# A New Strategy of Guidance Command Generation for Re-entry Vehicle 

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#### Abstract

Guidance command for re-entry vehicle can be in various formats, but the Euler angles can be provided directly by gyros, so designers used to develop autopilot with commands of Euler angles. After the generation of commands of angle of attack and sideslip angle, it's important to settle how to convert commands of angle of attack and sideslip angle to commands of Euler angles. Traditional conversion strategy relies on bank angle, but the solution to bank angle comprises complicated calculation and can't be precise. This paper introduces a new conversion strategy of guidance command. This strategy relies on the relative position and velocity measured by seeker, an auxiliary coordinate is established as a transition, the transformation matrix from launch coordinate to body coordinate is solved in a new way, then the commands of Euler angles are obtained. The calculation of bank angle is avoided. The autopilot designed with the converted Euler-angle commands, can track commands of angle of attack and sideslip angle steadily. The vehicle reaches the target point precisely. Simulation results show that the new conversion strategy based on seeker information from commands of angle of attack and sideslip angle to Euler-angle commands is valid.


Keywords: Re-entry vehicle, radar seeker, angle of attack commands, sideslip angle command, Euler angles, transformation matrix

## 1. INTRODUCTION

Guidance commands for re-entry vehicles are generated based on the line-of-sight (LOS) information. If the guidance command is proper, the vehicle will reach the target point precisely, but the premise is that the autopilot can track guidance command steadily. Usually, the error between guidance command and real state is used to design autopilot. Guidance commands can be in lots of formats, such as acceleration, Euler angles, Euler rate, angle of attack, sideslip angle and so on ${ }^{1}$. For re-entry vehicles, $\mathrm{Lu}^{2}$ derived body acceleration command, Daigoro ${ }^{3}$ developed guidance command in the format of angle of attack and sideslip angle.

The advantage of Euler angle command is that the feedback of the controlled variables (Euler-angle body attitude) can be directly provided by inertial measurement unit (IMU) or gyros ${ }^{3}$. The guidance system will determine the Euler angle commands to be tracked so that the velocity vector is steered according to the guidance law. So designers used to convert angle of attack command and sideslip angle to command of Euler angles ${ }^{3-5}$.

After command of angle of attack and sideslip angle are generated, the next problem is how to realize the conversion from commands of angle of attack and sideslip angle to command of Euler angles (yaw, pitch, roll). There exists between velocity coordinate, launch coordinate and body coordinate. Based on the information of heading angle, flight path angle, and bank angle $(\theta, \sigma, v)$, commands of Euler angles can be derived from transformation matrix. Above method is feasible, but the problem need to be emphasized is: heading
angle, flight path angle, and bank angle are derived based on the navigation data, the navigation data have error after long time flying during booster phase and middle phase, so the value of heading angle, flight path angle, and bank angle have error. If we use above conversion method, commands of Euler angles have error. Furthermore, computation of the bank angle comprises complicated triangular function. In a real flight process, to alleviate the operation burden, bank angle is set to be zero considering that bank angle is always small. Bank angle is treated as zero factitiously, the calculation process is simplified, but error is brought in.

Reentry homing vehicle is equipped with inertial measurement unit and radar seeker, the latter is used for terminal guidance. Dowdle ${ }^{6}$ estimated target maneuver using EKF with the on-board active rardar seeker measurements. The measurements were relative range, range rate, line-of-sight (LOS) angles and LOS rates. Balakrishnan ${ }^{7,8}$ has estimated LOS rates from passive measurements of LOS angles alone using modified Polar coordinates. With the method of Carlson filter or Kalman filter ${ }^{9-13}$, it is possible that relative distance and velocity between vehicle and target can be estimated precisely.

This paper introduces a new strategy of guidance command generation for re-entry vehicle. Transformation matrix from launch coordinate to velocity coordinate is solved relying on the relative position measured by the seeker, instead of the traditional way using heading angle, flight path angle, and bank angle. Through this strategy, commands of angle of attack and sideslip angle are converted into commands of Euler angles, avoiding the calculation process of heading angle, flight path angle, and bank angle.

## 2. GUIDANCE COMMAND GENERATION OF ANGLE OF ATTACK AND SIDESLIP ANGLE

### 2.1 Coordinate System Definition

As the downrange of the re-entry phase is short, the earth is modeled as flat. As Fig. 1 shows, $O_{E}-x_{E} y_{E} z_{E}$ is earth-centred-earth-fixed (ECEF) frame. Define launch coordinate(L): The origin is located at the launch point $O_{L}$. The $O_{L} x_{l}$-axis is contained in the horizontal plane at launch point and positive to the launching direction. The $O_{L} z_{l}$-axis is perpendicular to the horizontal plane at launch point and positive to the up. $O_{L} y_{l}$ -axis completes a right-handed set. $A_{0}$ is launching direction, $\phi_{0}$ is latitude of launch point, $\lambda_{0}$ is longitude of launch point.


Figure 1. Definition of ECEF, launch and body coordinate frames.

Define body coordinate system ( $B$ ): The origin is located at the vehicle center of mass $O_{1}$ and the axes are fixed relative to the vehicle body. The $O_{1} x_{b}$-axis is along the center line of the vehicle body and positive forward. The $O_{1} z_{b}$ -axis is perpendicular to the $O_{1} x_{b}$-axis and contained in the longitudinal plane of vehicle. It is positive to the downward. The $O_{1} y_{b}$-axis is mutually perpendicular to the $O_{1} x_{b}$ and $O_{1} z_{b}$ axes. $u, v, w$ are projections of the velocity in body coordinate, $V_{M}=\sqrt{u^{2}+v^{2}+w^{2}}$. The angle of attack and sideslip angle are written as
$\alpha=\operatorname{asin}(w / u)$
$\beta=\operatorname{atan}\left(v / V_{M}\right)$
As Fig. 2 shows, $O_{1}$ is the vehicle center of mass, $O_{2}$ is the projection of $O_{1}$ on the local horizontal plane. $v$ is the target position, $V_{M}$ is the velocity vector.

Target coordinate $(T)$ is local geographic frame (east-north-up) whose origin is located at the target point ${ }^{14}$. $\phi_{t}$ is latitude of target point, $\lambda_{t}$ is longitude of target point.

Define auxiliary coordinate system (A): The origin is located at the vehicle center of mass. The $O_{1} z_{f}$-axis is parallel to the $O_{1} z_{T}$-axis and positive up. $O_{1} x_{A}$-axis is parallel to $x_{T} O_{T} y_{T}$ plane and positive to the target. $O_{1} y_{f}$-axis completes a right-handed set.

Define velocity coordinate system (V): The origin is located


Figure 2. Definition of target, velocity, auxiliary and control coordinate frames.
at the vehicle center of gravity. The $O_{1} x_{V}$-axis is aligned with the vehicle velocity vector. The $O_{1} z_{V}$-axis is perpendicular to the $O_{1} x_{V}$-axis and contained in the longitudinal symmetry plane of vehicle. It is positive to the up. The $O_{1} y_{V}$-axis completes a right-handed set.

Define control coordinate system (C): The origin is located at the target point. The $O_{T} x_{C}$-axis is aligned with the target to vehicle range vector. The $O_{T} y_{C}$-axis is perpendicular to the $O_{T} x_{C}$-axis and contained in the horizontal plane. It is positive to the right. The $O_{T} z_{C}$-axis completes a right-handed set.

Components of re-entry position in target coordinate system are $x_{T}, y_{T}, z_{T}$; components of re-entry velocity are $V_{x T}, V_{y T}, V_{z T}$. Relative position and velocity vectors are determined by combining navigation with radar seeker. The included angle between $\mathrm{O}_{2} \mathrm{O}_{T}$ and $\lambda_{e}$ is elevation angle $\lambda_{e}$, and the included angle between $O_{2} O_{T}$ and, $\lambda_{a}$ is azimuth angle $\lambda_{a}$.

$$
\begin{align*}
& \lambda_{e}=\arcsin \left[z_{T} / \sqrt{x_{T}^{2}+y_{T}^{2}+z_{T}^{2}}\right]  \tag{1}\\
& \lambda_{a}=\arctan \left[y_{T} / x_{T}\right]
\end{align*}
$$

See Fig. 1, $O_{2} O_{T}$ is the projection of $O_{1} O_{T}$ on the horizontal plane $x_{T} O_{T} y_{T}$. Note the definition of control coordinate (C) and auxiliary coordinate (A), $O_{1} y_{f}$ and $O_{T} y_{\mathrm{s}}$ both are vertical to $O_{1} O_{2} O_{T}$ plane. The transformation matrix from auxiliary coordinate to control coordinate is $C_{F}^{S}=R_{y}\left(-\lambda_{e}\right) . R_{y}$ indicates rotation about y-axis for the specified angle.
$V_{x C}, V_{y C}, V_{z C}$ are projections of the velocity vector in control coordinate; it is expressed as in Eqn (2) where the transformation matrix $C_{T}^{F}$ from target coordinate to auxiliary coordinate is expressed in later section 3 in detail.

$$
\left[\begin{array}{c}
V_{x s}  \tag{2}\\
V_{y s} \\
V_{z s}
\end{array}\right]=C_{F}^{S} C_{T}^{F}\left[\begin{array}{c}
V_{x T} \\
V_{y T} \\
V_{z T}
\end{array}\right]
$$

$\gamma_{D}$ is the angle between the velocity vector and local horizontal plane, $\gamma_{T}$ is the angle between velocity vector and local vertical plane (usually called diving plane in re-entry guidance).


Figure 3. Definition of $\gamma_{D}$ and $\gamma_{T}=-\arctan \left[V_{z s} / \sqrt{V_{x s}^{2}+V_{y s}^{2}}\right]$.

Note that $O_{1} O_{Y} y_{C}$ plane is vertical to $O_{1} O_{2} y_{C}$ plane is horizontal. So we can white $\gamma_{D}$ and $\gamma_{T}$ as follows:

$$
\begin{align*}
& \gamma_{D}=\operatorname{atan}\left(V_{z C} / V_{x C}\right)+\lambda_{e} \\
& \gamma_{T}=\operatorname{atan}\left[V_{y C} / \sqrt{V_{x C}^{2}+V_{z C}^{2}}\right] \tag{3}
\end{align*}
$$

### 2.2 Commands of Angle of attack and Sideslip Angle

Optimal guidance law based on LQR theory is given by ${ }^{15,16}$

$$
\left\{\begin{array}{l}
\dot{\gamma}_{D}=-4 \dot{\lambda}_{e}-2\left(\lambda_{e}+\gamma_{D F}\right) / T_{g}  \tag{4}\\
\dot{\gamma}_{T}=3 \dot{\lambda}_{a} \cos \lambda_{e}
\end{array}\right.
$$

where $T_{g}$ is time-to-go, $\gamma_{D F}$ represents the terminal value of $\gamma_{D}$ specified a priori by guidance system requirement, which means that the designers desire the vehicle to impact the target in a particular direction in diving plane.

We obtain the corresponding commanded body accelerations (not including gravity) in the $y_{v}$ and $z_{v}$ axes as ${ }^{2}$

$$
\begin{align*}
& \dot{\mathrm{W}}_{y v}=-v \dot{\gamma}_{T}-g_{y v} \\
& \dot{\mathrm{~W}}_{z v}=v \cos \gamma_{T} \dot{\gamma}_{D}-g_{z v} \tag{5}
\end{align*}
$$

where $V_{M}$ is the velocity; $g_{y v}, g_{z v}$ are components of gravity in the $y_{v}$ and $z_{v}$ axes. Eqn (5) indicate that for $\gamma_{D}$ and $\gamma_{T}$ to change as specified by the guidance law, the apparent acceleration components $\dot{\mathrm{W}}_{y v}$ and $\dot{\mathrm{W}}_{z v}$ should be applied on the $y$ and $z$ direction of the velocity coordinate frame.

The corresponding guidance commands of angle of attack $\alpha_{c}$ and sideslip angle $\beta_{c}$ are

$$
\begin{align*}
& \alpha_{c}=-\frac{\alpha_{T O T} \dot{\mathrm{~W}}_{y v}}{\sqrt{\dot{\mathrm{~W}}_{y}^{2}+\dot{\mathrm{W}}_{z}^{2}}} \\
& \beta_{c}=-\frac{\alpha_{T O T} \dot{\mathrm{~W}}_{z v}}{\sqrt{\dot{\mathrm{~W}}_{y}^{2}+\dot{\mathrm{W}}_{z}^{2}}} \tag{6}
\end{align*}
$$

where $\dot{\mathrm{W}}_{y}$ and $\dot{\mathrm{W}}_{z}$ are projections of current apparent acceleration in velocity coordinates, $\alpha_{\text {тот }}$ is total angle of attack, $\alpha_{\text {Тот }}=\sqrt{\alpha^{2}+\beta^{2}}$.

## 3. TRADITIONAL SOLUTION TO COMMANDS OF EULER ANGLES

The transformation matrix between launch coordinate $(L)$, velocity coordinate $(V)$ and body coordinate $(B)$ can be expressed as

$$
\begin{equation*}
C_{L}^{B}\left(\varphi_{c}, \psi_{c}, \gamma_{c}\right)=C_{V}^{B}\left(\alpha_{c}, \beta_{c}\right) C_{L}^{V} \tag{7}
\end{equation*}
$$

Note Eqn (7), after the derivation of $\alpha_{c}$ and $\beta_{c}$, the key step in the solution to commands of Euler-angle body attitude $\psi_{c}, \theta_{\mathrm{c}}, \phi_{\mathrm{c}}$ (yaw command, pitch command, roll command) is the transformation matrix from launch coordinate to velocity coordinate $C_{L}^{V}$. The traditional solution strategy is

$$
\begin{equation*}
C_{L}^{V}=R_{x}(v) R_{z}(\sigma) R_{y}(\theta) \tag{8}
\end{equation*}
$$

$C_{L}^{V}$ is achieved through rotation by heading angle $\xi$, flight path angle $\gamma$, and bank angle $\mu$ in the sequence Z-Y-X(3-2-1).

Based on the velocity projection in launch coordinate $v_{L x}, v_{L y}, v_{L z}$ and the norm of velocity vector $V_{M}$, heading angle $\xi$ and, flight path $\gamma$ are expressed as

$$
\begin{align*}
& \xi=\arcsin \left(-v_{L y} / V_{M}\right) \\
& \gamma=\arctan \left(v_{L z} / v_{L x}\right) \tag{9}
\end{align*}
$$

Based on the current attitude $\varphi, \psi, \gamma$ (yaw, pitch, roll) and $\theta, \sigma$, bank angle $v$ is expressed as follows ${ }^{17}$.
$\sin \beta=\cos \theta[\cos \gamma \sin (\psi-\sigma)+\sin \varphi \sin \gamma \cos (\psi-\sigma)]-\sin \theta \cos \varphi \sin \gamma$
$\sin \alpha=\{\cos \theta[\sin \varphi \cos \gamma \cos (\psi-\sigma)-\sin \gamma \sin (\psi-\sigma)]-\sin \theta \cos \varphi \cos \gamma\} / \cos \beta$
$\sin n=(\cos \alpha \sin \beta \sin \varphi-\sin \alpha \sin \beta \cos \gamma \cos \psi)+\cos \beta \sin \gamma \cos \varphi] / \cos \theta$
Note that solution to $v$ is complicated, firstly we should get the value of angle of attack $\alpha$ and sideslip angle $\beta$.

See Eqn (7), the dimension of transformation matrix is $3 \times 3$, three Eqns can be obtained, then $\varphi_{c}, \psi_{c}, \gamma_{c}$ are solved. The solution Eqn is described in reference ${ }^{1}$.

## 4. A NEW SOLUTION STRATEGY TO COMMANDS OF EULER ANGLES

Without the information of $\theta, \sigma, \nu$, the transformation matrix $C_{L}^{V}$ from launch coordinate to velocity coordinate can't be described directly. Introduction of auxiliary coordinate (A) is just to realize this transformation.

Based on the defined auxiliary coordinate (A) and target coordinate (T), the transformation matrix $C_{L}^{V}$ is expressed in this paper as

$$
\begin{equation*}
C_{L}^{V}=C_{F}^{V} C_{T}^{F} C_{L}^{T} \tag{11}
\end{equation*}
$$

From Eqn (10), we note that auxiliary coordinate (A) is transition between target coordinate ( T ) and velocity coordinate (V).

### 4.1 Transformation Matrixes

The transformation matrix $C_{L}^{T}$ from launch coordinate to target coordinate is

$$
\begin{equation*}
C_{L}^{T}=R_{y}\left(-\phi_{t}\right) R_{x}\left(\lambda_{t}-\lambda_{0}\right) R_{y}\left(\phi_{0}\right) R_{z}\left(A_{0}\right) \tag{12}
\end{equation*}
$$

Based on the definition of $\gamma_{D}$ and $\gamma_{T}$, transformation matrix $C_{F}^{V}$ from auxiliary coordinate to velocity coordinate is as follows

$$
C_{F}^{V}=R_{z}\left(\gamma_{T}\right) R_{y}\left(\gamma_{D}\right)=\left[\begin{array}{ccc}
\cos \gamma_{T} \cos \gamma_{D} & \sin \gamma_{T} & -\cos \gamma_{T} \sin \gamma_{D}  \tag{13}\\
-\sin \gamma_{T} \cos \gamma_{D} & \cos \gamma_{T} & \sin \gamma_{T} \sin \gamma_{D} \\
\sin \gamma_{D} & 0 & \cos \gamma_{D}
\end{array}\right]
$$

Rotate velocity coordinate by angle $\beta_{c}$ and $\alpha_{c}$ in the sequence $\mathrm{Z}-\mathrm{Y}(3-2)$, body coordinate is obtained. So the transformation matrix $C_{V}^{B}$ from velocity coordinate to body coordinate is

$$
C_{V}^{B}=R_{y}\left(\alpha_{C}\right) R_{z}\left(\beta_{C}\right)=\left[\begin{array}{ccc}
\cos \alpha_{C} \cos \beta_{C} & \cos \alpha_{C} \sin \beta_{C} & -\sin \alpha_{C}  \tag{14}\\
-\sin \beta_{C} & \cos \beta_{C} & 0 \\
\sin \alpha_{C} \cos \beta_{C} & \sin \alpha_{C} \sin \beta_{C} & \cos \alpha_{C}
\end{array}\right]
$$

### 4.2 Euler-angle Commands Conversion-based on Relative Position Information

After the solution of $C_{L}^{T}$ and $C_{F}^{V}$, to get $C_{L}^{V}$, the transformation matrix $C_{T}^{F}$ from target coordinate to auxiliary coordinate is significant, which have been mentioned frontally in Section 2.1

See Fig 4, and also note the definition of auxiliary coordinate(A) and target coordinate(T), $O_{1} z_{f}$ is parallel to $O_{T} z_{T}$ axis, $O_{1} x_{\mathrm{A}}$ is parallel to $x_{T} O_{1} z_{T}$ plane, so $x_{f} O_{1} y_{f}$ plane is parallel to $x_{T} O_{T} y_{T}$ plane.
$O_{2} O_{T}$ is the projection of line-of sight $O_{1} O_{T}$ on horizontal plane $x_{T} O_{T} z_{T}$, and $O_{1} x_{\mathrm{A}}$ is toward the target, so $O_{1} x_{\mathrm{A}}$ is parallel to $O_{2} O_{T}$. By translation of $O_{1} x_{\mathrm{A}}$ axis to target point $O_{T}$, note that $O_{T} x_{f}^{\prime}$ locates at the extension of $O_{2} O_{T}$, and $O_{T} x_{f}^{\prime}$ also locates in the horizontal plane $x_{T} O_{T} y_{T} . \theta_{r}$ is defined as the included angle between $O_{2} O_{T}$ and $O_{T} y_{T}$, then the included angle between $O_{T} x_{f}^{\prime}$ and $O_{T} x_{T}$ is $\pi / 2-\theta_{r}$.

Rotate about $O_{T} z_{T}$ axis by an angle $\left(\pi / 2-\theta_{\mathrm{r}}\right)$ clockwise, the transformation matrix $C_{T}^{F}$ from target coordinate to auxiliary coordinate is obtained.

$$
\begin{align*}
C_{T}^{F} & =R_{z}\left(-\left(3 \pi / 2-\theta_{r}\right)\right) \\
& =\left[\begin{array}{ccc}
\cos \left(\theta_{r}-3 \pi / 2\right) & \sin \left(\theta_{r}-3 \pi / 2\right) & 0 \\
-\sin \left(\theta_{r}-3 \pi / 2\right) & \cos \left(\theta_{r}-3 \pi / 2\right) & 0 \\
0 & 0 & 1
\end{array}\right] \\
& =\left[\begin{array}{ccc}
-\sin \theta_{r} & \cos \theta_{r} & 0 \\
-\cos \theta_{r} & -\sin \theta_{r} & 0 \\
0 & 0 & 1
\end{array}\right]  \tag{15}\\
& =\left[\begin{array}{ccc}
-x_{T} / \sqrt{x_{T}^{2}+y_{T}^{2}} & y_{T} / \sqrt{x_{T}^{2}+y_{T}^{2}} & 0 \\
-y_{T} / \sqrt{x_{T}^{2}+y_{T}^{2}} & -x_{T} / \sqrt{x_{T}^{2}+y_{T}^{2}} & 0 \\
0 & 0 & 1
\end{array}\right]
\end{align*}
$$

In Eqn (15), $x_{T}$ and $y_{T}$ are re-entry position projections in target coordinate, it's measured by complex navigation with radar seeker and IMU data.

Substituting Eqns (12),(13), and (15) into Eqn (11), we have

$$
\begin{aligned}
C_{L}^{V} & =C_{F}^{V} C_{T}^{F} C_{g}^{T} \\
& =R_{z}\left(\gamma_{T}\right) R_{y}\left(\gamma_{D}\right) R_{z}\left(\theta_{r}-3 \pi / 2\right) R_{y}\left(-\phi_{t}\right) R_{x}\left(\lambda_{t}-\lambda_{0}\right) R_{y}\left(\phi_{0}\right) R_{z}\left(A_{0}\right)
\end{aligned}
$$



Figure 4. Transformation from target coordinate(T) to auxiliary coordinate(A).

Substituting Eqns (16) and (14) into Eqn (7), the transformation matrix $C_{L}^{B}$ from launch coordinate to body coordinate can be written as

$$
\begin{equation*}
C_{L}^{B}=C_{V}^{B} C_{L}^{V}=\left[d_{i j(i=j=3)}\right] \tag{17}
\end{equation*}
$$

Commands of Euler angles $\left(\varphi_{c}, \psi_{c}, \gamma_{c}\right)$ are attitudes that we desire the vehicle to achieve. The transformation $C_{L}^{B}$ is achieved through rotation by Euler-angle commands $\varphi_{c}, \psi_{c}, \gamma_{c}$ in the sequence $\mathrm{Z}-\mathrm{Y}-\mathrm{X}(3-2-1)$.

$$
\begin{align*}
C_{L}^{B} & =R_{x}\left(\gamma_{C}\right) R_{z}\left(\psi_{C}\right) R_{y}\left(\varphi_{C}\right) \\
& =\left[\begin{array}{cc}
{\left[\begin{array}{cc}
\cos \varphi_{C} \cos \psi_{C} & \sin \psi_{C} \\
-\cos \varphi_{C} \sin \psi_{C} \cos \gamma_{C}+\sin \varphi_{C} \sin \gamma_{C} & \cos \psi_{C} \cos \gamma_{C} \\
-\cos \varphi_{C} \sin \psi_{C} \sin \gamma_{C}+\sin \varphi_{C} \cos \gamma_{C} & -\cos \psi_{C} \sin \gamma_{C} \\
-\sin \varphi_{C} \cos \psi_{C} \\
\sin \varphi_{C} \sin \psi_{C} \cos \gamma_{C}+\cos \varphi_{C} \sin \gamma_{C} \\
-\sin \varphi_{C} \sin \psi_{C} \sin \gamma_{C}+\cos \varphi_{C} \cos \gamma_{C}
\end{array}\right]}
\end{array}\right] \tag{18}
\end{align*}
$$

Compare Eqn (18) with Eqn (17), the commands of Eulerangle body attitude in launch coordinate are expressed as

$$
\begin{align*}
& \varphi_{c}=\left\{\begin{array}{cc}
\arctan \left(-d_{13} / d_{11}\right) & d_{11}>0 \\
\pi \cdot \operatorname{sgn}\left(-d_{13}\right)+\arctan \left(-d_{13} / d_{11}\right) & d_{11}<0 \\
\pi \cdot \operatorname{sgn}\left(-d_{13}\right) / 2 & d_{11}=0
\end{array}\right.  \tag{19}\\
& \psi_{c}=-\arcsin \left(d_{12}\right) \\
& \gamma_{c}=\arctan \left(-d_{32} / d_{22}\right)
\end{align*}
$$

## 5. RESULTS AND DISCUSSION

The latitudes and longitudes of launch point and target point are presumed. The seeker starts work when the relative distance between vehicle and target is short than 150 km . In the re-entry phase, the vehicle has no thrust, the body attitude is changed by the control moment produced by the actuator.

The initial flight parameters are listed in Table 1. Simulation is realized with Matlab/Simulink software, and the simulation finishes when the flight height of the vehicle is

Table 1. Initial flight parameters of re-entry vehicle (in launch coordinate)

| Reentry parameters | Values |
| :--- | :--- |
| Position $(\mathrm{m})$ | $1588616.63186,-135167.42081$, |
|  | 104991.51512 |
| Velocity $(\mathrm{m} / \mathrm{s})$ | $2369.47627,-2836.55286,-55.18684$ |
| Angular rates $(\mathrm{rad} / \mathrm{s})$ | $-0.0053890820,0.00484471501$, |
|  | -0.00268215946 |
| Euler angles (rad) | $-0.9315217715,0.07032788799$, |
|  | -0.02611680311 |

below zero.
Noise of IMU and radar seeker are considered in simulation, the measurement noise of radar seeker is shown in Table 2. Estimation error of relative position is quite small

Table 2. Seeker measurement noise (one $\sigma$ )

| Seeker |  |  |
| :---: | :---: | :---: |
| Relative <br> range | LOS angle along | LOS rate along |
| 0.8 m | $0.2^{\circ}$ | $0.2^{\circ} / \mathrm{s}$ |

using complex navigation ${ }^{9-13}$.

### 5.1 Convert $\alpha_{c}, \beta_{c}$ to $\varphi_{c}, \psi_{c}, \gamma_{c}$

Guidance commands of angle of attack and sideslip angle are generated in earlier section. The maximum available angle of attack is 25 degree, and actuator deflection is limited to $\pm 30^{\circ}$.

In re-entry phase, the guidance and control system start work when the dynamic pressure is above a preset-value.

The commands of Euler angles using the new conversion strategy are shown in Fig. 5.

### 5.2 Tracking Performance of $\alpha_{c}$ and $\beta_{c}$

Next we will apply this new guidance command conversion strategy to the re-entry guidance and control system design, which mainly comprises three parts; commands of angle of attack $\alpha_{c}$ and sideslip angle $\beta_{c}$; command conversion from $\alpha_{c}, \beta_{c}$ to $\varphi_{c}, \psi_{c}, \gamma_{c}$; develop autopilot with Euler-angle commands $\varphi_{c}, \psi_{c}, \gamma_{c}$.

Note that commands of angle of attack and sideslip angle are proper, such guidance command will guidance the vehicle to target point. Figure 6 shows the tracking performance of Euler-angle command $\psi_{c}, \theta_{c}, \phi_{c}$, Figure 7 shows the response to Euler-angle command $\varphi_{c}, \psi_{c}, \gamma_{c}\left(0.5^{\circ}\right)$. The autopilot developed with commands of Euler angles $\psi_{c}, \theta_{c}, \phi_{c}$ can track the commands of Euler angles steadily.

Whether the vehicle can track guidance commands of angle of attack $\alpha_{c}$ and sideslip angle $\beta_{c}$, the focus is that the new conversion strategy from $\alpha_{c}, \beta_{c}$ to $\varphi_{c}, \psi_{c}, \gamma_{c}$ is right. To evaluate the correctness of this new conversion strategy, the tracking performance of commands of angle of attack and sideslip angle should be analyzed.

The tracking performance of command of angle of attack is shown in Fig. 8, $\alpha_{c}$ is command of angle of attack, $\alpha$ is the


Figure 5. Commands of Euler angles. (a) command of pitch angle; (b)command of yaw angle; (c)command of roll angle.


Figure 6. Tracking performance with commands of Euler angles. (a) pitch command (b) yaw command and (c) roll command.


Figure 7. Response to commands of Euler angles: (a) in pitch, (b) in yaw, and (c) in roll.
actual angle of attack of re-entry vehicle.
The tracking performance of command of sideslip angle is shown in Fig. 9, $\beta_{c}$ is command of sideslip angle, $\beta$ is the actual sideslip angle of re-entry vehicle.

The re-entry vehicle tracks the guidance commands of angle of attack and sideslip angle steadily, the tracking error is within $\pm 15^{\circ}$. Considering that there is a time lag between real body attitude and guidance commands, so the real tracking error is far smaller than $\pm 15^{\circ}$, which can be shown assuredly in Figs. 8(a) and 9(a).


Figure 8. Tracking performance with command angle of attack; (a) curves of $\alpha$ and $\alpha_{c}$, (b) tracking error with $\alpha_{c}$.

(a)

(b)

Figure 9. Tracking performance with command of sideslip angle; (a) curves of $\beta$ and $\beta_{c}$,(b) tracking error with $\beta_{c}$.

Euler-angle commands $\psi_{c}, \theta_{c}, \phi_{c}$ are obtained with the new strategy, and autopilot developed with $\psi_{c}, \theta_{c}, \phi_{c}$ can track $\alpha_{c}, \beta_{c}$ steadily. So the conversion strategy from $\alpha_{c}, \beta_{c}$ to $\varphi_{c}, \psi_{c}, \gamma_{c}$ is right. The precise tracking assures precise implement of guidance command, then the vehicle will reach the target point steered by the guidance law precisely.

The re-entry position in target coordinate $\left(x_{T}, y_{T}, z_{T}\right.$ ) is shown in Fig. 10, and the impact point error is $(-5.168$, $0.58,3.839) \mathrm{m}$. Radar seeker and IMU noises are considered in simulation, so there must be small deviation of $x_{T}, y_{T}, z_{T}$ from the true value. The errors in $x_{T}, z_{T}$ will influence the conversion process, so Euler angle command is a little different from the idealistic value, then re-entry trajectory is slightly separated from idealistic trajectory. The reentry velocity, curves of $\gamma_{D}$ and $\gamma_{T}$ are shown in Figs. 11-13.

Small impact point error also indicates that the new


Figure 10. Position of reentry vehicle in target coordinate system.


Figure 11. Reentry velocity varying with time.


Figure 12. Curve of $\gamma_{D}$ varying with time.


Figure 13. Curve of $\gamma_{T}$ varying with time.
guidance command conversion strategy is right.

## 6 CONCLUSIONS

For re-entry vehicle, a new conversion strategy from commands of angle of attack and sideslip angle to commands of Euler angles is brought in. This strategy relies on the relative position and velocity measured by seeker, an auxiliary coordinate is established as a transition, transformation matrix $C_{L}^{V}$ from launch coordinate to velocity coordinate is solved in a novel way.

After commands of angle of attack $\alpha_{c}$ and sideslip angle $\beta_{c}$ are generated, commands of Euler angles $\psi_{c}, \theta_{c}, \phi_{c}$ can be obtained with this new conversion strategy.

The main calculation in this strategy is matrix multiplication, and this strategy doesn't rely on the information of $\theta, \sigma, \nu$, so the complicated calculation of $\theta, \sigma, \nu$ are avoided.

The correctness of the new conversion strategy from $\alpha_{c}, \beta_{c}$ to $\varphi_{c}, \psi_{c}, \gamma_{c}$ can be testified from two aspects.

1. On the premise that $\alpha_{c}, \beta_{c}$ are right and the autopilot can track commands of Euler angles $\varphi_{c}, \Psi_{c}, \gamma_{c}$ steadily, the re-entry vehicle can track commands of angle of attack
$\alpha_{c}$ and sideslip angle $\beta_{c}$ steadily and precisely.
2. The vehicle reaches the target point precisely, which indicates that the guidance command is implemented fully, and the vehicle approaches the target steered by the guidance law.

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## Contributors


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