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Thermal Buckling and Free Vibration Analysis of Heated Functionally Graded Material Beams

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ABSTRACT

The effect of temperature dependency of material properties on thermal buckling and free vibration of functionally graded material (FGM) beams is studied. The FGM beam is assumed to be at a uniform through thickness temperature, above the ambient temperature. Finite element system of equations based on the first order shear deformation theory is developed. FGM beam with axially immovable ends having the classical boundary conditions is analysed. An exhaustive set of numerical results, in terms of buckling temperatures and frequencies, is presented, considering the temperature independent and temperature dependent material properties. The buckling temperature and fundamental frequency obtained using the temperature independent material properties is higher than that obtained by using the temperature dependent material properties, for all the material distributions, geometrical parameters in terms of length to thickness ratios and the boundary conditions considered. It is also observed that the frequencies of the FGM beam will reduce with the increase in temperature. This observation is applicable for the higher modes of vibration also. The necessity of considering the temperature dependency of material properties in determining thermal buckling and vibration characteristics of FGM beams is clearly demonstrated.

Keywords: Functionally graded material, finite element method, buckling, free vibration, Timoshenko beam

1. INTRODUCTION

The functionally graded material is an advanced composite material, realised by smoothly changing the composition of the constituent materials, in the preferred direction. Amongst many applications of FGM structures, the use of FGMs as thermal barriers is the most significant in the design of aerospace and nuclear structures. In these applications, the structures are subjected to severe thermal gradients. The metallic structures are quite inadequate to cope up with these requirements. The reinforced composites, though currently used, are susceptible to the debonding due to interlaminar stresses, stress concentrations and residual stresses. The FGM structures, with ceramic and metal as the constituent materials, have emerged as a new kind of materials, to circumvent these interface problems, with a gradual and continuous change in the material properties.

The buckling and free vibration aspects of the homogenous structures are exhaustively reported by many researchers. The buckling and free vibration analysis of the FGM beams is the current area of research[1-9]. In most of the above studies, the effect of the temperature dependency of the material properties is not considered to analyse thermal buckling and free vibration response. Further, there are very few papers which discusses about thermal buckling and free vibrations of FGM beams. Xiang[10], *et al.* have studied free and forced vibration of laminated FGM Timoshenko beam of variable thickness under heat conduction. Mahi[11], *et al.* used an analytical method for temperature dependent free vibration analysis of functionally graded beams with general boundary conditions. Wattanasakulpong[12], *et al.* have studied thermal buckling and elastic vibration of third-order shear deformable functionally graded beams. To the best of the authors' knowledge, finite element method is not used for the study of buckling and free vibrations of FGM beams in the thermal environment.

The material properties of constituent materials change with temperature. When a structure is used in thermal environment other than room temperature, the properties pertaining to that temperature (higher or lower than room temperature) should be used in the analysis. When the material properties at room temperature are used in the analysis of structure working in thermal environment, the analysis is with temperature independent material properties and the response of the structure is generally overestimated. When variation of material properties with temperature is considered in the analysis, the response of the structure is true response.

As well known, for a slender beam, the effect of transverse shear and rotary inertia on the buckling and free vibration response is negligible. For thick beams, however, the effect of transverse shear and rotary inertia are significant. The present

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formulation is based on Timoshenko beam formulation which includes shear flexibility and rotary inertia effects.

In the present study, a finite element formulation is developed by using the principle of virtual work and the weak forms of the governing equations of motion. The eigenvalue equations are solved to obtain the linear buckling and vibration response of heated FGM beams for various geometrical parameters, material distributions and support conditions. The material system considered in the analysis consists of zirconia and nickel as the constituent materials. The study is mainly focused on the importance of considering the temperature dependency of material properties, to get the true response of the FGM beams (in terms of buckling and free vibrations), operating in the thermal environment. The results are presented for FGM beam with various length to thickness ratios. The classical boundary conditions such as both ends hinged (H-H), both ends clamped (C-C) and one end hinged and the other end clamped (H-C) are considered with axially immovable ends.

2. FUNCTIONALLY GRADED BEAM

The variation of Young's modulus E, shear modulus G, coefficient of linear thermal expansion α across the thickness of FGM beam is governed by a power law distribution as given below.

$$E(z) = E_c(0.5+\frac{z}{h})^n + E_m\left\{1-(0.5+\frac{z}{h})^n\right\}$$
$$G(z) = G_c(0.5+\frac{z}{h})^n + G_m\left\{1-(0.5+\frac{z}{h})^n\right\} \quad (1)$$
$$\alpha(z) = a_c(0.5+\frac{z}{h})^n + \alpha_m\left\{1-(0.5+\frac{z}{h})^n\right\}$$

where h is the thickness of the beam and the thickness co-ordinate z varies from $-h/2$ to $h/2$. The volume fraction exponent n can take any value between 0 to ∞. Subscripts c and m denote ceramic and metal respectively. Here, a FGM beam, with the ceramic, zirconia on the top face and metal, nickel on the bottom face is considered as shown in Fig. 1. In other words, power law is utilized for achieving the compositional gradation between the complete zirconia top surface to complete nickel bottom surface. A typical variation of Young's modulus through thickness of FGM beam at 300 K for various values of volume fraction exponents is shown in Fig. 2. The properties of the constituent materials are assumed to be dependent on temperature. The property P of zirconia and nickel at any temperature T (in K) is expressed as a nonlinear

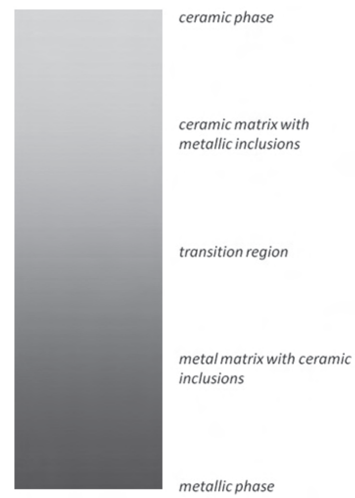

Figure 1. A typical cross section of FGM beam with ceramic and metal as constituent materials.

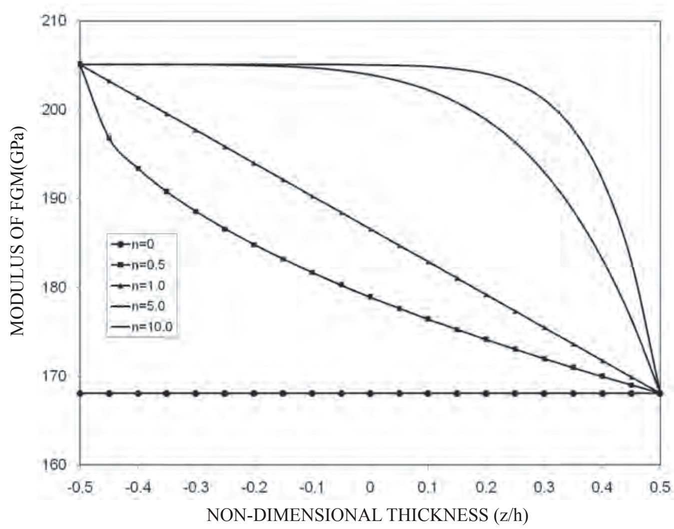

Figure 2. A typical variation of Young's modulus of FGM through thickness (at 300 K).

function of temperature as given in Shen[13], as

$$P = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \quad (2)$$

where P_0, P_{-1}, P_1, P_2, and P_3 are the coefficients of the temperature T as given in Table 1. The densities of zirconia and nickel are taken as 5680 kg/m³ and 8908 kg/m³, respectively.

Table 1. Coefficients of temperature for zirconia and nickel [Shen[13]]

Property	Material	P_0	P_{-1}	P_1	P_2	P_3
Young's Modulus E (Pa)	Zirconia	244.27 x 10^9	0	-1.371 x 10^{-3}	1.214 x 10^{-6}	-3.681 x 10^{-10}
	Nickel	223.95 x 10^9	0	-2.794 x 10^{-4}	-3.998 x 10^{-9}	0
Coefficient of Thermal Expansion α (K^{-1})	Zirconia	12.766 x 10^{-6}	0	-1.491 x 10^{-3}	1.006 x 10^{-5}	-6.778 x 10^{-11}
	Nickel	9.9209 x 10^{-6}	0	8.705 x 10^{-4}	0	0
Poisson Ratio ν	Zirconia	0.2882	0	1.133 x 10^{-4}	0	0
	Nickel	0.3100	0	0	0	0

3. FINITE ELEMENT FORMULATION

A finite element formulation is developed based on Timoshenko beam theory. The details of the formulation are available in Reddy[14], Anandrao[2] and will not be repeated here for the sake of brevity. A beam element with two nodes per element and having three degrees of freedom per node is considered. The lagrange linear interpolation functions are used for the axial displacement, transverse deflection and rotation. By using the principle of virtual work and the weak forms of the governing equations of motion, the element matrices are obtained.

3.1 Buckling of the FGM Beams

In the case of buckling of FGM beam, with axially immovable ends, subjected to a uniform temperature rise, above the ambient temperature (300 K) through the thickness, the following eigenvalue problem is obtained, as

$$[K]\{U\}-\lambda_b[G]\{U\}=0 \quad (3)$$

where $[K]$ is elastic stiffness matrix and $[G]$ is geometric stiffness matrix, λ_b is the eigenvalue indicating the buckling temperature and $\{U\}$ is the eigenvector (buckling mode shape).

3.2 Free Vibration of FGM Beams

In the case of free vibration, when the FGM beam is executing simple harmonic motion, the following eigenvalue equation is obtained, as

$$[K]\{V\}-\lambda_f[M]\{V\}=0 \quad (4)$$

where $[M]$ is the mass matrix, $\{V\}$ is the eigenvector (vibration mode shape) and $\lambda\rho_f$ denotes frequency parameters. For free vibration of a beam in a uniform temperature field, above the ambient temperature, the beam is subjected to an initial thermal load, due to the axially immovable ends. The following eigenvalue equation, for the heated FGM beam is written, as

$$[K+G]\{V\}-\lambda_f[M]\{V\}=0 \quad (5)$$

where $\{V\}$ is the eigenvector (vibration mode shape). A MATLAB code is developed based on the above formulation, to extract the eigenvalues, in terms of buckling load and the frequency parameters of the FGM beams with or without the initial load from Eqns. (3) to (5). The analysis is carried out for two conditions, namely

(1) Temperature independent material properties (TID): For this condition, the properties are evaluated at ambient temperature of 300 K and used in the analysis.
(2) Temperature dependent material properties (TD): For this condition, the properties corresponding to the actual temperature of FGM beam are used in the analysis.

4. CONVERGENCE STUDY

A convergence study is carried out with the different number of equal length finite elements (*NE*). The analysis is carried out for buckling as well as vibration. Table 2 shows the fundamental frequency obtained from free vibration analysis. The analysis is carried with different values of *L*/*h* ratios, where *L* is the length of FGM beam. It is to be noted that 160 equal length elements gives a converged solution.

Table 2. Convergence study: Fundamental frequency f (Hz) at T = 300K

NE	n = 0, C-C, L/h = 100	n = 0.5, H-H, L/h = 50	n = 1, H-C, L/h = 10
5	0.6372	0.9883	37.3417
10	0.5765	0.9518	35.1183
20	0.5631	0.9430	34.5979
40	0.5598	0.9408	34.4699
80	0.5590	0.9403	34.4380
160	0.5588	0.9401	34.4301

5. VALIDATION OF PRESENT FORMULATION

5.1 Buckling

The critical temperature T_{cr} obtained from the present study is compared with the critical temperature obtained from the commercial finite element software ANSYS[15] for homogenous ceramic beam with different values of *L* / *h* ratios. Table 3. gives the summary of comparison of dimensionless thermal buckling parameter, $\lambda = \Delta T_{cr} L^2 \alpha_m / h^2$, where Δ refers to temperature change from ambient temperature. The properties considered for the analysis correspond to those given in Table 1 evaluated at the ambient temperature of 300 K.

5.2 Free vibration

For the sake of comparison, the digital data (power law, single layer) giving the fundamental frequency ω, is available only at the ambient temperature. Table 4 shows the comparison of fundamental frequency parameter $\Omega = \omega L^2 \sqrt{\dfrac{I_0}{h^2 \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)dz}}$ obtained from the present study and the available literature for different values of *L*/*h* ratios. The term I_0 is given as

$$I_0 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)dz \quad (6)$$

and ρ is the density of constituent materials. The material properties are taken from the study by Sina[5], *et al.*

Table 3. Validation study: Comparison of non-dimensional thermal buckling parameter

	C-C			H-H			H-C		
L/*h*	100	50	10	100	50	10	100	50	10
Present, TID	2.212	2.206	2.009	0.553	0.553	0.540	1.132	1.130	1.071
ANSYS ,TID	2.212	2.205	2.004	0.553	0.553	0.540	1.132	1.130	1.070

Table 4. Validation study: Comparison of fundamental frequency parameter with literature (n = 0.3, Wattanasakulpong[12] *et al.*)

Support condition	Source	L/h = 10	L/h = 30	L/h = 100
C-C	Present	5.869	6.176	6.214
	Simsek[6] [PSDBT]	5.881	6.177	6.214
	Simsek[6] [ASDBT]	5.884	6.177	6.214
	Sina[5], *et al.* [FSDT2]	5.811	6.167	6.212
H-H	Present	2.737	2.774	2.779
	Simsek[6] [PSDBT]	2.702	2.738	2.742
	Simsek[6] [ASDBT]	2.702	2.738	2.742
	Sina[5], *et al.* [FSDT2]	2.695	2.737	2.742

5.3 Free vibration of pre-stressed beam

To validate the formulation for the heated beam, a study is carried out for the C-C homogenous ceramic beam at various uniform temperatures. The fundamental frequency parameter obtained from the present study is compared with that obtained by using ANSYS. The analysis using the present formulation and ANSYS is carried out for the two cases, namely, TID and TD. Figure 3 shows the comparison of the frequency parameter $\left[\bar{\omega} = \omega\left(\frac{L}{h}\right)^2 \sqrt{\frac{\rho_m h^2}{E_m}}\right]$.

For all the cases considered for validation, an excellent match is found between the results obtained with present formulation and with those of the literature/ANSYS.

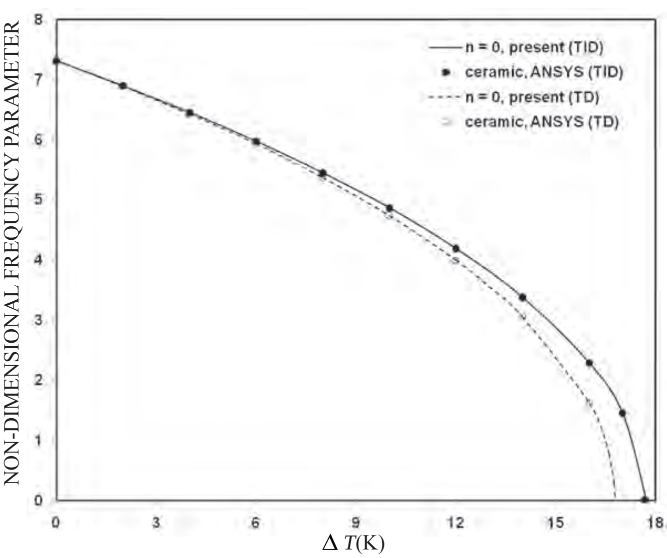

Figure 3. Fundamental frequency *vs* FGM beam temperature (*L* / *h* = 100, n = 0, C-C).

6. BUCKLING OF FGM BEAMS

The main aim of the present study is to determine the free vibration characteristics of the FGM beam in different constant temperature fields, above the ambient temperature of 300 K. The uniformly heated FGM beam will be subjected to initial thermal loads, due to the axially immovable ends. It becomes essential to obtain the critical temperatures of the FGM beam for the support conditions, geometrical parameters and material distributions considered in the study, to know the extent of temperature range for linear vibration analysis. A linear buckling analysis is carried out for the FGM beam to determine the temperature at which the beam buckles under thermal load.

Initially, the critical temperature T_{cr} is determined with the TID material properties. The temperature T_{cr} above the initial stress free temperature of 300 K is observed to be very large, especially at lower L/h ratios and higher values of volume fraction exponent n. These high temperatures will change the Young's modulus and linear coefficient of thermal expansion of the constituent materials. It is felt necessary at this stage, to determine the critical temperature with TD material properties. An iterative analysis is carried out to obtain the critical temperature.

The steps followed in this analysis are as follows.

(1) Carry out the buckling analysis with the material properties corresponding to the ambient temperature of 300 K, to determine the critical temperature T_{cr}.

(2) The properties are now updated to a temperature of 300 K + T_{cr}. The buckling analysis is carried out with these properties, to determine a new value of critical temperature T_{cr}^{new}.

(3) Step 2 is repeated till $(T_{cr}^{new} - T_{cr}) \leq tolerance(0.0001)$ and the solution converges to the critical temperature with TD material properties.

A typical graph indicating the convergence of critical temperature with number of iterations is shown in Fig. 4 for the homogenous ceramic beam with L/h = 10 and clamped boundary conditions. Tables 5-7 shows the thermal buckling load parameter, $\lambda = \Delta T_{cr} L^2 \alpha_m / h^2$ for various boundary conditions, volume fraction exponents and geometrical

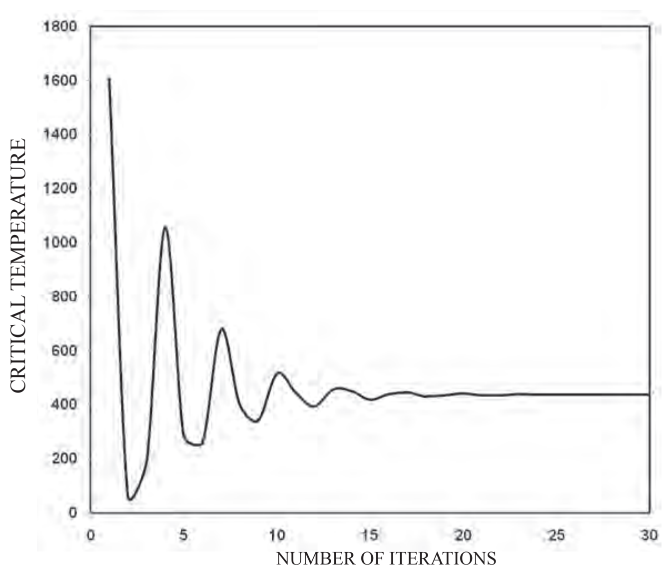

Figure 4. Critical temperature *vs* Number of iterations (*L*/*h* = 10, n = 0, C-C).

Table 5. Thermal buckling parameter $\lambda = \Delta T_{cr} L^2 \alpha_m / h^2$ (C-C)

N		L / h		
		100	50	10
0	TID	2.2124	2.2056	2.0085
	TD	2.0989	1.8299	0.5477
0.5	TID	2.5074	2.4996	2.2751
	TD	2.3912	2.1056	0.6412
1	TID	2.6533	2.6452	2.4095
	TD	2.5410	2.2578	0.7067
10	TID	3.0963	3.0870	2.8160
	TD	3.0192	2.8084	1.1190

Table 6. Thermal buckling parameter $\lambda = \Delta T_{cr} L^2 \alpha_m / h^2$ (H-H)

N		L / h		
		100	50	10
0	TID	0.5534	0.5530	0.5397
	TD	0.5459	0.5246	0.2708
0.5	TID	0.6283	0.6279	0.6127
	TD	0.6208	0.5989	0.3174
1	TID	0.6655	0.6650	0.6490
	TD	0.6582	0.6370	0.3474
10	TID	0.7749	0.7743	0.7560
	TD	0.7699	0.7551	0.4971

Table 7. Thermal buckling parameter $\lambda = \Delta T_{cr} L^2 \alpha_m / h^2$ (H-C)

N		L / h		
		100	50	10
0	TID	1.1319	1.1299	1.0708
	TD	1.1012	1.0193	0.3990
0.5	TID	1.2832	1.2810	1.2136
	TD	1.2520	1.1666	0.4674
1	TID	1.3581	1.3558	1.2850
	TD	1.3280	1.2445	0.5136
10	TID	1.5842	1.5815	1.5003
	TD	1.5637	1.5038	0.7806

parameters. A substantial difference is observed in critical temperatures obtained using the TID and TD, especially at lower L/h. The analysis with the TID over estimates the critical temperature than the realistic temperature obtained using TD. For higher values of volume fraction exponents, the critical temperature increases for all the L/h ratios and the boundary conditions. This tendency is expected since with the higher values of volume fraction exponent, the effective modulus $E = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)dz$ at any cross section of the beam will be higher. The highest critical temperatures for any L/h ratio are observed for the C-C beam followed by the H-C beam and the H-H beam. The critical temperatures, for these boundary conditions, are higher for lower L/h ratios.

7. FREE VIBRATION OF FGM BEAMS

A free vibration analysis of the FGM beam is carried out to determine the effect of temperature on the natural frequencies. The analysis is carried out with TID as well as TD material properties. The beam is assumed to be in a uniform temperature field of ΔT above the ambient temperature, so that the temperature across the thickness and length of the beam attain uniform value of 300 + ΔT. Fig. 5 shows the variation of the fundamental frequency parameter $\left[\bar{\omega} = \omega \left(\frac{L}{h}\right)^2 \sqrt{\frac{\rho_m h^2}{E_m}}\right]$ with temperature change ΔT for the clamped, homogenous (n = 0) and FGM (n = 0.5) beam for L/h = 50. For the non-zero value ΔT, an axial compressive stress is developed in the beam because of the axial constraints at the two ends. This will reduce the effective transverse stiffness of the beam and hence the fundamental frequency. As the temperature increases, the fundamental frequency decreases further. This trend continues till the transverse stiffness of the beam becomes zero at the critical temperature. The analysis with TID material properties over estimates the critical temperature for the FGM beam. At a particular temperature, the fundamental frequency predicted by considering the TID properties is higher than that predicted by using the TD material properties. From this study the necessity of considering the TD material properties in determining the vibration characteristics is clearly seen.

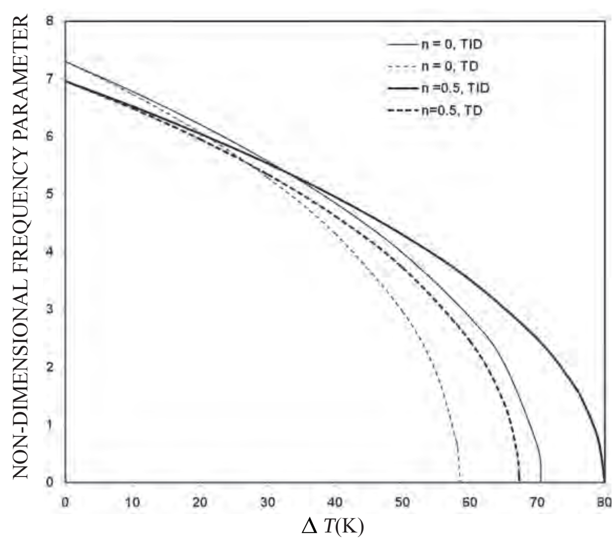

Figure 5. Effect of temperature dependent material properties on fundamental frequency (L/h = 50, C-C).

Figure 6 shows the variation of the fundamental frequency parameter with the temperature change for the clamped FGM beam with L/h = 10 and the TD material properties. As the volume fraction exponent is increased, the fundamental frequency is reduced at the ambient temperature. As the temperature is increased above the initial stress free temperature, the frequency starts decreasing for all volume fraction exponents. However, it is observed that, with the constituent materials considered in the analysis, the fall in the fundamental frequency with temperature is sharp for

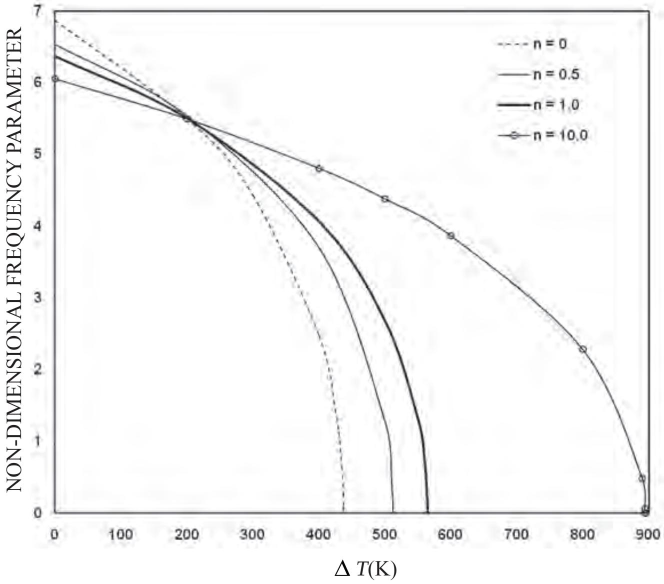

Figure 6. Effect of material distribution on fundamental frequency of FGM beam in uniform temperature field (*L*/*h* = 10, C-C, TD).

homogenous (n = 0) beam. As the volume fraction increases, the curve becomes more flat. This is due to the higher critical temperatures at higher volume fraction exponents.

Figure 7 shows the variation of the fundamental frequency parameter with the temperature change for hinged FGM beam with L/h = 50 and temperature dependent material properties. A similar behavior is observed for this support condition also. As the volume fraction exponent is increased, the fundamental frequency is reduced at the ambient temperature. As the temperature is increased above the initial stress free temperature, the frequency starts decreasing for all volume fraction exponents. It is observed that the fall in the fundamental frequency with temperature is sharp for homogenous ceramic (n = 0) beam. As the volume fraction increases, the curve becomes more flat.

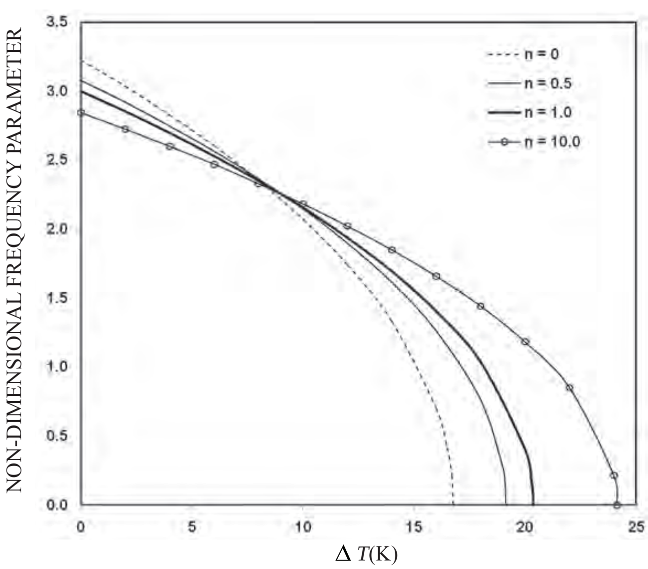

Figure 7. Effect of material distribution on fundamental frequency of FGM beam in uniform temperature field (*L*/*h* = 50, H-H, TD).

This is due to the higher critical temperatures at higher volume fraction exponents.

Figure 8 shows the effect of boundary conditions on fundamental frequency in uniform temperature field for n = 0.5 and L/h = 50. The behavior is observed to be identical for all the boundary conditions. At ambient temperature, the lowest fundamental frequency is observed for the H–H FGM beam and highest fundamental frequency is observed for C-C FGM beam. For each boundary condition considered in the analysis, the fundamental frequency is observed to be highest at ambient temperature. As the temperature increases above ambient temperature, axial compressive stresses are developed in the beam and the fundamental frequency starts decreasing till it becomes zero at critical buckling temperature for each boundary condition. At critical buckling temperature, the transverse stiffness of beam is zero and the corresponding fundamental frequency is also zero. The critical temperature is observed to be lower for the H-H beam and higher for C-C FGM beam. The behavior of H-C FGM beam is observed to be in between that of H-H and C-C FGM beam.

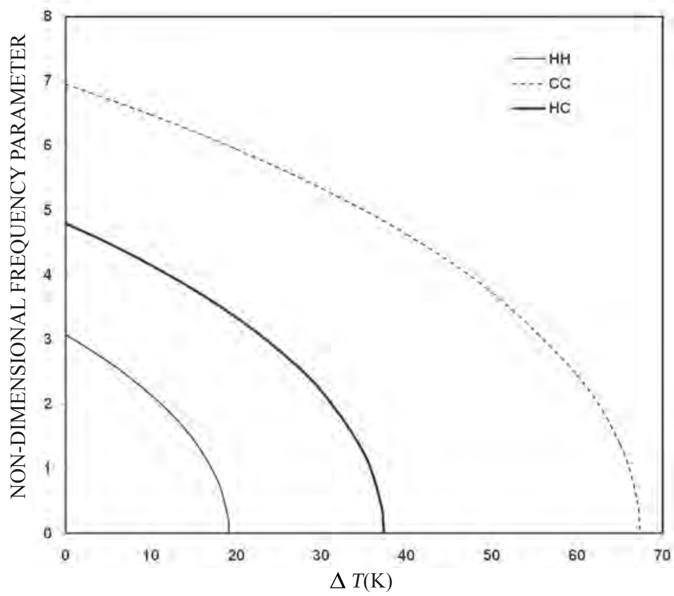

Figure 8. Effect of boundary conditions on fundamental frequency of FGM beam in uniform temperature field (*L*/*h* = 50, n = 0.5, TD).

Figure 9 shows the effect of boundary conditions on fundamental frequency of FGM beam for n = 1.0 and L/h = 10. A similar behavior as that of Fig. 8 is observed for this material distribution and geometrical parameters also. The authors aim of putting two different figures (Fig. 8 and 9) is to demonstrate that even with different material distributions (n = 0.5 and n = 1.0) and different geometrical parameters (L/h = 50 and L/h = 10), the trend of variation of fundamental frequency with increase in temperature is identical for various boundary conditions. Further, the trend of results for slender beams (L/h= 50) and thick beams (L/h = 10) are identical showing the applicability of present formulation for slender as well as thick beams.

The present study is further extended to the higher modes of vibration. Table 8 gives a summary of the effect of the TD material properties on the first five natural frequencies, for

Table 8. First five natural frequencies in terms of non-dimensional frequency parameter $\left[\bar{\omega} = \omega\left(\frac{L}{h}\right)^2 \sqrt{\frac{\rho_m h^2}{E_m}}\right]$

	C-C L/h = 50, ΔT = 50 n = 0.5		H-H L/h = 50, ΔT = 10 n = 1		H-C L/h = 10, ΔT = 300 n = 10	
	TID	**TD**	**TID**	**TD**	**TID**	**TD**
1	4.30253	3.72342	2.18351	2.15568	3.71954	3.27966
2	15.96483	15.20076	11.23280	11.17911	12.45236	11.49468
3	33.99335	32.90029	26.11799	26.01454	24.68399	23.00940
4	57.87905	56.33750	46.77949	46.60599	31.68084	30.00979
5	87.43828	85.33825	73.09521	72.83332	39.29881	36.78323

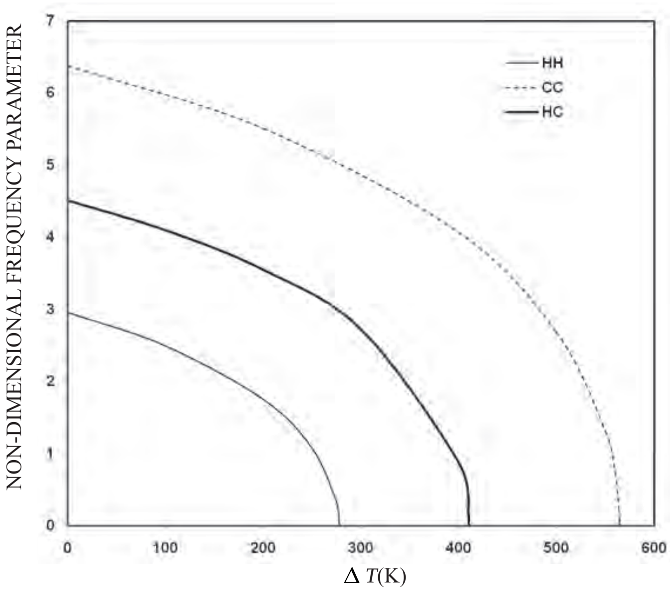

Figure 9. Effect of boundary conditions on fundamental frequency of FGM beam in uniform temperature field (*L*/*h* = 10, n = 1.0, TD).

boundary conditions considered, by using the representative geometric parameters, temperature increments and volume fraction exponents. It is observed from this study that the higher frequencies are also over predicted when the TD material properties are not considered.

8. CONCLUSION

FGM beam, considering Timoshenko formulation, is studied here, in terms of the critical temperature and the natural frequencies, based on the linear eigenvalue analysis. It is observed that neglecting temperature dependency of the material properties significantly over predicts the critical temperatures for uniformly heated FGM beams. This observation is applicable for all the boundary conditions, geometrical parameters and constituent material distributions across the thickness of the beam. With the increase in the temperature, from the ambient temperature, for the axially constrained FGM beam, the natural frequencies show a decreasing tendency. The natural frequencies predicted by considering the TID material properties are higher than those predicted using the TD material properties. The present study shows that for the analysis of the heated FGM beam, it is necessary to consider the temperature dependency of the material properties to obtain the realistic values of the buckling temperatures and the natural frequencies with initial thermal loads.

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