Prediction of Damping in Three-phase Fibre-reinforced Composites

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ABSTRACT

Fibre-reinforced composite materials are considered to comprise three components, i.e., fibre, matrix, and the interphase. The interphase is the part of the matrix placed in the vicinity of fibre surface all along its length and has distinct properties from that of the fibre and the matrix. In the present study, it is assumed that the interphase is homogeneous and isotropic. A 2-D and 3-D finite element modelling (micromechanical modelling) of the three-phase system is conducted to predict the four damping coefficients—longitudinal, transverse, longitudinal shear, and transverse shear loss factors. Effect of interphase loss factor in the range 0.002 to 0.500 and the interphase volume fraction on the loss factors have been studied.

Keywords: Fibre-reinforced composites, finite element modelling, damping, interphase, three-phase composites

1. INTRODUCTION

The three-phase model for fibre-reinforced materials account for the existence of a third phase interphase surrounding the fibres all along the length, having different mechanical properties than the properties of the two main constituent phases, i.e., the fibre and the matrix. Interphase may possess a reasonable thickness, and its properties are different from those of embedded fibres and bulk matrix. The nature of interphase and its properties in relation to that of fibre and matrix affects the mechanical properties, and in turn, the damping of the fibre-reinforced composites. Micromechanical methods/approaches adopted to model the composites with interphase are mostly based on the mechanics of material approach, method of cells, generic unit cell method, Airy's stress function micromechanics, Eshelby's/Mori-Tanaka modified approach, and finite element modelling (FEM). These models evaluate either the elastic moduli of three-phase composites, or the stress field within its constituents. Parametric studies for the interphase are related to the effect of the interphase material properties, interphase thickness, and fibre volume fraction on the elastic moduli of the composites.

The studies on the three-phase composites show that for the accurate prediction of effective
properties of composites, it is necessary to exactly characterise physical and elastic properties of the interphase region. Exact experimental data for the effective properties of the interphase in composites is not available. Interphase region is either considered homogeneous or heterogeneous while modelling it in the composites. A homogeneous interphase includes the properties of interphase either less than fibre and matrix, or intermediate to fibre and matrix. Heterogeneous interphase involves spatial variation of elastic properties of linear or parabolic variation type.

A few investigations related to the prediction of loss factor of three-phase composites are available in the literature. Chaturvedi and Tzeng12 developed three micromechanical models for the prediction of damping in aligned short fibre-reinforced three-phase polymer composites. The elastic modulus and damping properties of the interphase material appear to be dominant parameters, which significantly influence the dynamic stiffness and viscoelastic damping properties of the three-phase composite system. The main drawback of the model is that the input data for the interphase, i.e., geometric and material properties of interphase have been interpolated13 and no experimental values are available. Vantomme14 developed two-phase and three-phase model using an energy balance approach to obtain closed-form relationship between the material properties and the design parameters. Longitudinal, transverse, and shear loss factors for the composite materials (E-glass fibre epoxy) are predicted for each mode in terms of percentage contribution of damping due to fibre, matrix, and interphase. Finegan and Gibson15 proposed strain energy/finite element model to study the contribution of interphase to overall damping in coated fibre-reinforced composites.

In the present study, homogeneous interphase is considered in estimation of the damping of three-phase fibre-reinforced composites having unidirectional aligned fibres. Loss factors for longitudinal, transverse, longitudinal shear, and longitudinal shear loadings are predicted using 2-D and 3-D finite element models considering initially two types of interphases, one hard interphase and the other soft interphase. Parametric study for the soft interphase for interphase volume fraction over a wide range of interphase loss factors (0-0.5) is also conducted to predict their effect on the overall loss factor of the unidirectional fibre-reinforced composites.

2. FEM MODELLING WITH INTERPHASE

The presence of interphase between the fibre and the matrix is assumed in the formulation of finite element models to study its effect on various damping coefficients. Composite material is considered to comprise three phases, i.e., fibre, matrix, and interphase. The interphase is a part of the matrix placed in the vicinity of fibre surface all along its length, and has distinct properties from the fibre and the matrix. In the present work, it is assumed that the interphase is homogeneous and isotropic. Perfect bonding between fibre-interphase and interphase-matrix is assumed. In addition, fibre and matrix are also assumed to be homogeneous and isotropic. The survey of the work related to theoretical and experimental investigations on the role of interphase in composites shows that its elastic as well as dynamic properties are not standardised as in the case of fibre and matrix. Chandra16, et al., Tzeng13, Vantomme14 and, Chaturvedi and Tzeng13 used interpolated material properties of the homogeneous interphase in their studies to predict damping properties of three-phase composite system.

The strain energy method established by Ungar and Kerwin17 expresses for a given loading, the composite loss factor as the ratio of summation over all elements of the structure of the product of the loss factor for each element and the strain energy for each element to the total strain energy. Applying strain energy approach, static analysis of the unit cell can be used to evaluate the damping capacity of the composite from those of its constituents. The loss factor can be expressed as

\[ \eta = \frac{\sum_{i=1}^{n} \eta_i W_i}{\sum_{i=1}^{n} W_i} \]  

The loss factor for three-phase model, considering the effect of interphase, can be expressed as
The strain energy stored in the composite under loading can be written as

\[ \eta_c = \frac{\left( \eta_f W_f + \eta_m W_m + \eta_i W_i \right)}{W_c} \]  

The strain energy stored in the composite under loading can also be expressed as sum of the contributions from its constituents as

\[ W_c = W_f + W_m + W_i \]  

The strain energy stored in the constituents i.e., fibre, matrix, and interphase in a unit cell of a certain volume is given as

\[ W_f = \frac{1}{2} \sum \sigma_y^f (S_y^f) \delta V_f \]  

\[ W_m = \frac{1}{2} \sum \sigma_y^m (S_y^m) \delta V_m \]  

\[ W_i = \frac{1}{2} \sum \sigma_y^i (S_y^i) \delta V_i \]  

Percentage contribution of fibre = \( \frac{\eta_f W_f}{\eta_c W_c} \times 100 \)  

Percentage contribution of interphase = \( \frac{\eta_i W_i}{\eta_c W_c} \times 100 \)  

Percentage contribution of matrix = \( \frac{\eta_m W_m}{\eta_c W_c} \times 100 \)  

2.1 Finite Element Models with Interphase

The 2-D finite element models with interphase volume fraction, \( V_i = 0.02, 0.04, 0.08 \) and 0.10 for transverse and transverse shear loading with single-cell square array packing are analysed. Figures 1(a) and 1(b) show 2-D finite element models for transverse and transverse shear loadings, respectively. The models consist of 1-64 four-noded quadrilateral and three-noded triangular elements in fibre, 64-96 four noded elements in interphase, and 97-130 elements in matrix for both the transverse and the transverse shear loadings.

These models are constructed for fixed fibre volume fractions, \( V_f = 0.4 \), considering the interphase to be (i) hard: Interphase properties are taken as average of the elastic properties of the fibre and matrix; or (ii) soft: Interphase properties are lower than that of the matrix (Table 1). This assumption is made because a precise estimation of the properties of interphase is not available in the reported literature. Similarly, the loss factor for both the soft and the hard interphases is assumed as average of the loss factors of the fibre and the matrix.
The 3-D finite element models to predict loss factors in transverse and transverse shear modes are shown in Figs 2 and 3. Hexahedron eight-noded elements are used to construct the finite element model for fibre-reinforced composite with fibre volume fraction, $V_f = 0.4$ and interphase volume fraction, $V_i = 0.1$. Figures 2 and 3 also show the corresponding constraints and the loading for the particular mode for the single-cell quarter domain model. The model consists of 1-270 eight-noded elements of hexahedron-type in fibre, 271-510 elements in interphase, and 511-690 elements in matrix for both the transverse and the transverse shear loadings.

The total thickness of the interphase ($T_i$) is so worked out that the maximum value of $V_i$ is 0.1, and it is further subdivided into number of layers to obtain the $V_i = 0.02, 0.04, 0.08$. Four cases with different $V_i$ shown in Table 2 have been studied. Finer mesh size is used in the interphase near the fibre as well as near the matrix to correctly predict stresses in interphase region. The details of the several 3-D finite element models so constructed are given in Table 2.

### Table 1. Basic properties of glass fibre-reinforced epoxy (GFRP) constituents: Three-phase composite

<table>
<thead>
<tr>
<th>Properties</th>
<th>E-glass fibre</th>
<th>Epoxy matrix</th>
<th>Hard interface</th>
<th>Soft interface</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (Gpa)</td>
<td>72.4000</td>
<td>2.760</td>
<td>37.5800</td>
<td>0.5000</td>
</tr>
<tr>
<td>$G$ (Gpa)</td>
<td>30.2000</td>
<td>1.020</td>
<td>15.6100</td>
<td>0.1780</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.2000</td>
<td>0.350</td>
<td>0.2040</td>
<td>0.4000</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.0018</td>
<td>0.015</td>
<td>0.0084</td>
<td>0.0084</td>
</tr>
</tbody>
</table>

### Table 2. Details of 3-D FEM models with interphase for $V_f = 0.4$ (fixed)

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Fibre diameter ($\mu$m)</th>
<th>$V_i$</th>
<th>$T_i$ ($\mu$m)</th>
<th>Number of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Fibre</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($N_f$)</td>
</tr>
<tr>
<td>Case 1</td>
<td>9.144</td>
<td>0.10</td>
<td>0.736808</td>
<td>1-270</td>
</tr>
<tr>
<td>Case 2</td>
<td>9.144</td>
<td>0.08</td>
<td>0.597854</td>
<td>1-270</td>
</tr>
<tr>
<td>Case 3</td>
<td>9.144</td>
<td>0.04</td>
<td>0.308091</td>
<td>1-270</td>
</tr>
<tr>
<td>Case 4</td>
<td>9.144</td>
<td>0.02</td>
<td>0.156555</td>
<td>1-270</td>
</tr>
</tbody>
</table>

Both soft and hard interphases are incorporated in the finite element models given in Figs 2 and 3.
The output of the static analysis of these finite element models is obtained in the form of strain energy for the finite elements of fibre, matrix, and interphase in the respective modes of loadings. The strain energy of the constituent elements is used in Eqn (2) and (4) to predict corresponding loss factors.

These 3-D FEM models were also analysed for other loading conditions, i.e., longitudinal and longitudinal shear by replacing the nodal loads to extensional and longitudinal shear in the models shown in Figs 2 and 3, respectively. During the analysis, the loads were so selected that the stresses remain within the elastic limits.

Figure 2. Three-dimensional FEM model for composite with interphase: Transverse loading for $V_f = 0.4$ (fixed), $V_i = 0.1$, $N_f = 1-270$, $N_i = 271-510$, and $N_m = 511-690$. 
3 RESULTS & DISCUSSION

The loss factors $\eta_{11}$, $\eta_{22}$, $\eta_{33}$, and $\eta_{44}$ are evaluated for fixed fibre volume fraction ($V_f = 0.4$) by changing the interphase volume fraction between the range 0.02 and 0.10.

Figure 4 shows variation of longitudinal loss factor $\eta_{11}$ for composites with both hard and soft interphases. The trend for the loss factor $\eta_{11}$ for the composite predicted by FEM/strain energy approach with soft interphase is similar to the one obtained by Vantomme model, although it is on the higher side in case of FEM approach. Similarly for the composite with hard interphase, there is an increasing trend of $\eta_{11}$ in both the approaches.

Difference in loss factors predicted by FEM approach and Vantomme model is due to the fact that the former is based on the condition of variable stress distribution within composite, whereas the later is based on uniform state of stress. The magnitude of $\eta_{11}$ is higher by FEM approach. Further, overall increase in $\eta_{11}$ is only marginal when compared to the value for the case without interphase. This shows that $\eta_{11}$ is not much affected by the presence of interphase in the composite. The percentage contribution of individual constituents, i.e., fibre, matrix, and interphase to $\eta_{11}$ wrt interphase volume fraction is shown in Fig. 5. The contribution of fibre, matrix, and interphase to $\eta_{11}$ is maximum to minimum for the soft interphase. The percentage...
The contribution of soft interphase increases only marginally (from about 6% to 10%) with increase in $V_i$ from 0.02 to 0.10. On the other hand, percentage contribution of hard interphase to $\eta_{11}$ increases from about 10 per cent to 40 per cent as $V_i$ increases from 0.02 to 0.10. For the hard interphase, the contribution of matrix to overall damping decreases appreciably from 37 per cent to 20 per cent approximately with an increase in $V_i$ from 0.02 to 0.10. In hard interphase, the contribution of fibres is maximum and the contribution of interphase exceeds that of the matrix for values of $V_i$ greater than 0.06.

Figure 6 shows comparison of loss factor $\eta_{22}$ for FEM and Vantomme models considering the composite with soft and hard interphases. The presence of hard interphase does not have any effect on $\eta_{22}$ with the increase in $V_i$. In composite with soft interphase, $\eta_{22}$ decreases as $V_i$ increases. The values of $\eta_{22}$ predicted by two models are slightly different. Both the models, however, predict the same trend, i.e., no change in $\eta_{22}$ for hard interphase and continuous and gradual decrease for soft interphase with an increase in $V_i$.

Figure 7 shows the percentage contribution of the constituents to the loss factor $\eta_{22}$ as function of $V_i$. Contribution of soft interphase to $\eta_{22}$ shows increasing trend i.e., (from 11.6 to 22% corresponding to $V_i = 0.04$ to 0.10. Hard interphase has an insignificant contribution in the entire range of $V_i$ up to 0.1. As expected major contribution to $\eta_{22}$ is from the matrix, which is as high as 98-99 per cent for hard interphase and 88-78 per cent for soft interphase with interphase volume fraction varying in the range 0.04-0.10. Also the contribution...
from fibre, for both the cases of soft and hard interphases, is negligibly small.

Loss factor $\eta_{12}$ of three-phase composite is predominantly dependent upon the status of interphase as is clear from the Fig. 8. It is observed from the Fig. 8 that $\eta_{12}$ for composite with soft interphase decreases appreciably with an increase in $V_I$ as compared to the case of hard interphase. A marginal reduction in $\eta_{12}$ with hard interphase is due to small variation in the strain energy component of the interphase region. It may be noted that the overall loss factor of composite from FEM model

![Diagram](soft_interphase.png)

**Soft Interphase**
- Fibre
- Interphase
- Matrix

![Diagram](hard_interphase.png)

**Hard Interphase**
- Fibre
- Interphase
- Matrix

Figure 8. Effect of modulus and volume fraction of interphase on $\eta_{12}$.

$V_I$ increases from 0.02 to 0.10, when the composite with soft interphase is considered. This happens because the strain energy within the constituents

![Diagram](fig7.png)

**Figure 7. Percentage contribution of constituent of composite to $\eta_{12}$.**

![Diagram](fig9.png)

**Figure 9. Percentage contribution of constituents of composite to $\eta_{12}$.**

is dependent on the assumed values of interphase properties, i.e., loss factor and elastic modulus. Percentage contribution of fibre, interphase, and matrix to $\eta_{12}$ and their dependence on interphase volume fraction is shown in Fig. 9. Contribution of fibre to $\eta_{12}$ is negligible for both cases of hard and soft interphases. Percentage contribution of interphase increases in the range 13 to 29 per cent as
gets redistributed with soft interphase. Consequently, the contribution of the matrix to $\eta_{12}$ decreases with interphase $V_i$. It may be noted that Vantomme model is not capable of predicting $\eta_{12}$.

Loss factor $\eta_{23}$ is evaluated for the fibre-reinforced composite with soft and hard interphases, considering the interphase volume fraction in the range 0.02 to 0.10. Loss factor $\eta_{23}$ for composite with soft interphase predicted using 2-D and 3-D finite element models has been compared in Fig. 10.

Results from both models show good correlation, indicating that predictions made are independent of type of finite element model. There is a slight decrease in $\eta_{23}$ with increase in $V_i$.

![Figure 10. Effect of volume fraction of interphase on $\eta_{23}$](image)

Figure 10 shows that the interphase contribution by 3-D model is 13-27 per cent for $V_i$ from 0.02 to 0.10. Contribution of matrix to $\eta_{23}$, as predicted by 3-D model, varies between 85.5 per cent to 72.0 per cent over $V_i$ ranging 0.02 to 0.10. The contribution from fibre is marginal, i.e., of the order of few per cent. Results clearly show a gradual increase in the percentage contribution to $\eta_{23}$ from interphase, as it is increased from 0.02 to 0.1. This is at the expense of corresponding decrease in the contribution of matrix. Some difference in the percentage contributions to $\eta_{23}$ of various constituents is observed by 2-D and 3-D modelling. The 3-D modelling predicts a higher contribution of interphase as compared to 2-D modelling. Results from 3-D modelling are more realistic, since the corresponding stress field is more realistic. Comparison of loss factor $\eta_{23}$ for composite with hard interphase predicted using 2-D and 3-D models (Fig. 12) shows marginal difference. A gradual decrease in the value of $\eta_{23}$ is predicted with increasing $V_i$. Similarly, contributions of individual constituents of composite to loss factor $\eta_{23}$ by both the 2-D and the 3-D models (Fig. 13) compare well. Contribution of hard interphase varies approximately from 2 per cent to 6 per cent for $V_i$ varying in the range 0.02 to 0.10. Comparing with results of Fig. 11, it is clearly seen that the contribution of hard interphase to matrix is much smaller compared to that of soft interphase. Major contribution to $\eta_{23}$
Effect of interphase (soft) loss factor on the loss factor of three-phase composite has been studied. The interphase loss factor is varied over a wide range, from 0.002 to 0.500. The effect of this variation of $\eta_i$ can be better understood in two different ranges: 0.002 to 0.015 (from a low value to that of matrix loss factor) and 0.015 to 0.500 (higher than the matrix loss factor). Results showing the variation of loss factor as function of $\eta_i$ for various loadings are presented for $V_i = 0.02, 0.04,$ and 0.10 in Figs 14 to 17.
and the loss factor of the composite corresponds to that of a two-phase model. For \( \eta > 0.015 \), the value of \( \eta \) for all the loading conditions increases in the increasing order of \( V_i = 0.02, 0.04, \) and 0.10, whereas this trend reverses when \( \eta_i < 0.015 \). Further during the study, it was observed that in all the cases of loadings, the loss factor increases rapidly for higher values of \( \eta_i \) (i.e., 0.1 to 0.5).

4 CONCLUSIONS

Damping coefficients for fibre-reinforced composites with hard and soft interphases in longitudinal, transverse, longitudinal and transverse shear loading conditions are predicted using finite element models/strain energy approach, employing both the 2-D and the 3-D finite element models. A parametric study of soft interphase considering the effect of interphase loss factor and volume fraction, both varied over a wide range, on loss factor of three-phase composite has also been conducted. A comparison of results from the present model with those existing in the literature has been made for longitudinal and transverse loss factors.

The longitudinal loss factor shows increasing trend for composites with both hard and soft interphases, whereas transverse, transverse shear, and longitudinal shear loss factors show a decreasing trend in case of soft interphase wrt interphase volume fraction. Percentage contribution of hard interphase is more than the soft interphase to the longitudinal loss factor. For all the other three loss factors, i.e., transverse, transverse shear, and longitudinal shear, soft interphase has larger contribution as compared to hard interphase.

Perceptible increase in loss factors under all loadings is observed in loss factors under different loadings with increase in interphase volume fraction when interphase loss factor is higher than matrix loss factor. However loss factors decrease with increase in interphase volume fraction for the condition, when interphase loss factor is less than matrix loss factor.

Thus by optimising the properties/parameters of interphase (i.e., moduli, loss factors/size and
physical state) appropriate level of damping can be achieved in fibre-reinforced composites under different conditions. However, it requires an exact characterisation of properties and parameters of the interphase in the fibre-reinforced composites.

REFERENCES


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