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Effects of Velocity-slip and Viscosity Variation for Lubrication of Roller Bearings

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ABSTRACT

A generalised form of Reynolds equation for two symmetrical surfaces is derived considering velocity-slip at the bearing surfaces. This equation is applied to study the effects of velocity-slip for the lubrication of roller bearings under lightly loaded conditions. Expressions for the point of cavitation, load capacities, and coefficient of friction obtained are also studied theoretically for various parameters.

Keywords: Reynolds equation, velocity-slip, viscosity, roller bearings, cavitation, load capacity, coefficient of friction

NOMENCLATURE

F_{r1}	Frictional force
h_0	Minimum film thickness
h	Total film thickness
k	Ratio of the peripheral layers
a	Thickness of the peripheral layer
P	Hydrodynamic pressure
r	Radius of each cylinder
U	Rolling velocity of the cylinder
V	Squeezing velocity of the cylinder
W_x, W_z	Load components in x and z–directions
x, z	Cartesian coordinates
$(-x_1)$	Point of maximum pressure
x^*	Point of cavitation
μ	Viscosity of the base lubricant
η_1, η_2	Viscosities of the lubricant
$\bar{\beta}$	Dimensionless-slip parameter
λ & δ	Molecular mean free paths for gas lubrication

1. INTRODUCTION

Hydrodynamically lubricated cylindrical roller bearings are widely used in a variety of applications involving severe operating conditions of speeds, loads, etc. Consequently, considerable research effort has been made towards their analysis to develop a better understanding of the performance of such systems. The rollers have been selected for analysis since many real contacts in machinery can be adequately represented by equivalent cylinder-cylinder contact. Also, it operates at a very low coefficient of friction[1]. Martin analysed the roller bearings using deterministic approach[2]. Floberg studied the roller bearings using cavitation boundary condition [3].

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Sinha and Singh[4] considered lightly loaded bearings lubricated by non-newtonian fluids. Thermal effects on combined rolling and squeezing motion was also investigated by Prasad and Chabra [5]. The effects of couple stresses on the roller bearings were also considered by Sinha[6,7], et al. under the lightly and heavily loaded conditions.

In general, a small amount of additives were added to the lubricant to increase its efficiency. Usually, these additives are the long-chain organic compounds. It has been proved experimentally that the additives added to the base lubricant, attach themselves to the surface, and thus, the viscosity of the lubricant varies across, as well as along the film[8].

Less attention has been paid to study of effects of velocity-slip at the surface, although it may be of importance in the flow behaviour of gases and liquids, particularly, when the film is thin[9-11], the surface is smooth[12] and at the porous boundary [13-18]. In this study, the effects of velocity-slip and viscosity variation on roller bearings under lightly loaded conditions using cavitation boundary conditions has been discussed.

2. BASIC EQUATIONS

Consider the laminar flow of a fluid between two symmetric surfaces, whose physical configuration is shown in Fig.1. Considering the variation of fluid properties across as well as along the film thickness, the basic equations of motion and equation of continuity in their general form for a newtonian fluid can be written as

$$\rho \frac{Du}{Dt} = \rho X - \frac{\partial P}{\partial x} + \frac{2}{3}\frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\right\} + \frac{2}{3}\frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}\right)\right\} + \frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right\} + \frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right\}$$

$$\rho \frac{Dv}{Dt} = \rho Y - \frac{\partial P}{\partial y} + \frac{2}{3}\frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right)\right\} + \frac{2}{3}\frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z}\right)\right\} + \frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right\} + \frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right\}$$

$$\rho \frac{Dw}{Dt} = \rho Z - \frac{\partial P}{\partial z} + \frac{2}{3}\frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial w}{\partial z} - \frac{\partial u}{\partial x}\right)\right\} + \frac{2}{3}\frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial w}{\partial z} - \frac{\partial v}{\partial y}\right)\right\} + \frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right\} + \frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right\} \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \tag{2}$$

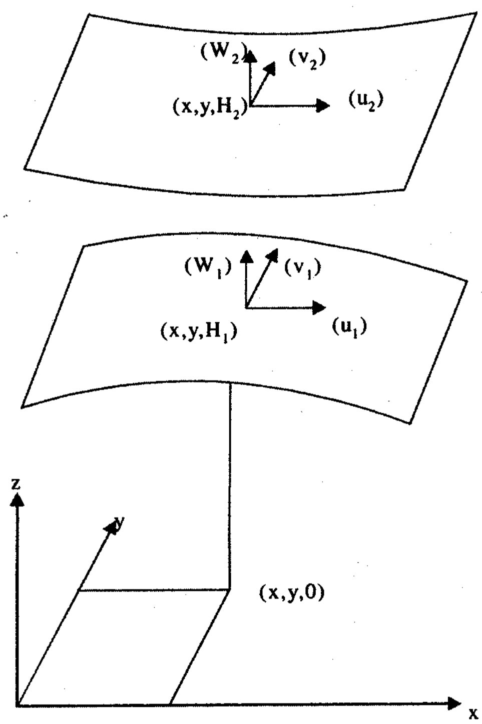

Figure 1. Coordinate system

with the following usual assumptions of lubrication theory:

(a) Inertia and body force values are negligible compared to the pressure and viscous values

(b) No variation of pressure across the fluid film, which means $\frac{\partial \rho}{\partial z} = 0$

(c) No slip in the fluid-solid boundaries

(d) No external forces act on the film

(e) The flow is viscous and laminar

(f) Due to the geometry of fluid film, the derivatives of u and v wrt z are larger than other derivatives of velocity components

(g) The height of the film h is very small compared to the bearing length 1. Typical value of h/l is about 10^{-3}.

The Navier–Stoke's Eqn (1) can be simplified as

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial z}\left[\eta \frac{\partial u}{\partial z}\right]$$

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial z}\left[\eta \frac{\partial v}{\partial z}\right] \quad (3)$$

where P = P (x,y) is the pressure in the film and η is the viscosity.

The boundary conditions considering velocity-slip at the surfaces[19] are:

$$u = (u)_1 = (\lambda_1)\left[\frac{\partial u}{\partial z}\right]_1 + U_1$$

$$v = (v)_1 = (\delta)_1\left[\frac{\partial v}{\partial z}\right]_1 + V_1 \qquad \text{at } z = H_1$$

$$u = (u)_2 = -(\lambda)_2\left[\frac{\partial v}{\partial z}\right]_2 + U_2$$

$$v = (v)_2 = -(\delta)_2\left[\frac{\partial v}{\partial z}\right]_2 + V_2 \qquad \text{at } z = H_2 \quad (4)$$

where $(\ )_1$, $(\ )_2$ denote the value at z = H, and $z = H_2$. Here λ's and δ's are molecular mean free paths for gas lubrication, and depend upon the lubricant's temperature, pressure, and viscosity. In liquid lubrication, λ and 6 depend on the viscosity and the coefficient is sliding friction. However, with porous bearings, λ and δ are functions of velocity-slip coefficient at the wall and the permeability parameter of the porous facing.

Integrating Eqn (3) and using boundary conditions [Eqn (4)], expressions for the fluid film velocities are obtained as

$$u = U_1 + \left[\alpha_1 H_1 + \int_{H_1}^{z}\frac{z\,dz}{\eta}\right]\frac{\partial P}{\partial x} + \left[\frac{U_2 - U_1}{F_0} - \frac{F_1}{F_0}\frac{\partial P}{\partial x}\right]\left[\alpha_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right] \quad (5)$$

$$v = V_1 + \left[\beta_1 H_1 + \int_{H_1}^{z}\frac{z\,dz}{\eta}\right]\frac{\partial P}{\partial y} + \left[\frac{V_2 - V_1}{F_0^1} - \frac{F_1^1}{F_0^1}\frac{\partial P}{\partial y}\right]\left[\beta_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right]$$

where

$$F_0 = \alpha_1 + \alpha_2 + \int_{H_1}^{z}\frac{dz}{\eta}, \quad F_0^1 = \beta_1 + \beta_2 + \int_{H_1}^{z}\frac{dz}{\eta}$$

$$F_1 = \alpha_1 H_1 + \alpha_2 H_2 + \int_{H_1}^{H_2}\frac{z\,dz}{\eta}$$

$$F_1^1 = \beta_1 H_1 + \beta_2 H_2 + \int_{H_1}^{z}\frac{z\,dz}{\eta}$$

$$\alpha_1 = \frac{(\lambda)_1}{(\eta)_1}, \alpha_2 = \frac{(\lambda)_2}{(\eta)_2}, \beta_1 = \frac{(\delta)_1}{(\eta)_1}, \beta_2 = \frac{(\delta)_2}{(\lambda)_2} \quad (6)$$

Integrating the equation of continuity [Eqn (2)] wrt z and taking limits from z = H_1 to $z = H_2$ gives:

$$\int_{H_1}^{H_2} \frac{\partial \rho}{\partial t} dz + \int_{H_1}^{H_2} \frac{\partial}{\partial x}(\rho u) dz$$

$$+\int_{H_1}^{H_2} \frac{\partial}{\partial y}(\rho v) dz + (\rho w)_{H_1}^{H_2} = 0 \tag{7}$$

The integrals of (ρu) and (ρv) are evaluated by partial **integration.** Introducing the expressions for (ρu) and (ρv) and their derivatives in Eqn (7) gives:

$$\frac{\partial}{\partial x}\left\{(F_2 + G_1)\frac{\partial P}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{(F_2^1 + G_1^1)\frac{\partial P}{\partial y}\right\}$$
$$= H_2\left\{\frac{\partial}{\partial x}(\rho u)_2 + \frac{\partial}{\partial y}(\rho v)_2\right\}$$
$$- H_1\left\{\frac{\partial}{\partial x}(\rho u)_1 + \frac{\partial}{\partial y}(\rho v)_1\right\}$$
$$- \frac{\partial}{\partial x}\left\{\frac{(U_2 - U_1)(F_3 + G_2)}{F_0} + U_1 G_3\right\}$$
$$- \frac{\partial}{\partial y}\left\{\frac{(V_2 - V_1)(F_3 + G_2^1)}{F_0^1} + V_1 G_3\right\} \tag{8}$$
$$+ \int_{H_1}^{H_2} \frac{\partial \rho}{\partial y} dz + (\rho w)_{H_1}^{H_2}$$

where

$$F_2 = \int_{H_1}^{H_2} \frac{\rho z}{\eta}\left[z - \frac{F_1}{F_0}\right] dz$$

$$F_2^1 = \int_{H_1}^{H_2} \frac{\rho z}{\eta}\left[z - \frac{F_1^1}{F_0^1}\right] dz, \quad F_3 = \int_{H_1}^{H_2} \frac{\rho z}{\eta} dz$$

$$G_1 = \int_{H_1}^{H_2}\left[z\frac{\partial \rho}{\partial z}\left\{\alpha_1 H_1 + \int_{H_1}^{z}\frac{z dz}{\eta} - \frac{F_1}{F_0}\left[\alpha_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right]\right\}\right] dz$$

$$G_1^1 = \int_{H_1}^{H_2}\left[z\frac{\partial \rho}{\partial z}\left\{\beta_1 H_1 + \int_{H_1}^{z}\frac{z dz}{\eta} - \frac{F_1^1}{F_0^1}\left[\beta_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right]\right\}\right] dz$$

$$G_2 = \int_{H_1}^{H_2}\left\{z\frac{\partial \rho}{\partial z}\left[\alpha_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right]\right\} dz$$

$$G_2^1 = \int_{H_1}^{H_2}\left\{z\frac{\partial \rho}{\partial z}\left[\beta_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right]\right\} dz$$

$$G_3 = \int_{H_1}^{H_3} z\frac{\partial \rho}{\partial z} dz \tag{9}$$

Equation (8) represents a generalised form of Reynolds equation for compressible fluid film lubrication considering velocity-slip at the bearing surfaces. The two sets of functions F and G depend upon the variation of fluid properties both along as well as across the film and on the velocity-slip conditions at the surfaces, i.e.

$$(\lambda)_1 = (\lambda)_2 = (\delta)_1 = (\delta)_2 = 0$$

$$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$$

The viscosity of the lubricant can vary across the film and may be different near the bearing surfaces owing to the reaction of additives and surfactants with the **surfaces**[20, 12, 8, 21-24].

Considering a reasonable case where the density and viscosity of the lubricant near the bearing surfaces may be different from the central region, one can have:

$$\rho = \rho_1(x,y), \quad \eta = \eta_1(x,y) \qquad H_1 \le z \le H_1 + h_1$$

$$\rho = \rho_2(x,y), \quad \eta = \eta_2(x,y) \qquad H_1 + h_1 \le z \le H_1 + h_1 + h_2$$

$$\rho = \rho_3(x,y), \quad \eta = \eta_3(x,y) \qquad H_1 + h_1 + h_2 \le z \le H_1 + h_1 + h_2 + h_3 \tag{10}$$

This introduces the concept of multiple layer lubrication. By taking

$$U_1 = U \qquad U_2 = V_1 = V_2 = 0$$

$$\alpha_1 = \beta_1, \quad \alpha_2 = \beta_2$$

$$\frac{\partial \rho_i}{\partial z} = 0 \qquad i = 1, 2, 3, ... \tag{11}$$

The generalised equation with velocity-slip reduces to the following form:

$$\begin{aligned}\frac{a}{\partial x}\left[F_2 \frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[F_2 \frac{\partial P}{\partial y}\right] &= H_2 \left\{\frac{\partial}{\partial x}(\rho u)_2 + \frac{\partial}{\partial y}(\rho v)_2\right\} \\ &- H_1 \left\{\frac{\partial}{\partial x}(\rho u)_1 + \frac{\partial}{\partial y}(\rho v)_1\right\} \\ &+ U \frac{\partial}{\partial x}\left[\frac{F_3}{F_0}\right] + [\rho w]_{H_1}^{H_2}\end{aligned} \tag{12}$$

where

$$F_0 = \alpha_1 + \alpha_2 + \frac{h_1}{\eta_1} + \frac{h_2}{\eta_2} + \frac{h_3}{\eta_3}$$

$$\begin{aligned}F_1 = \alpha_1 H_1 + \alpha_2 H_2 &+ \frac{h_1 (2H_1 + h_1)}{2\eta_1} \\ &+ \frac{h_2 (2H_1 + 2h_1 + h_2)}{2\eta_2} \\ &+ \frac{h_3 (2H_1 + 2h_1 + 2h_2 + h_3)}{2\eta_2}\end{aligned}$$

$$\begin{aligned}F_2 = \frac{\rho_1}{3\eta_1}&\left\{(H_1 + h_1)^3 - H_1^3\right\} \\ &+ \frac{\rho_2}{3\eta_2}\left\{(H_1 + h_1 + h_2)^3 - (H_1 + h_1)^3\right\} \\ &+ \frac{\rho_3}{3\eta_3}\left\{H_2^3 - (H_1 + h_1 + h_2)^3\right\} - \frac{F_1 F_3}{F_0}\end{aligned}$$

$$\begin{aligned}F_3 = \frac{\rho_1 h_1}{2\eta_1}&(2H_1 + h,) \\ &+ \frac{\rho_2 h_2}{2\eta_2}(2H_1 + 2h_1 + h_2) \\ &+ \frac{\rho_3 h_3}{2\eta_3}(2H_1 + 2h_1 + 2h_2 + h_3)\end{aligned}$$

$$(\rho u)_1 = \rho_1 \alpha_1 \left[H_1 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial x} + \rho_1 U \left[1 - \frac{\alpha_1}{F_0}\right]$$

$$(\rho u)_2 = -\rho_3 \alpha_2 \left[H_2 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial x} + \rho_3 U \frac{\alpha_2}{F_0}$$

$$(\rho v)_1 = \rho_1 \alpha_1 \left[H_1 - \frac{F_1}{F_0}\right]\partial y$$

$$(\rho v)_2 = -\rho_3 \alpha_2 \left[H_2 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial y} \tag{13}$$

$$\begin{aligned}[\rho w]_{H_1}^{H_2} &= (\rho u)_2 \frac{\partial H_2}{\partial x} + (\rho v)_2 \frac{\partial H_2}{\partial y} \\ &- (\rho u)_1 \frac{\partial H_1}{\partial x} - (\rho v)_1 \frac{\partial H_1}{\partial y} - V_s\end{aligned}$$

Here V_s is the resultant velocity of the film.

To see the effect of velocity-slip, three symmetrical incompressible layers between two solid boundaries are considered:

$$\eta_1 = r12 \qquad \rho_1 = P2 = P3$$

$$H, = 0 \qquad H_2 = (h + a) = h$$

$$h, = h_3 = a/2, \qquad h_2 = (h - a)$$

$$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1/\beta \tag{14}$$

The Reynolds equation applicable to this case can be written from Eqn *(12)* as follows:

$$\frac{\partial}{\partial x}\left[F_4 \frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[F_4 \frac{\partial P}{\partial y}\right] = U \frac{\partial}{\partial x}(h) - V \tag{15}$$

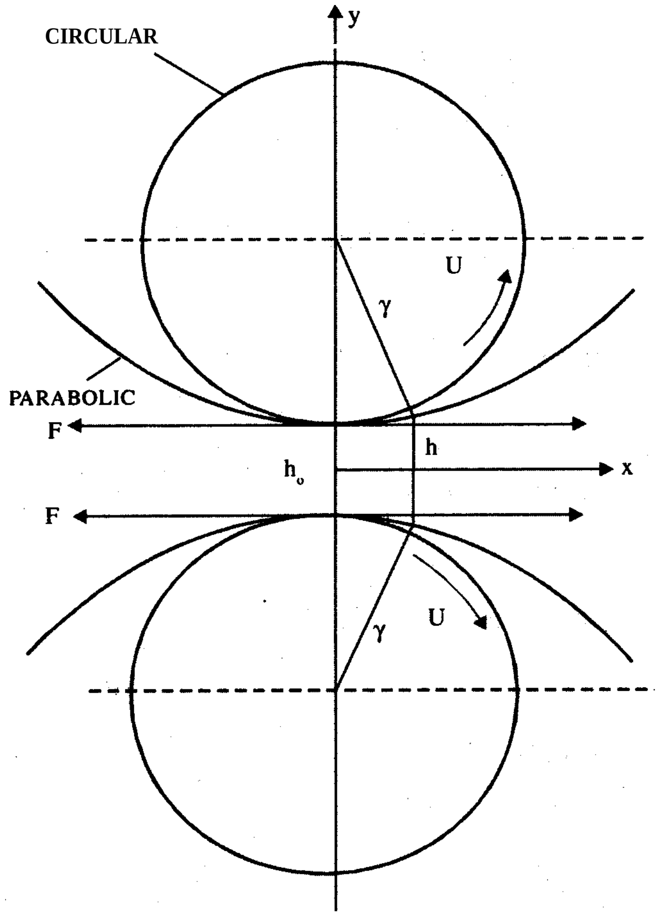

Figure 2. Roller bearing configuration

where

$$F_4 = \frac{(h-a)^3}{12\eta_2} + \frac{a^3 + 3a^2(h-a) + 3a(h-a)^2}{12\eta_1} + \frac{h^2}{2\beta}$$

taking $\beta = \frac{\eta_1}{\lambda}$ as the slip parameter.

3. LUBRICATION OF TWO SYMMETRICAL CYLINDERS UNDER LIGHTLY LOADED CONDITIONS

Consider the flow of an incompressible lubricant between two symmetrical rollers rotating with the same velocity U, as shown in Fig. 2.

The one-dimensional form of equation governing the pressure in the fluid film, taking $\eta_1 = kp$, $\eta_2 = \mu$ from the Eqn (15) is:

$$\frac{d}{dx}\left\{F_4 \frac{dP}{dx}\right\} = U \frac{d}{dx}(h) \tag{16}$$

where

$$F_4 = \frac{h^3}{12\mu}\left\{\frac{(1-a/h)^3(k-1)+1}{k} + \frac{6\mu}{h\beta}\right\} \tag{17}$$

here β represents the slip parameter, k the viscous layer parameter, a the thickness of the peripheral layer, h the total film thickness of the lubricant, and μ the viscosity of the middle layer.

The total film thickness h is given by

$$h = h_0 + \frac{h^2}{2R} \tag{18}$$

where R = $r/2$.

The boundary conditions for Eqn (16) considering the cavitation are:

$$p = 0 \qquad \text{at } x = -\alpha \tag{19}$$

$$\frac{dP}{dx} = 0 \qquad \text{at } x = -x_1 \tag{20}$$

$$P = 0, \qquad \frac{dP}{dx} = 0 \qquad \text{at } x = x^* \tag{21}$$

where $x = -x_1$ is the location of maximum pressure and $x = x^*$, the point at which cavitation starts.

Now integrating Eqn (16), and using the boundary conditions [Eqns (20) and (21)], one gets:

$$\frac{d\bar{P}}{dx} = 6\frac{(\bar{h} - \bar{h}^*)}{\bar{h}^3 \bar{F}_4} \tag{22}$$

where $$\bar{F}_4 = \left[\frac{(1-\bar{a}/\bar{h})^3(k-1)+1}{k} + \frac{6}{h\bar{\beta}}\right] \tag{23}$$

$$\bar{P} = \frac{P h_0^{3/2}}{2\mu U R^{1/2}} \qquad \bar{x} = \frac{x}{\sqrt{R h_0}} \qquad \bar{x}_1 = \frac{x_1}{\sqrt{R h_0}}$$

$$\bar{h} = \frac{h}{h_0} \qquad \bar{h}^* = \frac{h^*}{h_0} \qquad h = 1 + \frac{(\bar{x})^2}{2}$$

$$h^* = 1 + \frac{(\bar{x}_1)^2}{2} \qquad \bar{a} = \frac{a}{h_0} \qquad \bar{\beta} = \frac{h_0 \beta}{\mu}$$

Using the boundary conditions [Eqn (21)] in Eqn (22), one obtains:

$$x^* = x_1 \tag{24}$$

Thus, it may be noted that the point of cavitation and the point of maximum pressure lie equidistant from the point of minimum film thickness.

Integrating Eqn (22) and using the condition [Eqn (19)], one obtains:

$$\bar{P}(\bar{x}) = 6 \int_{-\alpha}^{\bar{x}} \frac{(\bar{h} - \bar{h}^*)}{\bar{h}^3 \bar{F}_4} \, d\bar{x} \tag{25}$$

Substituting boundary condition [Eqn (21)] in Eqn (25) one gets:

$$\int_{-\alpha}^{\bar{x}^*} \frac{(\bar{h} - \bar{h}^*)}{\bar{h}^3 \bar{F}_4} \, d\bar{x} = 0 \tag{26}$$

Equation (26) is used to determine the point $\bar{x} = \bar{x}_1^*$.

The load component along x–direction is given by

$$\bar{W}_x = -\int_{h_1}^{h_2} P\,dh - \int_{-\alpha}^{x_1} \frac{x^2}{2R} \frac{dP}{dx}\, dx \tag{27}$$

Substituting (22) in equation (27), one obtains:

$$\bar{W}_x = 3 \int_{-\alpha}^{x^*} \frac{(\bar{h} - \bar{h}^*)}{\bar{h}^3 \bar{F}_4} (\bar{x})^2 \, d\bar{x} \tag{28}$$

where

$$\bar{W}_x = \frac{W_x}{2\mu U} \left[\frac{h_0}{R}\right]^{1/2}$$

The load component (per unit width) in the z-direction is given by

$$W_z = \int_{-a}^{X} P\,dx \tag{29}$$

which on using the Eqn (25) in Eqn (29) yields:

$$\bar{W}_z = \frac{W_z h_0}{2\mu U R} = -6 \int_{-\alpha}^{x^*} \frac{(\bar{h} - \bar{h}^*)}{\bar{h}^3 \bar{F}_4} \bar{x}\, d\bar{x} \tag{30}$$

where

$$\bar{F}_4 = \left[\frac{(1 - \bar{a}/\bar{h})^3 (k-1) + 1}{k} + \frac{6}{\bar{h}\bar{\beta}}\right]$$

The frictional force on cylinder at z = H, is given by

$$F_{r1} = \int_{-\alpha}^{\alpha} \eta_1 \left[\frac{\partial u}{\partial z}\right]_{z=H_1=(-h/2)} dx \tag{31}$$

Integrating and using the condition $U_1 = U_2 = U$, one obtains:

$$F_{r1} = \frac{W_x}{2} \tag{32}$$

Similarly, the frictional force on the surface at $z = H_2$ is given by

$$F_{r2} = \frac{W_x}{2} \tag{33}$$

From the Eqns (28), (32), and (33), one gets:

$$\left(F_{r1} + F_{r2}\right) = \bar{W}_x \tag{34}$$

which is the equilibrium condition.

Equation (26) is solved numerically for x_1 and substituted in Eqns (28) and (30) to find $\bar{W}_x$ and $\bar{W}_z$.

4. RESULTS & DISCUSSION

4.1 Dimensionless Parameters

The bearing characteristics are dependent on the parameters $\bar{\beta}$, k and $\bar{a}$.

4.1.1 Velocity-slip Parameter

$\bar{\beta}$ represents the velocity-slip parameter. As $\bar{\beta}$ tends to infinity, it indicates velocity-no-slip at the surface and as $\bar{\beta}$ tends to zero, the velocity-slip becomes maximum, which means that as $\bar{\beta}$ increases, the velocity-slip decreases. So, the lower values of $\bar{\beta}$ indicate high velocity-slip and the higher values of indicate low velocity-slip.

4.1.2 Viscous Layer Parameter

It is mentioned earlier that the viscosity near the surface is different as compared to the viscosity of the middle layer. This is taken into account by the parameters k and $\bar{a}$. When k > 1, the viscosity near the periphery is more than the viscosity of the middle layer. k = 1 indicates that the viscosity is same everywhere.

When k < 1, the viscosity at the periphery is less than the viscosity of the middle region. Thus the difference in the viscosity of the middle and the peripheral regions is indicated by the parameter k.

Another parameter is $\bar{a}$ which indicates the thickness of the peripheral layer caused due to the presence of additives. When $\bar{a}$ = 0, there is no peripheral layer. As the peripheral layer thickness is small, normally there will exist small values of $\bar{a}$.

4.2 Bearing Characteristics

Equations (26), (28) and (30) are integrated numerically and appropriate graphs have been plotted with these parameters. The load capacities $\bar{W}_x$, and $\bar{W}_z$ are plotted with $\bar{\beta}$ for various values of k in the Figs 3 and 4. It has been found that these parameters increase rapidly as $\bar{\beta}$ increases up to certain level, and afterwards, the increase is very low and it becomes steady. This shows that the load capacities increase as the velocity-slip decreases, up to a certain point which become the result of no-slip case afterwards.

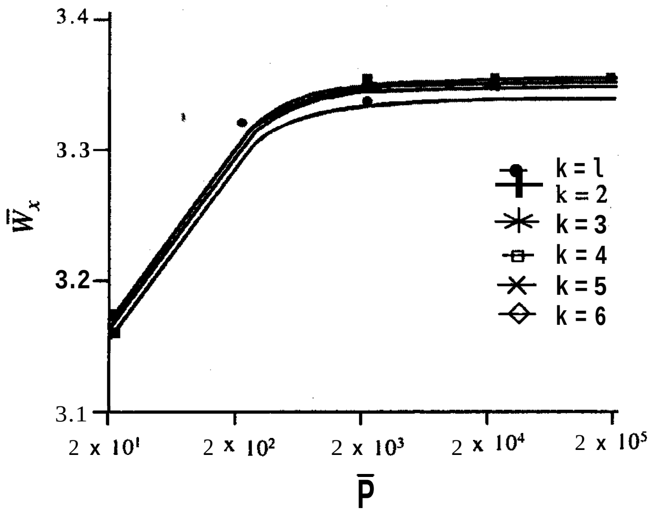

Figure 3. Variation of $\bar{W}_x$ versus $\bar{\beta}$ for various values of k.

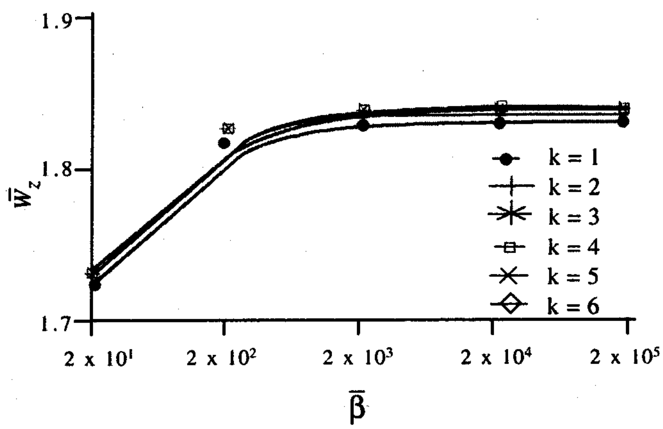

Figure 4. Variation of $\bar{W}_z$ versus $\bar{\beta}$ for various values of k.

It has also been observed that when $k > 1$, the load capacities increase due to the high viscous layer present near the periphery. This increase is more as its thickness increases and when $k < 1$, the load capacities decrease as $\bar{a}$ increases, the load capacities being more for higher values of k. Thus, when the viscosity of the peripheral layer is less than the viscosity of the middle layer, the load capacity decreases and this decrease is more as its thickness increases.

In Fig. 5, the coefficient of friction $\bar{C}_f$, is plotted with $\bar{\beta}$ for various values of k. It has been found that the coefficient of friction decreases as slip decreases. The values of the coefficient of friction are more for higher values of k, thus due to slip, coefficient of friction increases. For $k = 1$, which represents a single layer case, the coefficient of friction on $\bar{a}$ is zero.

For $k > 1$, it has been found that the coefficient of friction decreases as the thickness of the peripheral layer increases and for $k < 1$, the coefficient of friction increases as the thickness of the peripheral layer $\bar{a}$ increases and it is also more for higher values of k. In other words, the effect of high viscous layer near the periphery is to decrease the coefficient of friction and this decrease is more pronounced on the viscosity of peripheral layer, which is favourable for lubrication.

In Fig. 6, the point of cavitation, x^* is plotted with $\bar{\beta}$ for various values of k. It has been found that the point of cavitation decreases as the velocity-slip decreases up to a certain limit and becomes steady afterwards, i.e., if the velocity-slip increases, the point of cavitation moves away from the centre.

Thus when $k = 1$, the point of cavitation does not change as $\bar{a}$ increases. When $k > 1$, the point of cavitation decreases, as $\bar{a}$ increases and this decrease is more pronounced for the higher values of k, and when $k < 1$, the point of cavitation increases as $\bar{a}$ increases and this increase is more for the higher values of k. Thus, due to the presence of high viscous layer, the point of cavitation moves away from the centre and this is more pronounced as the thickness of the high viscous layer increases. The values of point of cavitation is also presented in Tables 1 and 2.

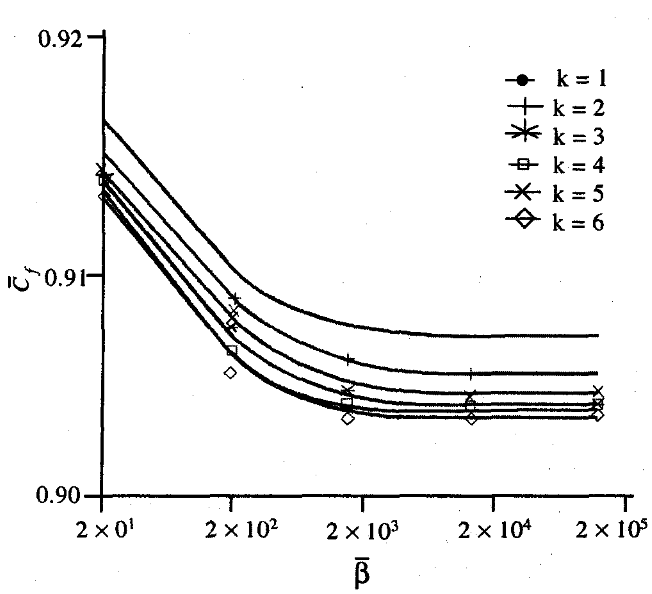

Figure 5. Variation of $\bar{C}_f$ versus $\bar{\beta}$ for various values of *k*.

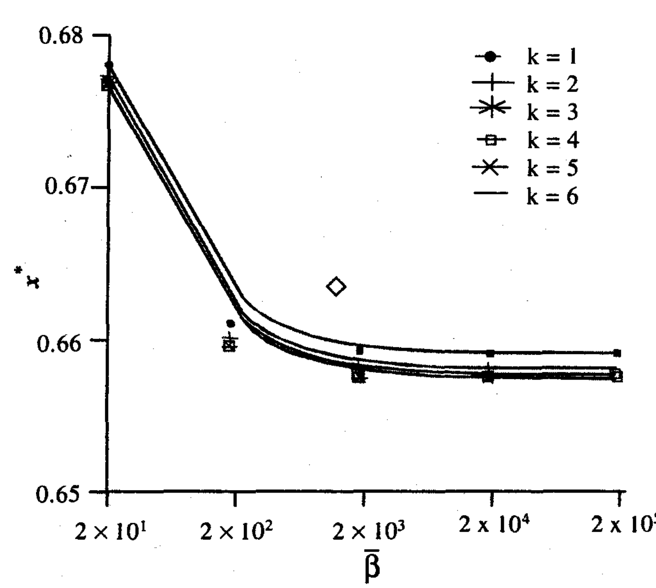

Figure 6. Variation of x^* versus $\bar{\beta}$ for various values of *k*.

Table 1. Position of the film rupture point x^* (cavitation point) for various values of $\bar{a}$ and k

k ↓ $\bar{a}$ →	0.1	0.3	0.5	0.7	0.9
0.2	0.71459970	0.74760740	0.74838870	0.73793940	0.72407230
0.4	0.68500980	0.70678720	0.71098630	0.70698250	0.69946300
0.6	0.67250980	0.68481450	0.68842780	0.68696300	0.68315440
0.8	0.66557620	0.67084970	0.67270510	0.67241220	0.67084970
1.0	0.66108410	0.66108410	0.66108410	0.66108410	0.66108410
2.0	0.65161140	0.63735350	0.62954100	0.62768560	0.63061530
3.0	0.64819340	0.62768560	0.61499030	0.61059560	0.61362310
4.0	0.64653320	0.62241210	0.60659190	0.59995120	0.60239260
5.0	0.64545910	0.61909180	0.60102540	0.59272460	0.59428710
6.0	0.64477550	0.61684570	0.59721680	0.58735350	0.58823250

Table 2. Position of the film rupture point x^* (cavitation point) for various values of $\bar{\beta}$ and k (k > 1)

k ↓ $\bar{\beta}$ →	2 x 10^1	2 x 10^2	2 x 10^3	2 x 10^4	2 x 10^5
1.0	0.67817390	0.66108410	0.65922850	0.65903320	0.65903320
2.0	0.67729500	0.66010740	0.65825190	0.65805660	0.65805660
3.0	0.67700200	0.65981440	0.65786130	0.65766600	0.65766600
4.0	0.67680670	0.65961920	0.65766600	0.65747060	0.65747060
5.0	0.67670910	0.65952150	0.65756830	0.65737300	0.65737300
6.0	0.67670910	0.65942380	0.65756830	0.65737300	0.65727540

5. CONCLUSION

A generalised form of Reynolds equation applicable to fluid film lubrication was derived considering the variation of fluid properties, both across and along the film thickness, with velocity-slip at the bearing surfaces. The effects of velocity-slip and viscosity variation for the lubrication of roller bearings under lightly loaded conditions have been studied. The beneficial result for hydrodynamic lubrication due to the presence of increased viscosity near the bearing surface was indicated. However, although, the effect of velocity-slip at the bearing is to decrease both the frictional force and the load capacity, the coefficient of friction increases, which leads to unfavourable results. For a gas-lubricated hydrostatic bearing, the gas film pressure and load decrease with increasing molecular mean free path.

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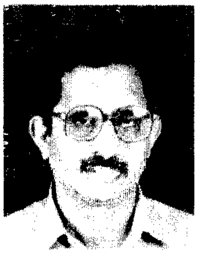

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