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Dynamic Analysis of Offshore Spar Platforms

A.K. Jain and A.K. Agarwal

Indian Institute of Technology Delhi, New Delhi – 110 016

ABSTRACT

Offshore spar platform is a compliant offshore floating structure used for deep water applications for drilling, production, processing, storage and offloading of ocean deposits. The offshore spar platform is modelled as a rigid body with six degrees-of-freedom (DOFs), connected to the sea bed by multicomponent catenary cables which consist of a mooring line, a clump weight, and an anchor line attached to the fairleads. The response-dependent stiffness matrix consists of three parts: (i) the restoring hydrostatic force, and the stiffness due to cables. (ii) nonlinear horizontal springs, and (iii) nonlinear vertical springs. A unidirectional regular wave model is used for computing the incident wave kinematics by Airy's wave theory and hydrodynamic force by Morison's equation. The response analysis is performed in time domain using the iterative incremental Newmark's method. Numerical studies have been conducted for sea state conditions with and without coupling of DOFs.

Keywords: Offshore floating structure, spar platforms, hydrodynamic forces, floating structures, rigid bodies

NOMENCLATURE

$\{X\}$, $\{\dot{X}\}$, $\{\ddot{X}\}$	Structural displacement, velocity, and acceleration vector, respectively
$[M]$	Mass matrix $\{[M^{spar}] + [M^{added\ mass\ (am)}]\}$
$[K]$	Stiffness matrix $\{[K^{hydrostatic\ (hs)}] + [K^{horizontal\ spring\ (hs)}] + [K^{vertical\ spring\ (vs)}]\}$
$[C]$	Damping matrix
$\{F(t)\}$	Hydrodynamic force vector
ρ_w	Mass density of the fluid
D	Diameter of offshore spar platform
C_d, C_m	Drag and inertia coefficients, respectively
$\{\dot{u}\}$, $\{\ddot{u}\}$	Velocity and acceleration of the fluid, respectively
t, Δt	Time instant and increment in time, respectively.

1. INTRODUCTION

Many innovative floating structures have been proposed for cost-effectiveness of oil and gas exploration and production in water depths exceeding one thousand meters. Offshore spar platform is one such compliant offshore floating structure used for deep water applications–for drilling, production, processing, storage, and offloading of ocean deposits. It consists of a cylinder which floats vertically in the water. The structure floats at such a depth in the water that the wave action at the surface is dampened

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by the counterbalance effect of the structure weight. Fin like structures, called strakes, attached in a helical fashion around the exterior of the cylinder, act to break the water flow against the structure, further enhancing the stability. Station-keeping is provided by lateral multicomponent catenary cables attached to the hull near its centre of pitch for low dynamic loading. Most of the researchers have modelled the cables for offshore spar platform with multilinear segment for the force-excursion relationship for horizontal excursion of cables with only two and three DOFs, and no studies have been reported for the nonlinear force-excursion relationship for both horizontal and vertical excursions of cables with six DOFs. Keeping this in view, this study considers nonlinear force-excursion relationship for both horizontal and vertical excursions of the cables with six DOFs.

Some of the features of offshore spar platforms are:

- It can be operated till 3000 m depth of water for full drilling and production-to-production only.
- It is always stable because the centre of buoyancy is above the centre of gravity.
- It can be used as a mobile drilling rig.
- It has sea-keeping characteristics superior to all other mobile drilling units.
- The cable system is easy to install, operate, and relocate the risers.
- Drilling units are protected inside the offshore spar platform.
- Sea motion inside the offshore spar platform centre well would be minimal.

The concept of offshore spar platform as an offshore structure is not new. The offshore spar platform, buoy-type of structure, has been built in ocean before also. The Brent offshore spar platform was built by the Royal Dutch shell as a storage and offloading platform in the North Sea at intermediate water depth (Bax and de Werk[1], Van Santen and de Werk[2]). The use of offshore spar platform as a production platform is relatively recent. Glanville[3], *et al.* gave details of the concept, construction, and installation of offshore spar platform. Mekha[4], *et al.* modelled the offshore spar platform with three DOFs, i.e., surge, heave, and pitch, and used constant inertia coefficient, C_m, as in the standard Morison's equation, or used a frequency-dependent C_m coefficient based on the diffraction theory. The drag forces were computed using the nonlinear term of Morison's equation in both the cases. The analysis was performed in time domain. Halkyard[5] reviewed the status of several offshore spar platform concepts, emphasising the design aspect of these platforms. Ran and Kim[6] studied the nonlinear response characteristics of a tethered/moored offshore spar platform in regular and irregular sea waves. An efficient global coordinate-based dynamic finite-element program was developed to simulate the nonlinear tether/mooring responses. Fischer and Gopalkrishnan[7] highlighted the importance of heave characteristics of offshore spar platforms. Chitrapu[8], *et al.* studied the nonlinear response of an offshore spar platform under different environmental conditions, such as regular sea state, bi-chromatic, random waves, and water current using a time domain simulation model. The model can consider several nonlinear effects. Hydrodynamic forces and moments were computed using the Morison's equation.

2. STRUCTURAL MODEL

The offshore spar platform is modelled as a rigid cylinder with six DOFs [i.e., three displacement DOFs (surge, sway and heave along X, Y and Z- axes) and three rotational DOFs (roll, pitch and yaw about X, Y and Z-axes)] at its centre of gravity. The offshore spar platform is assumed to be closed at its keel. The stability and stiffness to the offshore spar platform is provided by a number of cables attached near the centre of gravity for low dynamic positioning of the offshore spar platforms. When the offshore spar platform deflects, the movement takes place in a plane of symmetry of the cable system and the resultant horizontal and vertical forces also occur in this plane and the behaviour is two-dimensional. It is the force and the displacement (excursion) at this attachment point that is of fundamental importance for the overall analysis of the platforms. It is assumed that the offshore spar platform is connected to the sea bed by four multicomponent catenary cables placed perpendicular to each other, and which are attached to the offshore spar platform at the fairleads. The force-excursion relationship is nonlinear and requires an iterative solution. The development of offshore spar platform model for dynamic analysis involves the formulation of a nonlinear stiffness matrix considering cable tension fluctuations due to variable buoyancy and other response-dependent nonlinearities. Figure 1 shows a schematic elevation of offshore spar platform.

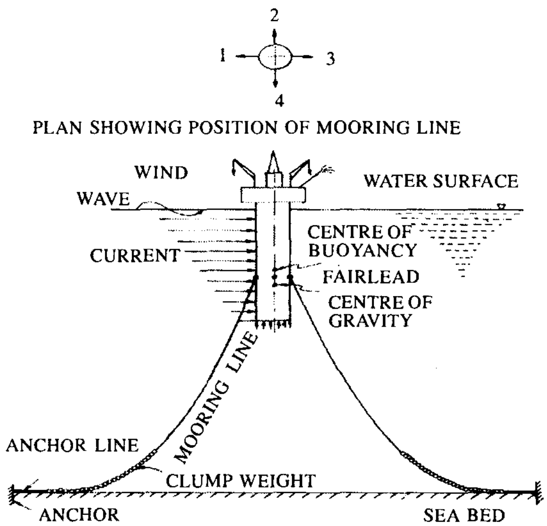

Figure 1. Schematic elevation of offshore spar platform.

3. ASSUMPTIONS & IDEALISATION

The offshore spar platform and the cable system are treated as a single unit, and the analysis is carried out for the six DOFs under the environmental loads. The following assumptions have been made in the analysis:

- Initial pretension in all cables is equal. However, total pretension changes with the motion of the offshore spar platform, change in pretension in cables is calculated at each time step, and writing the equation of equilibrium at that time step modifies the elements of the stiffness matrix.
- All the components of the cables move slowly inside the water, so that the generated drag forces on the line, due to the motion, can be treated as negligible.
- The clump weight for the cable segment is inextensible.
- Anchor point does not move in any direction, and both the horizontal and the vertical excursions of the catenary cable are considered.
- Change in pretension in cable system is calculated at each time step and the difference is considered with the force vector, keeping the stiffness matrix constant for each time period cycle of a regular sea wave.

4. EQUATION OF MOTION OF OFFSHORE SPAR PLATFORM

The equation of motion of the offshore spar platform under regular sea wave is:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{F(t)\} \qquad (1)$$

The mass matrix represents the total mass of the offshore spar platform, including the mass of the soft tanks, hard tanks, deck, ballast, and the entrapped water. The added mass matrix is obtained by integrating the added mass term of Morison's equation along the submerged draft of the offshore spar platform. Mass is taken as constant and it has been assumed that the masses are lumped at the centre of gravity. The structural damping matrix is taken to be constant and is dependent on the mass and initial stiffness of the structure. The stiffness matrix consists of three parts: (i) the restoring hydrostatic force and the stiffness due to cables, (ii) nonlinear horizontal springs, and (iii) nonlinear vertical springs. In this study, the cable is modelled as nonlinear, horizontal and vertical springs located at the fairleads along the offshore spar platform centre with no hydrodynamic forces applied on these. The coefficients of the stiffness matrix have nonlinear terms. Further, the cable tension changes due to the motion of the offshore spar platform in different DOFs make the stiffness matrix response-dependent.

5. HYDRODYNAMIC FORCE VECTOR

A unidirectional regular wave model is used for computing the incident wave kinematics. The kinematics of the water particles has been evaluated using Airy's wave theory. A simplified alternative proposed in this study is to predict the response of a deep-drafted offshore structure based on the slender body approximation, that is, without explicitly considering the diffraction and radiation potential due to the presence of the structure. For typical deep water offshore structures, such as offshore spar platforms, the ratio of the structure dimension to wave length is small. Hence, it is assumed that the wave field is virtually undisturbed by the structure and that the Morison's equation is adequate to calculate the wave exciting forces. The wave loads on the structure are computed by integrating forces along the height of the structure at the instantaneous displaced position. Total force per unit length is given by

$$\{F(t)\}=\frac{1}{2}\rho_w C_d D(\dot{u}-\dot{X})\,|\,\dot{u}-\dot{X}\,|+\frac{\pi D^2}{4}\rho_w C_m \ddot{u} \qquad (2)$$

The equation of motion has been solved by an iterative procedure using unconditionally stable Newmark's beta method. The displacement at time t has been calculated and the stiffness matrix, which is response-dependent, is re-evaluated at this time instant. The force vector is recalculated at time t, considering the response of the structure. Once the response converges at a time instant, structural acceleration, and velocity at that time is calculated. The same procedure is used for the next time step ($t + \Delta t$) till the steady state solution is achieved.

6. RESULTS & DISCUSSIONS

Table 1 gives the particulars of the multicomponent catenary cable and Table 2 gives the particulars of the offshore spar platform and wave data. The nonlinear cable behaviour is evaluated to study the movement of the moored offshore spar platform. The fairlead point is allowed to move in the horizontal and vertical directions. Figures 2 and 3 show the resultant force (horizontal, vertical)-excursion (horizontal) relationship at the top of the mooring line for initial horizontal forces equal to 2000 kN and 2500 kN. Figure 4 shows the resultant vertical force-vertical excursion relationship at the top of the mooring line for initial horizontal forces equal to 2000 kN and 2500 kN, when four cables are placed perpendicular to each other in opposite directions.

From Fig. 2 it has been observed that as the initial horizontal force at the top of mooring line increases, the resultant horizontal force increases

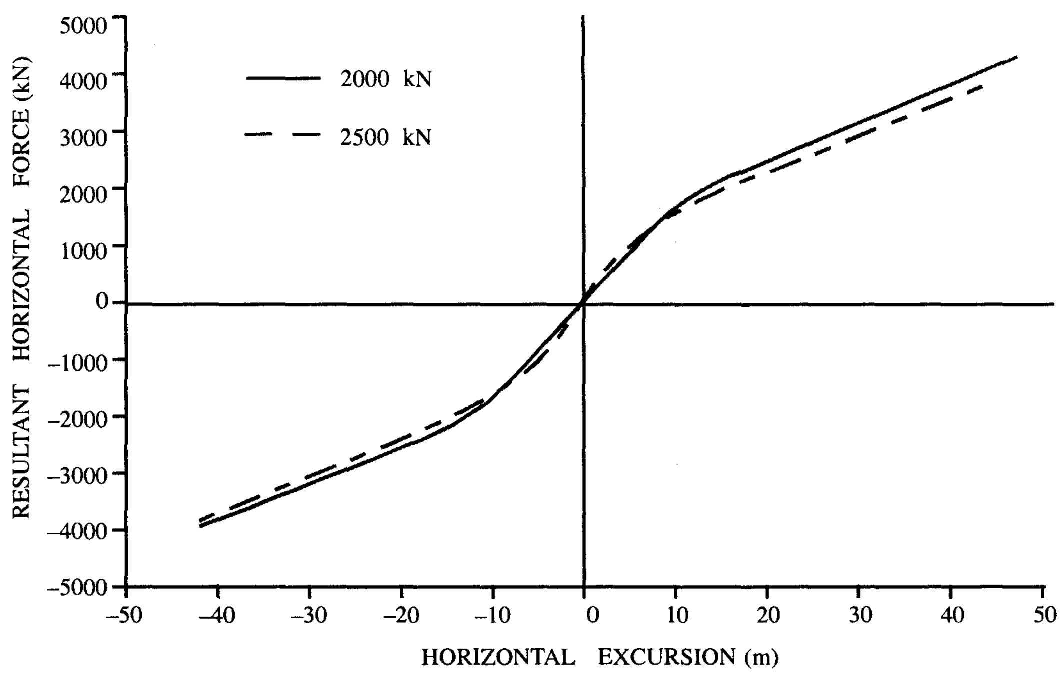

Figure 2. Resultant horizontal force–horizontal excursion for four cables for initial horizontal force

Table 1. Particulars of multicomponent catenary cable

Parameters	Values
Weight of clump (kN/m)	25
Effective area of mooring line and anchor line (m^2)	0.0032
Weight of mooring line and anchor line (N/m)	293.2000
Young's modulus of mooring line (E_c) and anchor line (E_a) (kN/m^2)	0.21E+9
Height of fairlead point (m)	808.8000
Initial horizontal forces (kN)	2500 and 2000
Length of clump (m)	40
Angle of inclination at the fairlead point (deg)	30
Length of anchor line (m)	800

till the clump weight lifts off. The resultant horizontal force decreases even though the initial horizontal force increases, at higher horizontal excursions. This is due to the net resultant horizontal force being more for lower initial horizontal force than for higher initial horizontal force.

From Fig. 3, it has been observed that as the initial horizontal force at the top of mooring line increases, the resultant vertical force for horizontal excursion increases. From Figs 2 and 3 for different values of initial horizontal forces equal to 2000 kN and 2500 kN, the behaviour changes from their point of lifting of the clump weight equal to 9.94 m and 2.04 m, respectively.

Table 2. Geometrical properties of offshore spar platform and wave data

Parameters	Values
Weight of the structure (kN)	2.6×10^6
Height of the structure (m)	216.40
Radius of the structure (m)	20.26
Distance of centre of gravity to buoyancy (m)	6.67
Distance of centre of gravity from keel (m)	92.40
Distance of centre of gravity to fairleads (m)	0.20
Structural damping ratio	0.05
Wave period (s)	12.50
Wave height (m)	7.00
Drag coefficients (C_d)	1.0 and 0.0
Inertia coefficients (C_m)	2.0 and 1.8
Water depth (m)	914.40
Draft of the structure (m)	198.12

From Fig. 4 it has been observed that as the initial horizontal force at the top of mooring line increases, the resultant vertical force-vertical excursion (+ve) relationship increases, and it is very high for

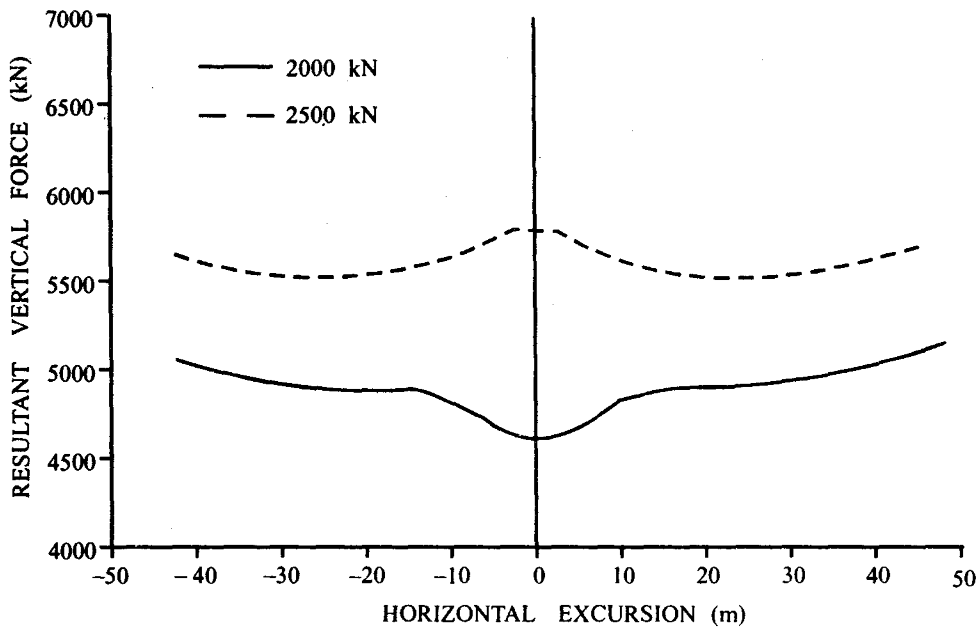

Figure 3. Resultant vertical force–horizontal excursion for four cables for initial horizontal force

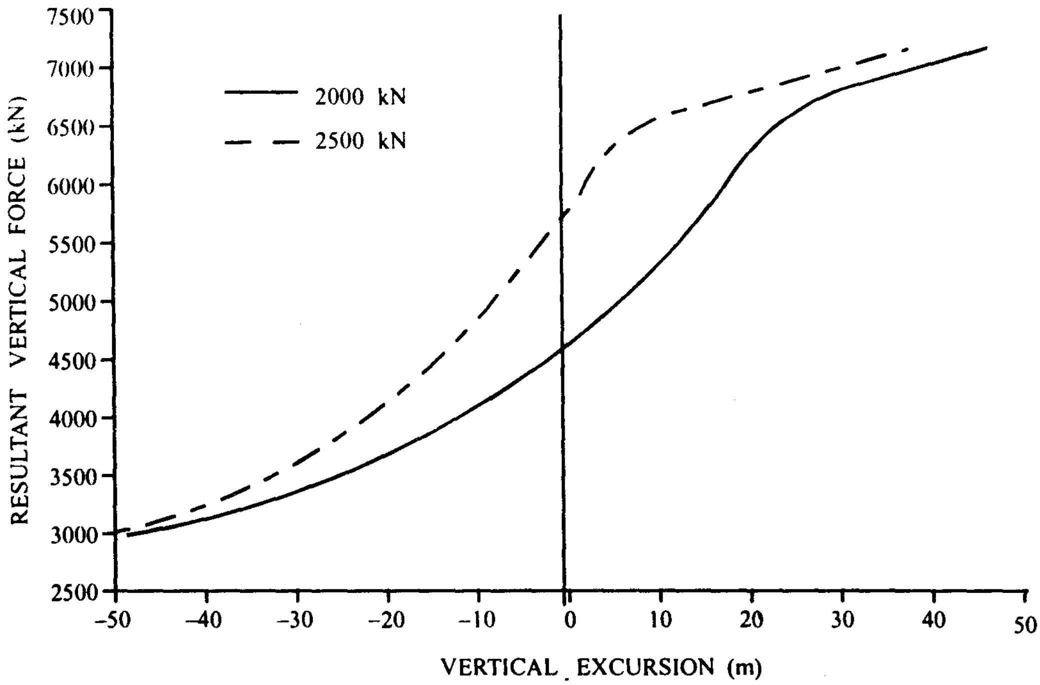

Figure 4. Resultant vertical force–vertical excursion for four cables for initial horizontal force

higher initial horizontal force, whereas resultant vertical force-vertical excursion (– ve) relationship increases up to –46 m. It is observed that when vertical excursion is higher than –46 m, it causes convergence of resultant vertical force to a value approx. equal to 3000 kN. The resultant vertical force increases for +ve vertical excursion because the cables become taut, whereas the resultant vertical force decreases for – ve vertical excursion because the cables become slack.

6.1 Effect of Inclusion of Vertical Excursion of Cables in Stiffness Matrix

Two cases have been taken for the initial horizontal force of 2500 kN: (i) for coupled stiffness matrix considering only horizontal excursion of cables and (ii) for coupled stiffness matrix considering both horizontal and vertical excursions of the cable. Table 3 shows the natural time period for both the cases at the time, $t = 0$ and at steady state response. In the calculation of natural frequency, only diagonal term of the stiffness matrix is effective, as the mass matrix is diagonal due to lumped mass assumption.

Table 4 shows the maximum value of steady state response for the above two cases. From Table 4, it is seen that there is a decrease of 6.32 per cent in surge, an increase of 50.51 per cent in heave, and an increase of 0.26 per cent in pitch responses when case (ii) is considered in comparison to case (i). Considering the effect of vertical excursion of the cable, the stiffness component in heave and pitch directions is reduced, so the displacement is more in comparison to when only the horizontal

Table 3. Natural time period (s) for offshore spar platform for cases (i) and (ii)

Time instant	Case No	Surge	Sway	Heave	Roll	Pitch	Yaw
Response at $t = 0$	Case (i)	215.43	215.43	28.04	50.84	50.84	102.97
Steady state response	Case (i)	392.23	215.43	39.79	50.84	50.84	102.97
Response at $t = 0$	Case (ii)	215.43	215.43	4.20	50.77	50.77	102.97
Steady state response	Case (ii)	386.80	209.25	43.51	50.77	50.77	102.97

Table 4. Response for inclusion of vertical excursion of cable in stiffness matrix

Stiffness matrix	Max displacement (m)		Max rotation (rad)
	Surge	Heave	Pitch
Case (i)	-15.769	-1.779	-0.0385
Case (ii)	-14.773	-2.678	-0.0386

excursion of the cable is considered, whereas, the stiffness component in surge direction is increased, which results in less displacement in comparison to when only horizontal excursion of the cable is considered. Inclusion of vertical excursion affects heave response significantly. In all further studies, stiffness matrix with both horizontal and vertical excursions of the cable have been considered.

6.2 Effect of Initial Horizontal Force

Two cases have been considered for the initial horizontal force: Case (iii) 2500 kN and case (iv) 2000 kN. Table 5 shows the maximum value of steady state response for the above two cases. From Table 5 , it is seen that there is a decrease of 14.5 per cent in surge, a decrease of 18 per cent in heave, and a decrease of 0.78 per cent in pitch responses when case (iv) is considered. An increase in initial horizontal force at the attachment point makes the system taut as it decreases the length of the mooring line. For having lesser mooring system stiffness in case (iv), the structure is more flexible and gives lower dynamic amplification of the response, although the static contribution of response being more for lower stiffness of the structure. Case (iii), on the other hand, gives higher response as the structure is stiff and produces comparatively more dynamic amplification of the static response which is lower than in the case (iv). This indicates that the better performance of offshore spar platforms can be achieved with lesser stiffness of mooring system, although not below a minimum value.

Table 5. Response for different initial horizontal forces

Initial horizontal force	Max displacement (m)		Max rotation (rad)
	Surge	Heave	Pitch
Case (iii)	-14.773	-2.678	-0.0386
Case (iv)	- 12.625	- 2.196	-0.0383

6.3 Effect of Coupling of Stiffness Matrix

Two cases have been considered: (i) for the coupled stiffness matrix and (ii) for the uncoupled stiffness matrix with the initial horizontal force of 2500 kN. Table 6 shows the maximum value of steady state response for the above two cases. From Table 6, it is seen that there is an increase of 6.6 per cent in surge response, a decrease of 83.01 per cent in heave response and a decrease of 1.55 per cent in pitch response, when uncoupled stiffness matrix is considered in comparison to the coupled stiffness matrix. The sway, roll, and yaw responses are zero for the uncoupled case as a unidirectional wave is taken, while for the coupled case, the responses are almost zero, that means in these DOFs there is no displacement and rotation. The result shows that coupling of DOFs has a significant effect on the response behaviour in surge, heave, and pitch directions. In all further studies, coupled stiffness matrix is considered. The stiffness matrix plays the most important role on the overall response analysis because it is response-dependent, and more so, it is based on the nonlinear cable force.

Table 6. Response for different stiffnesses

Stiffness matrix	Max displacement (m)		Max rotation (rad)
	Surge	Heave	Pitch
Coupled	-14.773	-2.678	-0.0386
Uncoupled	-15.749	-0.455	-0.0381

6.4 Effect of Inertia Coefficient

Two cases of inertia coefficient, $C_m = 2$ and $C_m = 1.8$ have been considered with the initial horizontal force of 2500 kN. Surge force, heave force, and pitch moment is reduced when C_m reduces from 2.0 to 1.8. Table 7 shows the maximum value of steady state response for the above two cases. From Table 7, it is seen that there is a decrease of 15.93 per cent in surge, an increase of 0.41 per cent in heave and a decrease of 10.36 per cent in pitch responses when C_m equal to 1.8, is considered instead of C_m equal to 2.0. An increase in the value of C_m increases inertia force, resulting in an increase in the total force and pitch moment. It

Table 7. Response for variation in inertia cofficient

Inertia coefficient	Max displacement (m)		Max rotation (rad)
	Surge	Heave	Pitch
$C_m = 2.0$	-14.773	-2.678	-0.0386
$C_m = 1.8$	-12.419	-2.689	-0.0346

has been observed that for a percentage decrease in the value of C_m, there is an almost equal percentage decrease in the maximum value of surge force, heave force, and pitch moment. The variation in C_m affects surge and pitch responses significantly.

6.5 Effect of Drag Coefficient

Two cases of drag coefficient $C_d = 1$ and $C_d = 0$ are taken with initial horizontal force of 2500 kN. Surge and heave forces are reduced as the total force decreases when drag coefficient is zero. Table 8 shows the maximum value of steady state response for the above two cases. From Table 8, it is seen that there is an increase of 4.82 per cent in surge, decrease of 0.07 per cent in heave, and an increase of 1.29 per cent in pitch responses when C_d equal to 0 is considered in comparison to C_d equal to 1. It is observed that surge response has –ve offset and also that when C_d increases, surge response decreases. Higher value of C_d causes a greater amount of hydrodynamic damping, thereby, for the similar force, the response in surge direction gets reduced for an increase in the C_d value. Although the offshore spar platform being a large diameter structure, exhibits inertia-dominated force regime, the influence of drag coefficient is appreciable in surge and pitch responses.

Table 8. Response for variation in drag coefficient

Drag coefficient	Max displacement (m)		Max rotation (rad)
	Surge	Heave	Pitch
$C_d = 1$	-14.773	-2.678	-0.0386
$C_d = 0$	-15.485	-2.676	-0.0391

7. CONCLUSIONS

Following conclusions have been drawn based on the numerical study conducted on the offshore spar platform:

(a) Considering the effect of vertical excursion of the cable, the heave natural period of the offshore spar platform is decreased, which makes offshore spar's natural periods in heave, at initial displacement of the structure, nearer to the frequently occurring wave periods.

(b) Considering the effect of vertical excursion of the cable, the stiffness component in heave and pitch directions is reduced, so the displacement is more in comparison to when only horizontal excursion of the cable is considered, whereas the stiffness component in surge direction is increased, which results in less displacement in comparison to when only horizontal excursion of the cable is considered.

(c) Inclusion of vertical excursion of the cable in the stiffness matrix plays an important role in the dynamic response of offshore spar platform.

(d) The horizontal force-excursion relationship of a single cable depends mainly on the initial horizontal force. Lesser the initial horizontal force in the cable, the lower the offshore spar platform response for particular cable modelling, due to the net force, given by the cable system, at lower initial horizontal force being more. Initial horizontal force affects the surge and heave responses significantly.

(e) The coupling of stiffness matrix of the offshore spar platform plays an important role in the dynamic behaviour of the offshore spar platform as the heave response is significantly affected.

(f) It is necessary to evaluate the appropriate value of C_m so that wave force can be suitably estimated, as it has a significant effect on the response of the offshore spar platform.

(g) The effect of C_d is important as drag force affects both the total wave force and the surge response, although the overall response is not much affected.

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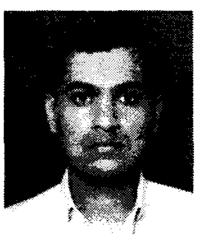
Contributors



Dr AK Jain is working as Professor in the Civil Engg Dept at the Indian Institute of Technology (IIT) Delhi. His areas of research include: Offshore structural dynamics and structural engineering.

Dr AK Agarwal has obtained his PhD in offshore structures. He is working as Consultant in structural design.