\boldsymbol{N}$ = 5 receive antenna system is considered to illustrate the performance of MIMO radar waveform with per antenna power constraint. Figs. 2(a) and 3(a) show the MI performance and MMSE performance, respectively, when there is no power constraint on the power amplifiers used in the transmit antennas. The graphs are plotted for the cases of sum power constraint (SPC), per antenna power constraint (PAPC) and equal power allocation (EPA). As observed in the figures the sum power constraint that results in waterfilling power allocation has the superior performance. But the amplifier in each antenna would have a maximum limit on the power that could be amplified. This practical consideration would result in clipping effect. The MI and MMSE performance in such a situation would be as shown in Figs. 2(b) and 3(b). The rate of improvement



(a) (b)

Figure 2. (a) MI Performance without constraint on the power amplifier and (b) MI Performance with constraint on the power amplifier.



(a) (b)

Figure 3. (a) MMSE Performance without constraint on the power amplifier and (b) MMSE Performance with constraint on the power amplifier.

of mutual information with respect to Signal-to-Noise Ratio (SNR) for SPC-clipping changes beyond SNR 2 dB as shown in Fig.2(b). Also, as per Fig. 3(b), rate of improvement of MMSE with respect to SNR improves beyond SNR of 4 dB. This is because of the relationship between SNR and MI/MMSE as given in Eqns. (15) and (20) respectively. So the peak power clipping would have different effect on MI and MMSE. It is observed that as the constraint on the power of the individual antenna is relaxed the MMSE and MI performance of the general case moves towards the optimal water filling solution.

6. CONCLUSIONS

As MIMO radar is a relatively new concept, there is much remaining to be exploited about its performance potential. This paper presents the key concepts of waveform design for MIMO radar. Waveform design in terms of the distribution of energy among target scattering modes is also dealt with. Apart from waveform design for MIMO radar the other signal design concepts for MIMO radar such as adaptive waveform design, beamforming, waveform synthesis, waveform optimization and code design are analyzed. MIMO communication is theoretically superior to conventional communication and also it appears to be practical and cost effective in the real world for some applications. The same is expected in the case of radar, but that is not fully explored. As the potential benefits offered by MIMO radar depend mostly on the signal design this paper is expected to create interest in the area of waveform design.

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